

# Home assignment sets 1 and 2: Traffic models and MAC

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## Traffic models

### Problem 1

a) *You would like to model the traffic flow a voice source with a two state Markov modulated fluid model. After extensive measurements, you see that high-bitrate and silent periods follow each other, and you have the following statistical data. In high-bitrate periods the source generates data with 64kbit/s. The average length of the high-bitrate periods is 10 sec, the average length of the silent periods is 30 sec.*

*Define the Markov modulated fluid model that describes the flow. Give the state transition intensities and the "fluid generation rate" in the two states. What is the probability that the source is silent at an arbitrary point of time?*

Let state 0 be the idle state and state 1 the high-bitrate state. Then, the state transition intensities are:  $q_{01} = 1/30 = 0.033$  transitions per second,  $q_{10} = 1/10 = 0.1$  transitions per second. The fluid generation rates are  $\lambda_0 = 0$  and  $\lambda_1 = 64$  kbit/s. The probability that the source is silent at an arbitrary point of time can be derived from the matrix equation  $\underline{Q} = \underline{p}\underline{Q}$ , from the balance equation  $p_0q_{01} = p_1q_{10}$  or from the ratio of average period lengths. The result is  $p_0 = 0.75, p_1 = 0.25$ . The probability that the source is silent at an arbitrary point of time is 0.75.

b) *Evaluate the effect of multiplexing such traffic flows. Increase the number of flows transmitted on a link, at the same time increasing the link transmission capacity. Assume, that there are no buffers, that is, data is lost, if the data generation rate is higher than the link transmission capacity. (This may sound stupid, but is rather realistic, if late packets needs to be dropped anyway).*

*Consider the specific case, where for  $n$  multiplexed flows the link capacity is set to  $n \cdot 32$  kbit/s. Prepare a graph that shows the probability that data is lost at an arbitrary point of time, as a function of the number of sources multiplexed.*

*(Note, from these results we actually do not know how much data is lost, but let us assume, we do not care about that.)*

As the source rate is 64kbit/s, there is packet loss at the multiplexer, if more than half of the flows are in active state. That is, for  $n$  sources:

$$P(\text{loss}) = \sum_{i=\lfloor n/2 \rfloor + 1}^n \binom{n}{i} p_1^i p_0^{n-i}.$$

## Problem 2

*In this assignment you need to prepare a statistical analysis of a packet arrival trace. The trace gives the packet arrival times in seconds to a 10 Mbps Ethernet link (it also gives the packet sizes in the second column, but we are not interested in that now.) Analyse the trace considering the properties we discussed in the first two lectures. You can select freely what properties you want to discuss and how you want to analyse them. See the link to the trace on the web.*

The three properties we discussed were heavy tailness, self-similarity and long range dependence. They are not simple to evaluate based on a small set of data, since, at least SS and LRD are asymptotic properties, and heavy tailness may need lots of data to contain the large samples.

Plotting e.g. Q-Q plot on the interarrival time seem to show light tail distribution (no strange deviance from the normal distribution).

SS is possible to show based on "visual inspection" of the histograms, or by plotting the autocorrelation for increasing lag. Note, that you can not use the asymptotic definition of the infinite sum of the autocorrelation values, since you have a finite sample (always finite result).

The demonstration long range dependence requires the estimation of the H parameter. According to Corentin, Matlab allows the estimation of the H parameter. He got a value of 0.75, which reflects long range dependence.

I attach some figures from earlier submitted home assignments.

## Problem 3

*When we discussed short range dependence, we have stated that for a two state, continuous time on-off Markov process with parameters  $\alpha$  and  $\beta$  the autocorrelation decays Exponentially, according to  $r(t) = e^{-(\alpha+\beta)t}$ . Present a derivation of this result. (Does not have to be your own derivation if you can find it in the literature.)*

The search continues.

# MAC

## Problem 1

*In a slotted ALOHA network delay sensitive traffic is transmitted as follows: once a node has a packet to transmit, it attempts transmission in the coming time slot. If collision happens, packets are lost and will not be retransmitted, since they would arrive late to the destination anyway. A large number of nodes are in the network, and altogether they generate packets according to a Poisson process with intensity  $\lambda$ . Packet transmission times (that are the same as the time slot lengths) are one time unit.*

*a) Assume, the sources can tolerate a packet loss probability of 0.1. What is the maximum allowed arrival intensity?*

$$P(\text{loss}) = P(\text{another arrival in the same timeslot}) = 1 - e^{-\lambda T}$$

For  $T = 1$  we have

$$1 - e^{-\lambda} \leq 0.1$$

$$\lambda \leq -\ln(0.9) = 0.105$$

b) To cope with the packet losses, so called forward error correction is introduced in the ALOHA system described above. It means that each piece of information is transmitted twice, in two separate packets with random delay. Due to the random delays the packet generation process can still be modeled as Poisson. Give an analytic model of the system, expressing the probability that the information is received. Can such a solution increase the probability that a piece of information is received by the destination?

This solution doubles the load in the system ( $2\lambda$ ), but now packets have two opportunities to get transmitted. They are lost, if transmission is unsuccessful in both cases:

$$P(\text{loss}) = P(\text{collision at transmission})^2 = (1 - e^{-2\lambda})^2$$

FEC is then efficient if:

$$(1 - e^{-2\lambda})^2 < 1 - e^{-\lambda}$$

$$\lambda < 0.48.$$

All in all, the FEC scheme is efficient for rather high load!

## Problem 2

Consider the slotted ALOHA model described from the bottom of page 50 to page 52 in the Rom-Sidi book. Assume that the time slots are 1ms, the packet arrival intensity is 0.2 packets per ms. The packet transmission time is one time slot. Calculate the average length of the idle periods, the average length of the busy periods, the average number of useful slots in a busy period, and finally the throughput. Double check the result using  $S = Ge^{-G}$ .

Following the presentation in the book, for  $T=1\text{ms}$ ,  $g=0.2$  packets/ms, we can express the average idle time as:

$$I = \frac{1}{1 - e^{-gt}} = 5.51,$$

the average busy time as:

$$B = \frac{1}{e^{-gt}} = 1.22,$$

the average number of useful (non-colliding) packets within a busy time as:

$$U = B \frac{gte^{-gt}}{1 - e^{-gt}} = 1.10.$$

For the throughput we get:

$$Th = B \frac{U}{B + I} = 0.16.$$

We get the same numerical result, if we use directly the final throughput equation of slotted ALOHA:

$$Th = gte^{-gt} = 0.16.$$

### Problem 3

*Describe the traffic models that have been used for the ALOHA and CSMA analysis in class as well as in the Rom-Sidi book. Compare it with the traffic model used for the CSMA/CA analysis in the Bianchi paper.*

*Considering the Bianchi paper, try to collect the assumptions that are made for the throughput analysis.*

ALOHA and CSMA in the book: large number of stations, Poisson arrival process of new and retransmitted packets, fixed packet size, constant propagation time. No hidden terminal, packets are lost even if the overlap is small.

CSMA/CA in the Bianchi paper: fixed (small) number of stations, backlogged traffic, transmission time defined by the back-off procedure, variable packet size, constant propagation time.

Assumptions for the analysis: the stations are independent from each other, the consecutive collision probabilities at a give station are independent from each other.