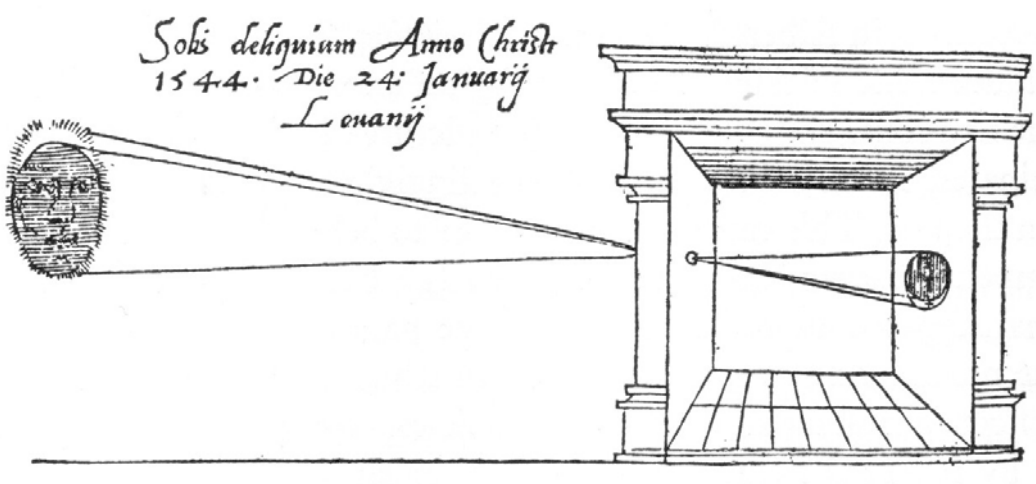
The Pinhole Camera Revisited

or

The Revenge of the Simple-Minded Engineer



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Based on a seminar given at KTH during the fall of 2004.

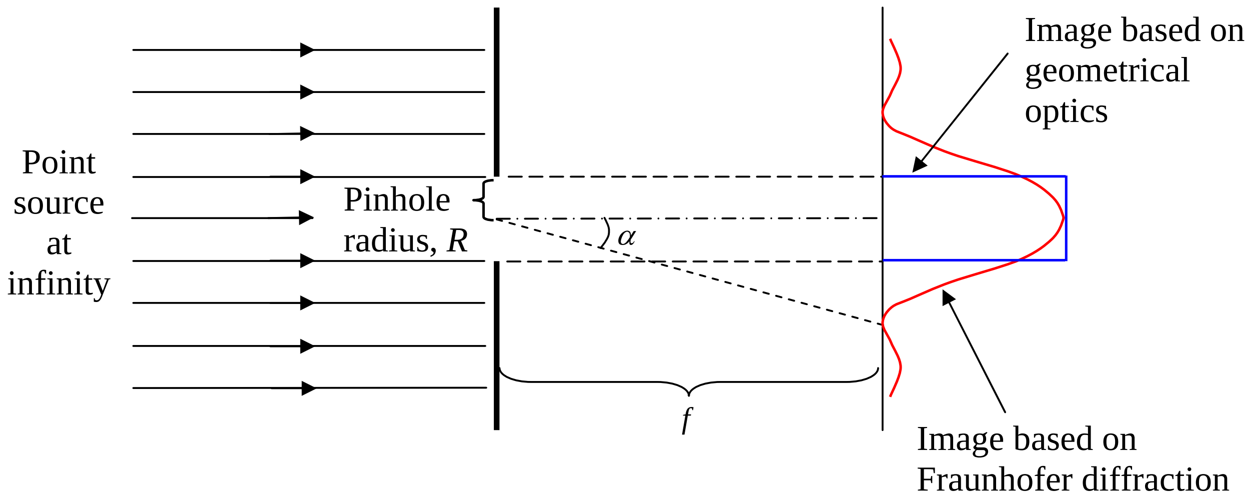
As a young amateur photographer, I was fascinated by pinhole cameras. Here are two images obtained with a home-built camera in the mid 1970s. The negative size was 55 mm x 80 mm, the "focal length" 40 mm, and the size of the pinhole $\frac{1}{4}$ mm. The images cover approximately 30 mm x 40 mm of the central part of the negative. No computer processing has been made to improve the sharpness.

