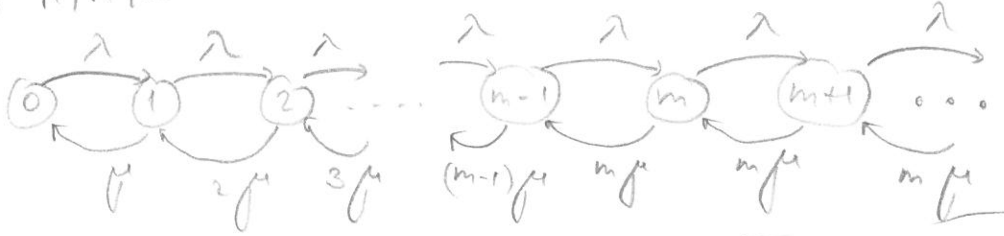
1. $\bar{x} = 3 min$

infinite capacity

$\lambda = 30 \frac{cust}{hour}$

$\mu = \frac{1}{3} \frac{cust}{min} \cdot 60 \frac{min}{hour} = 20 \frac{cust}{hour}$; $\rho = \frac{\lambda}{\mu} = \frac{3}{2}$

a.) $M/M/m$



b.) $m = 2 \Rightarrow M/M/2$

$\bar{W} = \frac{1}{m \cdot \mu - \lambda} \cdot P(wait)$

$P(wait) = \frac{m \cdot E_m(\rho)}{m - \rho \cdot (1 - E_m(\rho))}$

$E_m(\rho) = E_2(1.5) \approx 0.310345$

$P(wait) \approx 0.64$

$\bar{W} \approx 0.064$ hours $= 3.86$ min

$\bar{N} = \bar{N_S} + \bar{N_q} = \rho + \frac{\lambda}{m \cdot \mu - \lambda} \cdot P(wait) \approx 3.43$

c.) $P_0 = P$(bank is empty)

(1) $P_0 + P_1 = 1 - P(wait)$

(2) $\lambda \cdot P_0 = \mu \cdot P_1 \Rightarrow P_1 = \rho \cdot P_0$

(1) & (2) $\Rightarrow P_0 = \frac{1 - P(wait)}{1 + \rho} \approx \underline{\underline{0.14}}$

Free time $= (1 - \text{utilization}) \cdot T$

Utilization $= \frac{\lambda \cdot \bar{x}}{m} = 0.75$

$T = 8h$

Free time $= 0.25 \cdot 8h = \underline{\underline{2h}}$

d.) $P(W > 5 min) \leq 10\% = 0.1$

$P(W > 5 min) = 1 - P(W < 5 min)$ $\quad 1 - F_W(t) = P(wait) \cdot e^{-\mu \cdot (m - \rho) \cdot t}$

$m = 2$: $P(W > 5 min) = 27.9\% \Rightarrow m = 2$ is not good

$m = 3$: $P(W > 5 min) = 1.9\%$ ✓ The bank will need at least 3 tellers to meet this standard

e.) $\lambda_1 = \lambda_2 = \lambda = 15 \frac{cust}{hour}$; $\mu = 20 \frac{cust}{hour}$; $\rho = \frac{\lambda}{\mu} = \frac{3}{4}$

$2 \times M/M/1$ queues

$\bar{W_1} = \bar{W_2} = \frac{\rho}{\mu - \lambda} = 0.15$ $\quad \bar{N_1} = \bar{N_2} = \frac{\rho}{1 - \rho} = 3$ $\quad \bar{N}_{total} = \bar{N_1} + \bar{N_2} = 6$

The results are significantly worse than those from part b.)

The new system would not be good!

