

**SF2720**  
**Final exam**

Fall 2016

Please, **justify all your answers and claims** by either providing a proof or a counterexample. Claims without proof will not be awarded credit. Please write your answers clearly.

1. Consider  $F(x) = x + \frac{1}{4\pi} \sin 2\pi x$ .
  - a) Is  $F$  a lift of a circle homeomorphism?
  - b) If the answer in a) is yes, is the homeomorphism orientation preserving?
  - c) If the answer in b) is yes, what is the rotation number of the homeomorphism?
2. Let  $E_2 : x \mapsto 2x \pmod{1}$ , be the doubling map on the circle. Prove that  $E_2$  has a non-periodic orbit all of whose iterates lie in the left half of the unit interval.
3. Show that no increasing homeomorphism  $f : I \rightarrow I$ , where  $I$  is an interval in  $\mathbb{R}$ , is topologically transitive.
4. Determine whether or not there exists a homeomorphism  $f$  of the 2-torus such that  $f \circ T_A = T_B \circ f$ , where

a)

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$$

b)

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

c)

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$$

5. Let  $f$  be a smooth diffeomorphism of a 2-dimensional torus  $\mathbb{T}^2$ . Assume that the hyperbolic set is the whole torus.
  - a) Define the stable manifold through a point  $x \in \mathbb{T}^2$ .
  - b) Determine whether a stable manifold can contain two distinct periodic points.
  - c) Give an example of a smooth diffeomorphism of  $\mathbb{T}^2$  whose hyperbolic set is the whole torus, and describe the stable manifold through the point  $x = (0, 0)$ .
6. Recall that a homeomorphism  $f : X \rightarrow X$ , where  $X$  is a metric space, is called **minimal** if all the orbits of the dynamical system defined by  $f$  are dense in  $X$ .
  - a) If  $g : Y \rightarrow Y$  is a topological factor of  $f : X \rightarrow X$  and if  $f$  is minimal, is it true that  $g$  is necessarily minimal?
  - b) Give an example of  $f : X \rightarrow X$  such that:  $f$  has a minimal factor and  $f$  has positive topological entropy.