Pinhole cameras existed (at least) 1000 years ago.

They are still used today, because they work for all wavelengths!
(useful for X-rays and gamma-rays)

Ancient question: <u>What size should the pinhole be?</u>

<u>Simplistic view</u>



The image of a point object, the Point Spread Function (PSF), should be as small as possible to produce a sharp image.

Geometrical optics approximation:
Hole should be as small as possible

Fraunhofer diffraction approximation:
Hole should be as large as possible

No optimum size is found!

Conclusion:

The optimum size is found in a region where neither geometrical optics nor Fraunhofer diffraction is valid!

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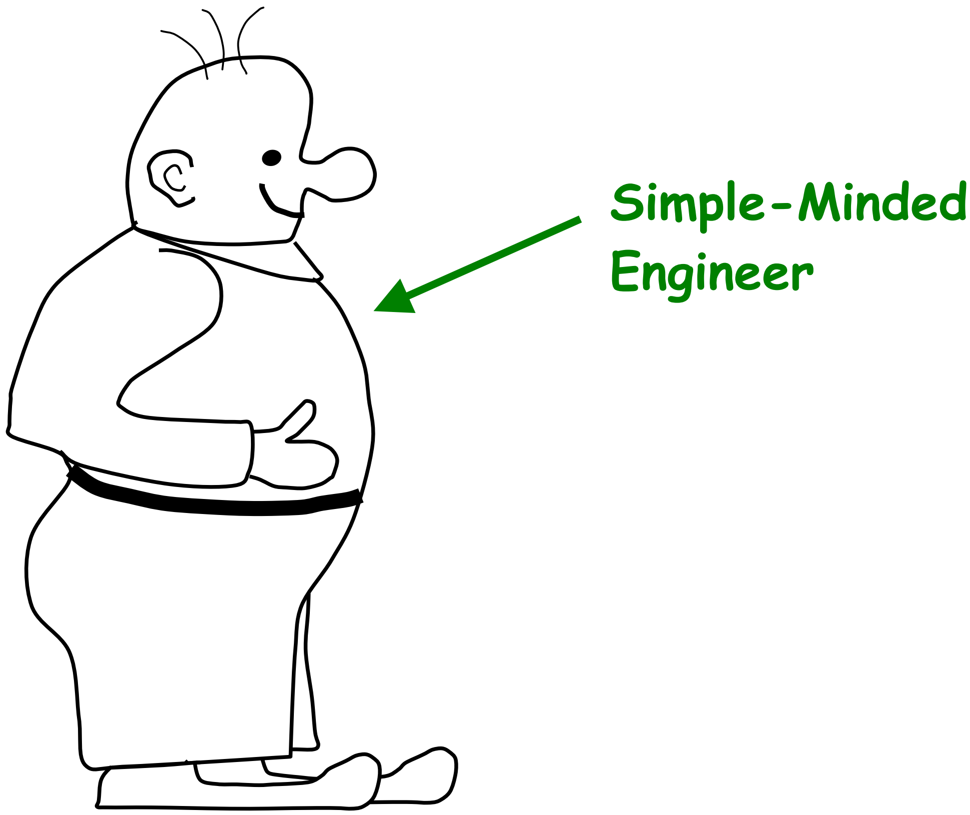

Solution (he says): Optimum hole size must be somewhere near where geometrical optics and Fraunhofer diffraction give the same size.

Result: $R = 0.78\sqrt{\lambda f}$

But we know better, don't we?

