Project 2
SF2568 Program construction in C++ for Scientific Computing

October 26, 2018

In this project you will implement a general class for matrices. The final aim is to provide all necessary functionality for computing elementary functions of matrices. You will learn details about the construction of classes in C++.

The exponential function has the series representation

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

This series converges for all $x \in \mathbb{R}$. Therefore, an exponential for square matrices $A \in \mathbb{R}^{m\times m}$ can be defined by

$$\exp(A) = \sum_{n=0}^{\infty} \frac{A^n}{n!}.$$

Task 1 Implement the evaluation of the exponential for real numbers! The calling sequence should be

```
double myexp(double x, double tol = 1e-10);
```

The parameter `tol` indicates the required accuracy.

Validate your routine by checking it against the exponential function from the standard library!

Task 2 Now we are interested in computing the matrix exponential. The idea consist of using the series for that. Therefore, you must construct a matrix class according to the following skeleton:

```
class Matrix {
public:
  Matrix(int m);
  Matrix(const& Matrix);
  Matrix& operator=(const Matrix&);
  Matrix& operator+=(const Matrix&);
  Matrix& operator*=(const Matrix&);
  Matrix& operator*=(const double);
  double norm() const;
  void printMatrix() const;
  void fillMatrix(....);
  ...
};
```

Add members as needed and convenient for the problem. For the implementation of the arrays you may use either C-style arrays or corresponding classes from the C++ standard library.

In order to check your computations, a C++ routine for computing the matrix exponential, `r8mat_expm1.cpp`, is provided. The latter implements Matlab's algorithm for `expm`. Show using a number of freely choosen matrices how accurate your matrix exponential is!

Note: The provided routines consist of two source files and two header files. The calling sequence is:

```
double* result = r8mat_expm1(int m, double a[])
```

Here, `m` is the dimension of the matrix and `a` denotes a C-style array where the matrix is stored in Fortran-style (column wise). The returned value `result` is a pointer to a C-style array holding the matrix exponential in Fortran style. Note: The header file `r8lib.h` assume that the namespace `std` has been opened. So do the following in your code:

```
using namespace std;
#include ‘‘r8lib.h’’
```

Note: I recommend to have a look at the following paper: Cleve Moler, Charles Van Loan: *Nineteen dubious ways to compute the exponential of a matrix, twenty-five years later*. SIAM Review **45**(2003)1, 3–49

The programming exercises should be done individually, or in groups of two. Hand in a report containing:

- Comments and explanations that you think are necessary for understanding your program.
- Results as indicated in the tasks. Don't forget to draw conclusions!
- Printout of your source code.
- E-mail the source code to `hanke@nada.kth.se`.