Microdisk resonator

a. Calculate the number of modes with frequencies lying between 0 and \( \nu \) in a two-dimensional square resonator (of size \( L \times L \), with \( L \) much greater than the wavelength), allowing two orthogonal polarizations per mode. Determine then the density of modes [number of modes per unit area per unit frequency] at a frequency \( \nu \) in the 2D resonator. [Assume a refractive index \( n = 1 \) in the derivations].

b. Light can be confined in a two-dimensional circular resonator by repeated reflections from its circular boundary, as sketched in the figure here on the right. Consider the case of light confined in a disk of diameter \( D \) and refractive index \( n \), as a result of \( N \) reflections with equal path lengths. Derive the expression for the frequency spacing of the optical modes supported by the resonator as function of \( D \) and \( N \) and calculate the lowest possible value of \( N \) for full-confinement of a mode in a disk made of silicon, with \( D = 10 \, \mu\text{m} \) and \( n = 3.5 \). Sketch the corresponding optical trajectory in the disk and comment on the result. [For the sake of simplicity, consider only: non-crossing ray trajectories - as the one illustrated by Fig.1 for the case \( N = 6 \), one polarization and one direction of propagation out of the two possible for the same ray trajectory].

c. Derive the expression of the intermodal frequency spacing in the circular resonator of task b) for \( N \to \infty \). Calculate its value for the same values of \( D \) and \( n \) as above. How do the optical trajectories look like in this case?