Matrix Algebra
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Homework # 8

1. Show that among the vectors in the definition of $F(A)$, only vectors with real nonnegative first coordinate need to be considered.

2. Let
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$$
Determine $F(A)$, $F(B)$, $F(A+B)$, and show that $F(A+B)$ is a proper subset of $F(A) + F(B)$.

3. Show that $A \in M_2(R)$ is positive stable if and only if $\mathrm{tr}(A) > 0$ and $\det(A) > 0$.

4. Let $B \in M_n$ be a matrix none of whose eigenvalues μ_i is equal to 1, and define
$$A = (B + I)(B - I)^{-1}$$
Show that the eigenvalues of A are given by
$$\lambda_i = \frac{\mu_i + 1}{\mu_i - 1}$$
Prove that $\mathrm{Re}(\lambda_i) < 0$ if and only if $|\mu_i| < 1$. Conclude that A is negative stable if and only if B is a convergent matrix.

5. Use the transformation in the previous exercise to derive the discrete time version of the Lyapunov equation from the continuous time counterpart. (That is, derive $A^*GA - G = -H$ from $GA + A^*G = -H$, where the last negative sign is because we discuss negative stability here. Note that the matrices in these equations are generic and not the same.)