(2.) 4 servers

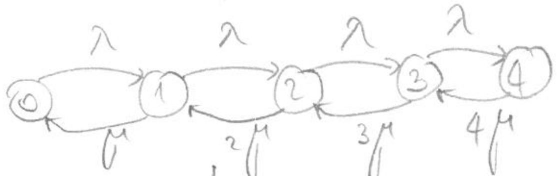
blocking system

$\bar{x} = 2.5$ min

$\lambda = 72 \frac{\text{users}}{\text{hour}}$

$\mu = \frac{2}{5} \frac{\text{users}}{\text{min}} \cdot 60 \frac{\text{min}}{\text{hour}} = 24 \frac{\text{users}}{\text{hour}}$; $\rho = \frac{\lambda}{\mu} = \frac{72}{24} = 3$

a.) $M/M/4/4$

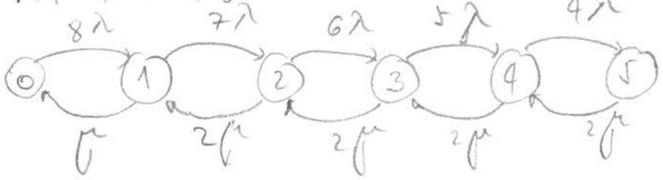


b.) $P_k = \frac{\rho^k / k!}{\sum_{k=0}^{m} \rho^k / k!}$

$P_2 = \frac{\rho^2 / 2!}{\sum_{k=0}^{4} \rho^k / k!} = \frac{36}{131}$ $\qquad P(\text{blocking}) = P_4 = \frac{\rho^4 / 4!}{\sum_{k=0}^{4} \rho^k / k!} = \frac{27}{131}$

$\bar{N}_{\text{comp-occupied}} = \rho \cdot (1 - P(\text{blocking})) \approx 2.38$

c.) $M/M/2/5/8$



$\left.\begin{array}{l} \lambda = 1 \frac{\text{user}}{\text{hour}} \\ \mu = 2 \frac{\text{users}}{\text{hour}} \end{array}\right\} \Rightarrow \rho = \frac{\lambda}{\mu} = \frac{1}{2}$

d.) $P_1 = 8\rho \cdot P_0 = 4 P_0 = \frac{32}{390}$

$P_2 = \frac{7}{2} \cdot 8\rho^2 \cdot P_0 = 7 P_0 = \frac{56}{390}$

$P_3 = 3 \cdot \frac{7}{2} \cdot 8\rho^3 \cdot P_0 = \frac{21}{2} P_0 = \frac{84}{390}$

$P_4 = \frac{5}{2} \cdot 3 \cdot \frac{7}{2} \cdot 8\rho^4 \cdot P_0 = \frac{105}{8} P_0 = \frac{105}{390}$

$P_5 = 2 \cdot \frac{5}{2} \cdot 3 \cdot \frac{7}{2} \cdot 8 \cdot \rho^5 \cdot P_0 = \frac{105}{8} \cdot P_0 = \frac{105}{390}$

Time blocking prob: $P_5 = \frac{105}{390} \approx 0.27$

Call blocking prob: $a_5 = \frac{\lambda_5 \cdot P_5}{\sum_{i=0}^{5} \lambda_i \cdot P_i} \approx 0.177$

$P_0 + P_1 + P_2 + P_3 + P_4 + P_5 = 1$

$\Rightarrow \boxed{P_0 = \frac{1}{1+4+7+\frac{21}{2}+\frac{105}{8}+\frac{105}{8}} = \frac{8}{390}}$

$\bar{W} = \frac{N_q}{\lambda_{eff}}$; $\lambda_{eff} = \sum_{i=0}^{4} \underbrace{(8-i) \cdot \lambda}_{\lambda_i} \cdot P_i = \frac{732}{390}$

$N_q = 0 \cdot P_0 + 0 \cdot P_1 + 0 \cdot P_2 + 1 \cdot P_3 + 2 \cdot P_4 + 3 \cdot P_5 = \frac{609}{390}$

$\Rightarrow \bar{W} \approx 0.83h$

(3) $L = 250B = 2000$ bits

$\lambda = 600\ /s$

$C = 2 \cdot 10^6\ b/s$

a) $\bar{x} = \frac{L}{C} = 10^{-3} s$ (deterministic) $\Rightarrow \bar{x^2} = \bar{x}^2,\ V[x]=0,\ C_x^2 = 0$

