KTH Teknikvetenskap

Suggested solutions for exam

Version Preparatory course in mathematics

SF0003 Introductory Course in Mathematics
Friday, August 25, 2017

1. Simplify $\frac{\frac{7}{16}-\frac{5}{8}}{\frac{7}{10}-\frac{3}{5}}$ by writing over a common denominator. The answered should be reduced as much as possible.

   Suggested solution: We have

$$\begin{aligned}\frac{7/16-5/8}{7/10-3/5} &= \frac{16}{16}\cdot\frac{7/16-5/8}{7/10-3/5} = \frac{7-10}{16(7/10-3/5)}\\ &= \frac{10}{10}\cdot\frac{-3}{16(7/10-3/5)} = \frac{-30}{16(7-6)} = -\frac{15}{8}.\end{aligned}$$

2. Simplify $\dfrac{2}{x^2-4}-\dfrac{1}{x^2+2x}$ as much as possible.

   Suggested solution: We have $x^2-4=(x+2)(x-2)$ and $x^2+2x=x(x+2)$, so the smallest common denominator is $x(x+2)(x-2)$. This gives

$$\begin{aligned}\frac{2}{x^2-4}-\frac{1}{x^2+2x} &= \frac{2}{(x+2)(x-2)}-\frac{1}{x(x+2)}\\ &= \frac{2x}{x(x+2)(x-2)}-\frac{x-2}{x(x+2)(x-2)}\\ &= \frac{2x-(x-2)}{x(x+2)(x-2)}\\ &= \frac{x+2}{x(x+2)(x-2)}\\ &= \frac{1}{x(x-2)}.\end{aligned}$$

3. Determine a quadratic equation on the form $ax^2+bx+c=0$ which has the roots$3+\sqrt{7}$ and $3-\sqrt{7}$.

Suggested solution: One such equation is $(x - (3 + \sqrt{7}))(x - (3 - \sqrt{7})) = 0$. To get the equation on the wanted form we multiply and simplify to get

$$\begin{aligned} 0 &= (x - (3 + \sqrt{7}))(x - (3 - \sqrt{7})) \\ &= ((x - 3) - \sqrt{7})((x - 3) + \sqrt{7})) \\ &= (x - 3)^2 - (\sqrt{7})^2 \\ &= x^2 - 6x + 9 - 7 \\ &= x^2 - 6x + 2. \end{aligned}$$

4. Solve the equation $e^{x^2+2x} = 1$.

   Suggested solution: The equation $e^{x^2+2x} = 1$ is satisfied precisely when $x^2 + 2x = 0$. This equation has the solutions $x = 0$ and $x = -2$.

5. Determine the centre and the radius of the circle which is given by the equation $x^2+6x+y^2-4y = -4$.

   Suggested solution: We complete the square and find

$$\begin{aligned} -4 &= x^2 + 6x + y^2 - 4y \\ &= x^2 + 6x + 9 - 9 + y^2 - 4y + 4 - 4 \\ &= (x + 3)^2 - 9 + (y - 2)^2 - 4, \end{aligned}$$

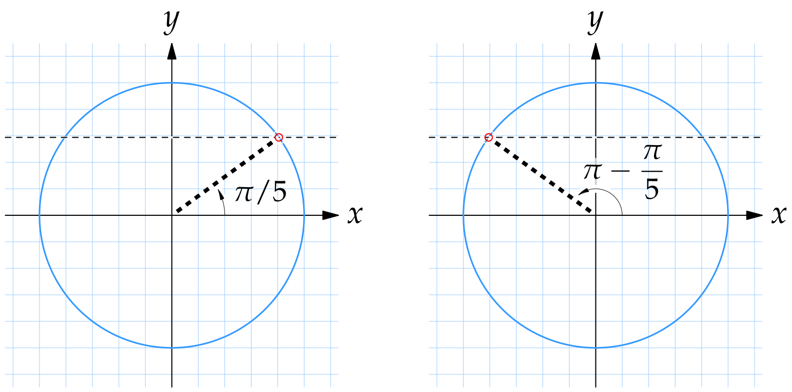
   or

$$9 = (x + 3)^2 + (y - 2)^2.$$

   This is the equation for a circle with centre $(x, y) = (-3, 2)$ and radius $r = 3$.

6. Solve the equation $\sin x = \sin \dfrac{\pi}{5}$.

   Suggested solution: We note immediately that $x = \pi/5$ is one solution to the equation, and considering the unit circle we conclude that $x = \pi - \pi/5 = 4\pi/5$ is the only further solution between 0 and 2π.



   We get all solutions to the equation by adding integer multiples of 2π,

$$x = \frac{\pi}{5} + 2n\pi \quad \text{and} \quad x = \frac{4\pi}{5} + 2n\pi,$$

   where n is an arbitrary integer.