Rigid Body Dynamics (SG2150)
Exam, 2018-10-25, 08.00-13.00

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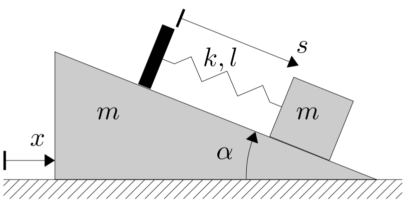
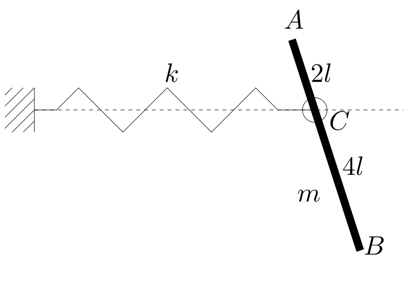
Each problem gives a maximum of 3 points, so that the total maximum is 18. Grading: 1–3 F; 4–5 FX; 6: E; 7–9 D; 10–12 C; 13–15 B; 16–18 A.

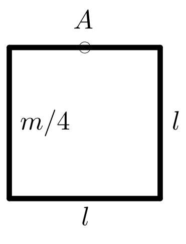
Allowed equipment: Handbook of mathematics and physics. One one-sided A4 page with your own compilation of formulae.



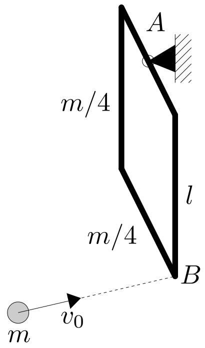
Problem 1. A triangular wedge of mass m is sliding smoothly on a horizontal plane. A block of mass m is sliding smoothly on the upper surface of the wedge, which is inclined an angle α from the horizontal. The block is attached to a linear spring with spring constant k and unstressed length l, which is parallel to the wedge surface.
Initially, the system is at rest with the spring having length $2l$. Compute the initial values of the coordinate second derivatives $\ddot{x}(0)$ and $\ddot{s}(0)$.



Problem 2. A thin homogeneous rod AB has mass m and length $6l$. The point C, which is located $2l$ from the end point A, can slide smoothly along a horizontal line and rotate smoothly in a vertical plane. The point C is also attached to a horizontal linear spring with spring constant $k = mg/l$.
Find a stable equilibrium solution for this planar system, and find the frequencies of small oscillations about this equilibrium.



Problem 3. A plane, square frame consists of four thin rods, each of mass $m/4$ and length l, and rigidly joined with each other. Compute all three principal moments of inertia about a point A that is the mid point of one of the rods.



Problem 4. The frame of Problem 3 is suspended with a smooth ball joint at the point A and is at rest. A particle of mass m and initial velocity v_0 perpendicular to the frame, hits the frame in the corner B. After the collision, the particle's velocity has slowed down to $(29/49)v_0$ but is still in the same direction.
Compute the angular velocity of the frame after the impact, and the (Newton) coefficient of restitution.

Problem 5. If the Lagrange function $L(q, \dot{q}, t)$ does not depend on some q_i (the coordinate q_i is *cyclic*), show that there is a quantity that is conserved along solutions to Lagrange's equations. If instead the Lagrange function $L(q, \dot{q})$ does not depend on the time t, show that the value of

$$\left(\sum_i \dot{q}_i \frac{\partial L}{\partial \dot{q}_i}\right) - L$$

is conserved along solutions to Lagrange's equations.

Problem 6. A system with two degrees of freedom q_1 and q_2 has the non-dimensionalised Lagrange function

$$L = \dot{q}_1^2 + \cos(\alpha)\dot{q}_1\dot{q}_2 + \frac{1}{2}\dot{q}_2^2 + \sin(\alpha)q_2 - \frac{1}{2}(q_2 - 1)^2$$

and initial conditions

$$q_1(0) = 0, \quad q_2(0) = 2, \quad \dot{q}_1(0) = 0, \quad \dot{q}_2(0) = 0.$$

The angle α is a constant.
Compute the maximal and minimal values of the coordinates q_1 and q_2 during the motion.