$\rho = \lambda x = 0.6$

$\boxed{M/D/1 \text{ queue}}$ (3)

$$W = \frac{\rho \cdot \bar{x}}{2(1-\rho)} = \frac{0.6 \cdot 10^{-3}}{0.8} = 0.75 \cdot 10^{-3} s$$

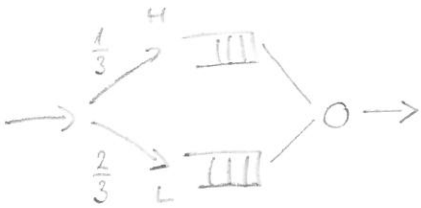
$$T = W + X = 1.75 \cdot 10^{-3} s$$

b) Time to next completed service in empty buffer case (T_c)

$$T_c = \underbrace{\{\text{time to next arrival}\}}_{Exp(\lambda)} + \underbrace{\{\text{time of service}\}}_{\text{Deterministic.}}$$ (2)

$$F_{T_c}(s) = \frac{\lambda}{s+\lambda} \cdot e^{-\bar{x}s}, \quad \lambda = 600,\ x = 10^{-3}$$

c)



$\lambda_H = 200/s \qquad \rho_H = 0.2$

$\lambda_L = 400/s \qquad \rho_L = 0.4$

$R = \frac{1}{2} \cdot (\lambda_H + \lambda_L) \cdot \bar{x^2} = 0.3 \cdot 10^{-3}$ (3)

$$W_H = \frac{R}{1-\rho_H} = \frac{0.3}{0.8} \cdot 10^{-3} s = \frac{3}{8} \cdot 10^{-3} s = 0.375 \cdot 10^{-3}$$

$$W_L = \frac{R}{(1-\rho_H)(1-(\rho_H+\rho_L))} = \frac{0.3}{0.8 \cdot 0.4} = \frac{3}{8 \cdot 0.4} \cdot 10^{-3}$$

d) $$W = \frac{1}{3} \cdot W_H + \frac{2}{3} \cdot W_L = \left[\frac{1}{8} + \frac{2}{8} \cdot \frac{10}{4}\right] 10^{-3} = \frac{24}{32} \cdot 10^{-3} = \underline{\frac{6}{8} \cdot 10^{-3} s}$$ (2)

The average waiting time has not changed.
The reason is, that all packet sizes (high and low priority) has the same distribution. From "outside" the service looks like FIFO.

(4)

$\bar{x} = 0.04$

$c_x^2 = \frac{1}{4}$

no buffer.

a) Arrival: Poisson

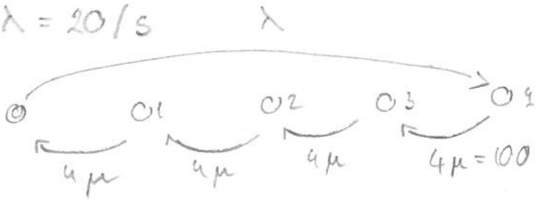
Service: since $c_x^2 = \frac{1}{4}$, we can assume Erlang-4

$E[x] = 0.04 \Rightarrow \mu = \frac{100}{4}$

Buffer: no buffer

$\Big\}$ $M/E_4/1/1$ (2)

b) $\lambda = 20/s$



$p_0 \cdot \lambda = p_4 \cdot 4\mu \Rightarrow p_4 = \frac{\lambda}{4\mu} p_0 = \frac{1}{5} p_0$

$p_1 \cdot 4\mu = p_2 \cdot 4\mu$

$\vdots$

$p_1 = p_2 = p_3 = p_4$

$p_0 \left(1 + 4 \cdot \frac{1}{5}\right) = 1$

$p_0 = \frac{5}{9}, \quad p_1 = p_2 = p_3 = p_4 = \frac{1}{9}$

$P(\text{drop}) = 1 - p_0 = \frac{4}{9}$

P(no measurement dropped under a service) = ?

