Rigid Body Dynamics (SG2150)
Exam, 2018-12-20, 08.00-13.00

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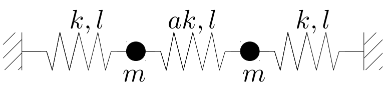
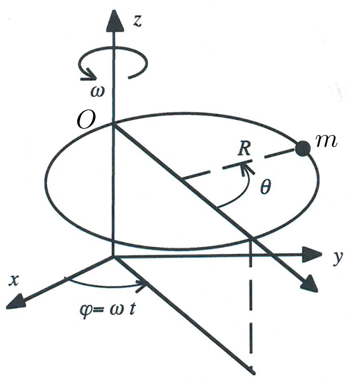
Each problem gives a maximum of 3 points, so that the total maximum is 18. Grading: 1–3 F; 4–5 FX; 6: E; 7–9 D; 10–12 C; 13–15 B; 16–18 A.

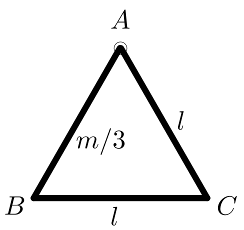
Allowed equipment: Handbook of mathematics and physics. One one-sided A4 page with your own compilation of formulae.



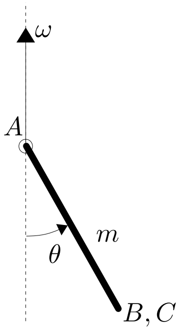
Problem 1. Two equal particles of mass m are constrained to move on a horizontal line. Three linear springs with spring constants k (outer two) and ak (middle) respectively, and natural lengths l are used to connect the masses with each other, and each mass to a fixed point, respectively. The fixed points are separated by a distance $3l$. Determine the two angular frequencies of small oscillations and the value of the number a, if one the the frequencies is twice as large as the other.



Problem 2. A circular, horizontal, ring of radius R is rotating with a given constant angular velocity ω about a verical axis through the point O on the ring. On the ring, a small particle of mass m can slide smoothly. The position of the particle relative to the ring is given by the angle θ. Derive the equation of motion for the particle (the differential equation for θ). Is it possible to have a motion with constant θ?



Problem 3. A plane, triangular frame consists of three thin rods, each of mass $m/3$ and length l, and rigidly joined with each other. Compute all three principal moments of inertia about the corner A.



Problem 4. The frame of Problem 3 is suspended with a smooth ball joint at the point A. Consider a motion where the frame is rotating with constant angular velocity ω upwards about the point A, the plane of the frame makes a constant angle θ with the rotation axis, and the lower part BC of the frame remains horizontal. Show that this motion is consistent with the equations of motion provided a certain relation between ω and θ is fulfilled, and compute the value of ω if the angle θ is $\pi/6$.

Problem 5. A system with two degrees of freedom θ and φ has the Lagrange function

$$L = \frac{1}{2}mr^2\left(\dot{\theta}^2 + \sin(\theta)^2\dot{\varphi}^2\right) - mgr\cos(\theta).$$

Explain why we can immediately conclude that the two expressions

$$mr^2\sin(\theta)^2\dot{\varphi}$$
$$\frac{1}{2}mr^2\left(\dot{\theta}^2 + \sin(\theta)^2\dot{\varphi}^2\right) + mgr\cos(\theta)$$

have constant values along solutions to Lagrange's equations.

Problem 6. Let $q_a(t)$ and $p_a(t)$ (for $a \in 1..n$) be *independent* functions of time. Suppose also that the first order variation of the integral

$$\int_{t_0}^{t_1}\left[\left(\sum_a \dot{q}_a p_a\right) - H(q,p,t)\right]dt$$

is zero when the values of t_0, t_1, $q_a(t_0)$, and $q_a(t_1)$ are kept fixed. Show that $q_a(t)$ and $p_a(t)$ then must satisfy Hamilton's equations:

$$\dot{q}_a = \frac{\partial H}{\partial p_a} \quad \dot{p}_a = -\frac{\partial H}{\partial q_a}.$$