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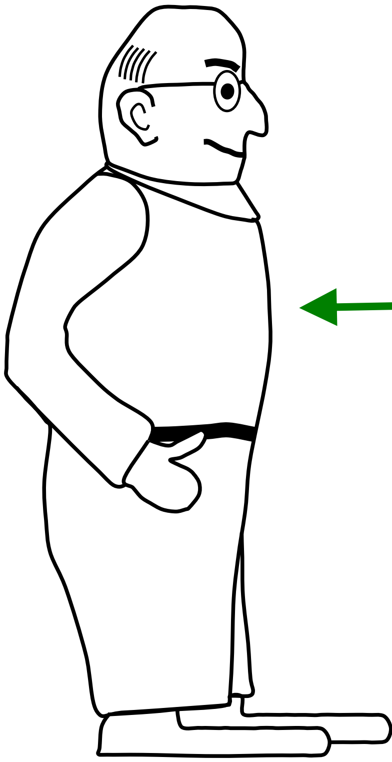


Able-Minded Engineer

Solution (he says): Hole should be the first zone of a zone-plate!! (Classic answer from Able-Minded Engineers!)

$$R = \sqrt{\lambda f}$$

In what respect is this optimal?
What do the PSF and MTF look like?

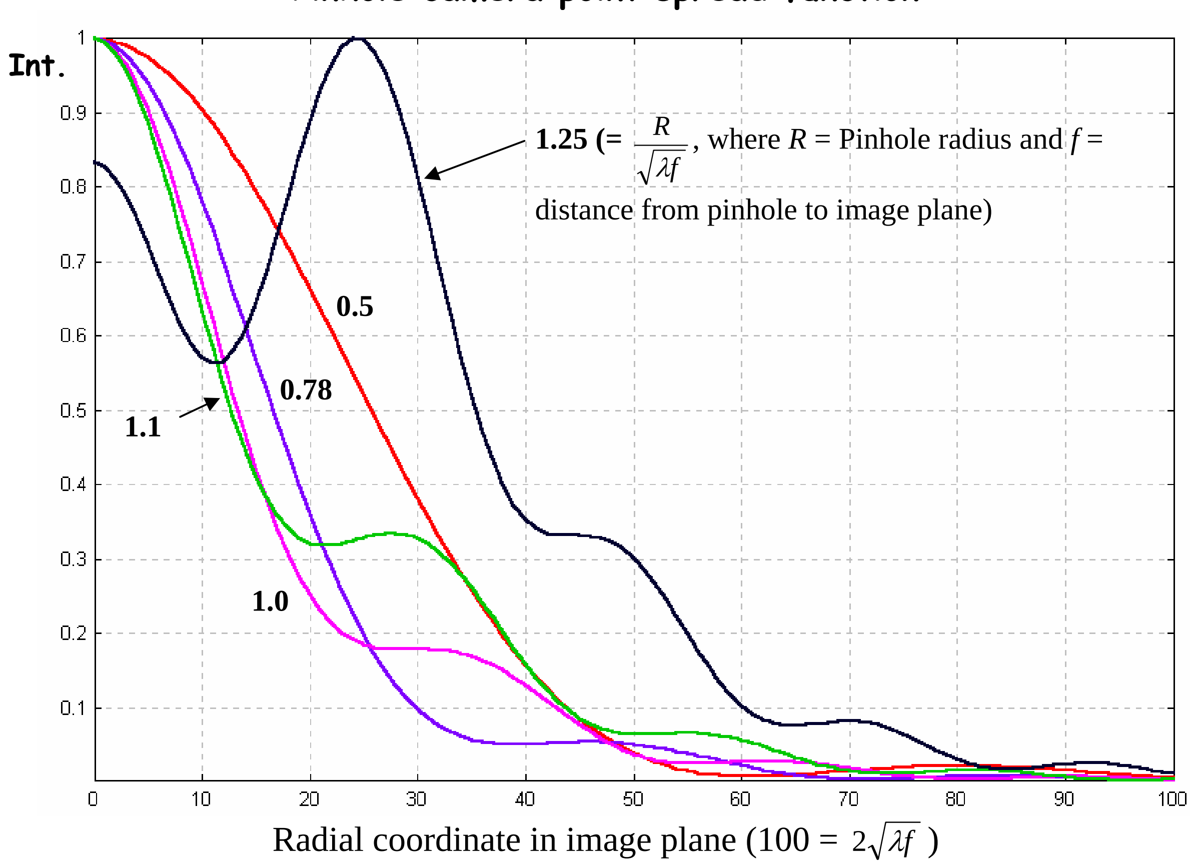
To investigate, we have to use

Fresnel-Kirchoff diffraction formula:

