EP2200 Queuing Theory and Teletraffic Systems

Saturday, June 10$^{\text{th}}$, 2017
Available teacher: Viktoria Fodor, 08 790 4268
Allowed help: Calculator, Beta mathematical handbook or similar, provided sheets of queuing theory formulas, Laplace transforms and Erlang tables.

1.
Vanja Spinner has just completed her degree in business administration and is proud to have earned a promotion to Vice President for Customer Services at her hometown bank. One of her responsibilities is to manage how tellers provide services to customers, so she is taking a hard look at this area of the bank's operations. Customers needing teller service arrive in a Poisson fashion with an intensity of 30 per hour. Customers wait in a single line and are served by the one of m tellers when they reach the front of the line. Each service takes a variable amount of time (assume an exponential distribution), but on average can be completed in 3 minutes. Suppose that the line has an infinite capacity.
a) Give the Kendall notation of the above system and draw the system diagram. (2p)
Consider the case when two tellers are working in the bank.
b) What will be the average waiting time for a customer before reaching a teller? On average, how many customers will be in the bank, including those currently being served? (2p)
c) Calculate the probability that the bank is complectly empty, and give the part of the time of the 8 hours working day when a teller is idle. (2p)
d) Company policy is to have no more than a 10% chance that a customer will need to wait more than 5 minutes before reaching a teller. How many tellers need to be used in order to meet this standard? (2p)

Consider now that the bank has two types of customers: merchant customers and regular customers. The mean arrival rate for each type of customer is 15 per hour. Both types of customers currently wait in the same line and are served by the same tellers with the same average service time. However, Vanja is considering changing this. The new system she is considering would have two lines with infinite capacities — one for merchant customers and one for regular customers. Both of the lines would have its dedicated teller, and the service time would remain exponential, with the average of 3 minutes.
e) What would be the average waiting time for each type of customer before reaching a teller? On average, how many total customers would be in the bank, including those currently being served? How do these results compare to those from part b)? Motivate the difference. (2p)

2.
In the school library there are four computers that can be used to obtain information about the available literature. If all computers are occupied when someone wants information, then that person will not wait but leave immediately. It takes on average 2.5 minutes for a person to get the required information, with an exponentially distributed time. The number of potential users is large, and the users arrive in a Poisson fashion with an intensity of 72 per hour.
a) Give the Kendall notation of the above system and draw the system diagram. (2p)
b) Calculate the probability that two computers are occupied. Calculate the probability that an arriving user will leave the library without getting the required information, and the expected number of computers that are occupied at an arbitrary point of time. (3p)

Assume now that the potential users are only eight students. The students work on a project for an exponentially distributed amount of time, with a mean of one hour. After that, they go to the library to check the literature that is relevant for the project. Now, it takes on average 30 minutes for a student to check the available literature and to find the required information, with an exponentially distributed time. Consider that in the library only two computers are working, and there is also one sofa with three places. If on arrival both computers are occupied, the student looks for a place on the sofa, and if the sofa is occupied as well, she leaves the library and she tries again later. After the student leaves the library, she continues working on the project, and comes back to the library after one hour in average, and so on. Consider the system in steady state.
c) Give the Kendall notation of the above system and draw the system diagram. (2p)
d) Calculate the mean waiting time of the user. Calculate the ratio of time the library is completely full,

e.g. an user arriving has to leave the library. Calculate the probability that an user finds the library completely full. (3p)

3.

Consider the output buffer of a router that receives packets of constant size, arriving according to a Poisson process. The size of the packets is 250 Bytes, and there are in average 12000 packets arriving per second. The transmission speed of the link is 2Mbits/s (that is, $2 \cdot 10^6$ bits/s.)

a) Calculate the transmission time of a packet, the utilization of the transmission link and the average waiting and system times the packets experience. (3p)

b) Consider the case when a packet is completely transmitted, and the transmission buffer is empty. Give the distribution of the time until the next completed service. You can give the distribution in time or in Laplace transform domain. (2p)

Assume now that some of the packets has to be transmitted with high priority. Specifically, each arriving packet is a high priority packet with probability 1/3, the other packets have low priority. High and low priority packets are served from two dedicated buffers, according to non-preemptive priority.

c) Calculate the average waiting time of the high priority, as well as of the low priority packets. (3p)

d) Calculate the average waiting time, considering all packets in the system. Compare the result to the average waiting time without priority. How does the average waiting time change? Why? (2p)

4.

Consider a sensor node that needs to transmit measurement information to the coordinator node. The measurements are generater randomly, according to the changes in the environment, and we can assume that the measurement interarrival times are exponentially distributed. The time needed to transmit the measurements over the wireless channel has a mean value of 0.04 second, and a coefficient of variation of 0.25. Consider first that the sensor has no memory available to store measurements, and new data is dropped, if the transmission of the previous measurement is not yet completed.

a) Define a Markovian queuing model of the sensor, give the value of all the required parameters. Motivate your decisions. (2p)

b) Consider that 20 measurements are generated per second. What is the probability that new measurement data has to be dropped? What is the probability that no measurement data has to be dropped during one transmission interval? (4p)

c) What is the utilization of the wireless channel, the distribution of the sensor's idle time and the sensor's busy time? (2p)

d) Assume now that the sensor can store the measurements waiting for transmission. Give the queueing model, the average number of measurements waiting for transmission and their average waiting time. (2p)

5)

Consider a queuing network that works as follows. Request arrive to the first queue with infinite buffer, where they are served by a single server. After service, a job is redirected to the same server with probability 0.5, where it needs to wait for service again. Otherwise the job is directed to the next queue. This second queue has two servers, but no buffer. Blocked jobs leave the network immediately, while the served ones leave the network after service. Jobs arrive with an intensity of 1 per time unit, according to a Poisson process. The service times are exponential. The mean service time is 0.2 time unit at both of the queues.

a) Draw a diagram of the system, and give all the parameters. Calculate the load and the utilization of the servers at the first and at the second queue. (3p)

b) Calculate the mean time jobs spend in the queuing system. (3p)

c) Calculate the probability that the queuing system is completely empty. (2p)

d) Consider the same service time distributions. What is the maximum arrival intensity that keeps the system stable? (2p)