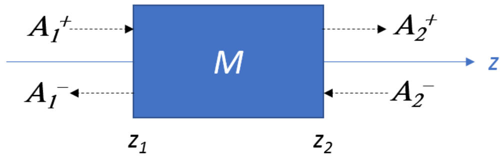
Task EO2 / Home Assignment

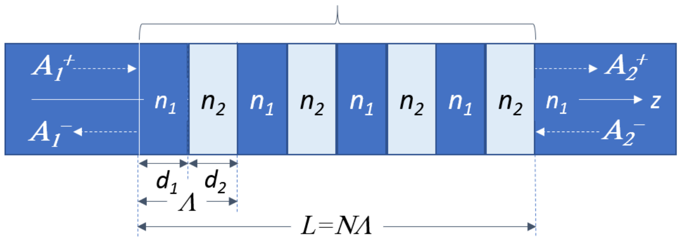
1. Consider the case of two monochromatic plane waves, A^+ and A^- (at a given wavelength λ), propagating along the same axis (z), in the positive and negative z-direction, respectively. A generic optical element acting on those waves can be described as a four-port device:



and characterized by a 2x2 transfer matrix M, which links the complex amplitudes of the two waves in z_1 and z_2 according to the relationship:

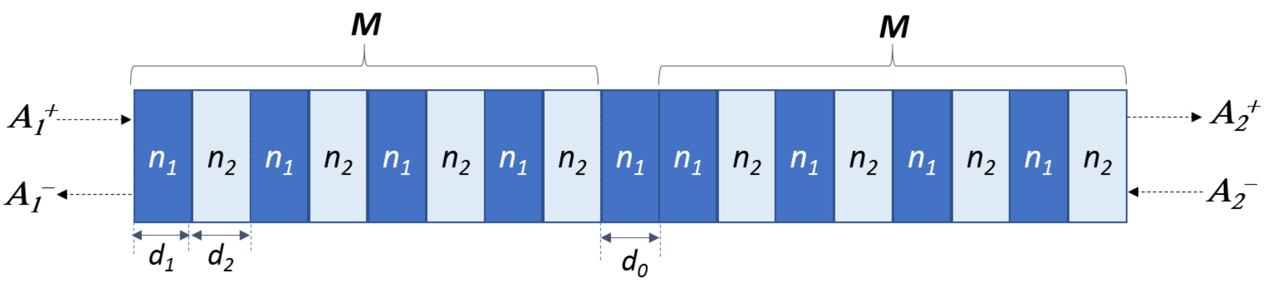
$$\begin{bmatrix} A^+ \\ A^- \end{bmatrix}_{z=z_2} = M \begin{bmatrix} A^+ \\ A^- \end{bmatrix}_{z=z_1}$$

Given this definition, determine the transfer matrix M corresponding to :
   - a single dielectric boundary, orthogonal to the z axis, separating two lossless materials of refractive indices n_1 and n_2;
   - a uniform section of length d in a material of refractive index n.
2. Consider now a periodic structure, of total length L and period Λ, consisting of N sections of alternating materials of refractive indices n_1 and n_2 with lengths d_1 and d_2, respectively ($\Lambda = d_1 + d_2$and $L = N\Lambda$), as shown in the figure here below:



Provide an expression for the transfer matrix M of the whole grating.
3. Using a perturbative development, derive the coupled-mode equations that describe the evolution along z of the forward and backward wave amplitudes (A^+ and A^-) in the grating considered at point 2, for the case where: N = 20, $\lambda = 1.53\ \mu m$, Λ = 450 nm, d_1= d_2=$\frac{\Lambda}{2}$, n_1= 1.8 and n_2=1.6 (neglecting dispersion). Define all parameters and determine the values of the relevant coupling coefficients and phase-mismatches. Finally determine the value of the grating reflectivity when only the A^+ wave is present as an input (i.e. $A_1^+ \neq 0,\ \ A_2^- = 0$).

BONUS question - Calculate and plot the transmission of the structure sketched below, where an additional section of length d_0 =d_1= d_2=$\frac{\Lambda}{2}$ and refractive index n_1 has been added between two uniform gratings of the type of point 3 and comment on the results.



Points 1