$$U_P = -\frac{ikU_0 e^{-i\omega t}}{4\pi} \iint \frac{e^{ik(r+r')}}{rr'} \left[\cos(\vec{n},\vec{r}) - \cos(\vec{n},\vec{r}')\right] dA$$

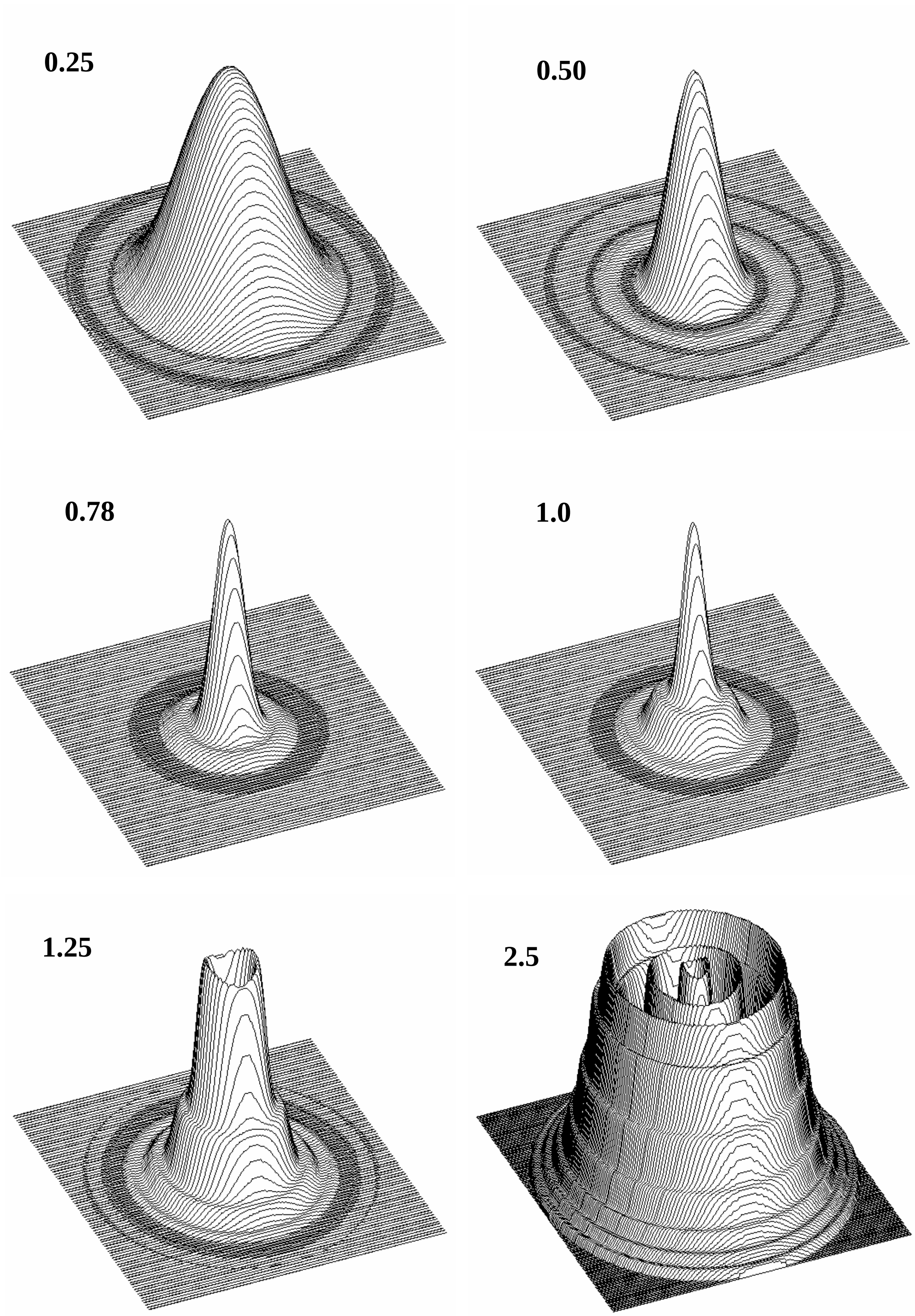
Brute force number-crunching with a computer will give us the PSF.