i) Brute force: P(no drop) = P(no arrival) =

$\int_0^\infty P(\text{no arrival before } \tau)\, P(\text{service time is } \tau)\, d\tau \Rightarrow$ complicated integral.

ii) Due to the memoryless nature of the interarrival time, the above is the same as:

$P(\text{no arrival}) = P(\text{no arrival in any stage}) = \left[\int_0^\infty e^{-\lambda t} \cdot 4\mu e^{-4\mu t}\, dt\right]^4$

$= \left[\frac{4\mu}{4\mu + \lambda}\right]^4 = \left[\frac{5}{6}\right]^4 = 0.482$ (4)

c) utilization $= 1 - p_0 = \frac{4}{9}$ (2)

Sensor idle time ~ $Exp(\lambda)$, Sensor busy time ~ Erlang-4 (μ)

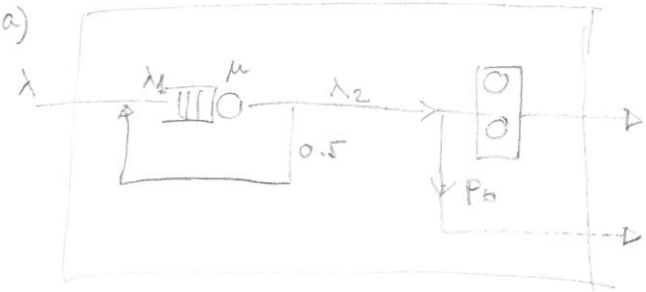
d) $M/G/1$ with $C_x^2 = \frac{1}{4}$, $\rho = \lambda x = 0.8$

$\bar{N}_q = \frac{1 + C^2}{2} \cdot \frac{\rho^2}{1-\rho} = 1.7$ (2)

$W = \frac{N_q}{\lambda} = 0.085\,s$

(5)

a)



$\lambda = 1$

$\bar{x} = 0.2 \Rightarrow \mu = 5$

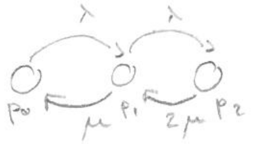
First queue: M/M/1

$\lambda_1 = \lambda + 0.5\cdot\lambda_1$ $\quad \rho_1 = \lambda_1 x = 0.4$

$\lambda_1 = 2\lambda = 2$ $\quad$ (Load = utilization)

Second queue: M/M/2/2

$\lambda_2 = 0.5\cdot\lambda_1 = \lambda = 1$



$p_0\cdot\lambda = p_1\cdot\mu$ $\quad p_1 = \frac{\lambda}{\mu}p_0$

$p_1\cdot\lambda = p_2\cdot 2\mu$ $\quad p_2 = \frac{\lambda^2}{2\mu^2}p_0$

$p_0 = \frac{50}{61},\ p_1 = \frac{10}{61},\ p_2 = \frac{1}{61}$

$P_b = p_2 = \frac{1}{61}$

Offered load = $\lambda\bar{x} = 0.2$

Effective load = $\lambda\bar{x}\cdot(1 - p_{block}) \approx 0.18$

Utilization = $\frac{\text{Effective load}}{2} = 0.09$ (3)

b) Time to pass the first queue:

$N = \frac{\rho}{1-\rho} = \frac{2}{3}, \quad T_1 = \frac{N}{\lambda} = \frac{2}{3} = 0.66$ $\quad \boxed{T_1 + T_2 = 0.84}$ (3)

Time to pass the second queue:

$T_2 = p_b\cdot 0 + (1-p_b)\cdot\bar{x} = 0.18$

c) P(queuing system is empty) = P(queue 1 is empty) P(queue two is empty) =

$= (1-\rho_1)\cdot p_0 = 0.6\cdot\frac{50}{61} \approx 0.5$ (2)

d) The second queue is always stable.

For the first queue: $\rho_1 < 1$ is needed.

$\Rightarrow \lambda_1\cdot 0.2 < 1$ (2)

$\lambda_1 < 5$

$\lambda < 2.5$