Matrix Algebra
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Homework # 9

1. Can the matrix transpose operation be represented by

$$X^T = AXB$$

   That is, are there fixed matrices $A, B \in M_{n,m}$ such that the above equation holds for all $X \in M_{m,n}$

2. Let $\lambda_1, \dots, \lambda_n$ be the eigenvalues of a given matrix $A \in M_n$. Show that the matrix equation $AX - XA = \lambda X$ has a nontrivial solution $X \neq 0 \in M_n$ if and only if $\lambda = \lambda_i - \lambda_j$ for some i, j.

3. The orthogonal projection matrix on the range space of a full rank matrix $A \in M_{m,n}$, $(m > n)$, can be written as $\Pi = AA^\dagger$. Here, $A^\dagger = (A^*A)^{-1}A^*$ is the Moore-Penrose pseudo inverse of A. Assume now that A depends on some parameter x (real scalar). Derive an expression for $d\Pi/dx$ in terms of dA/dx.

4. Let A, B, X be real matrices of appropriate dimensions. Derive

$$\frac{\partial \operatorname{tr}(X^T A X^{-1} B)}{\partial X}$$