Pinhole camera point spread function

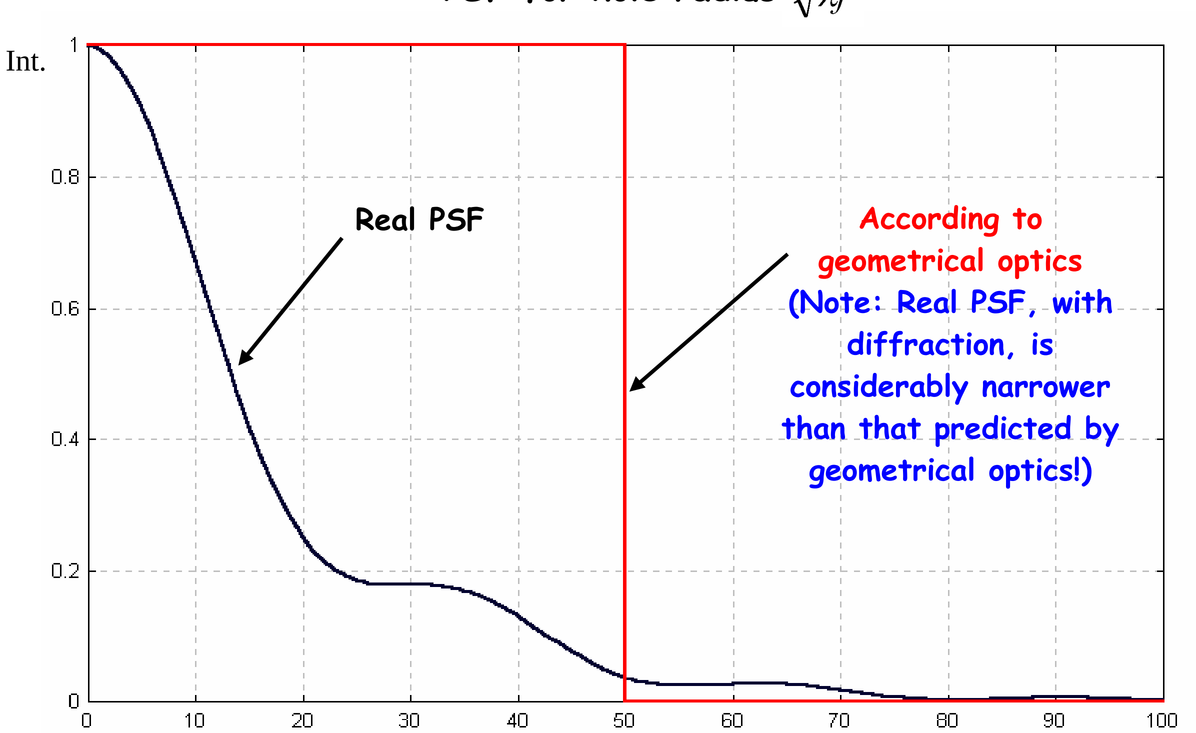


On next page we can see some 3D plots of PSFs for different pinhole radii.

PSFs for different pinhole radii (in units of $\sqrt{\lambda f}$)

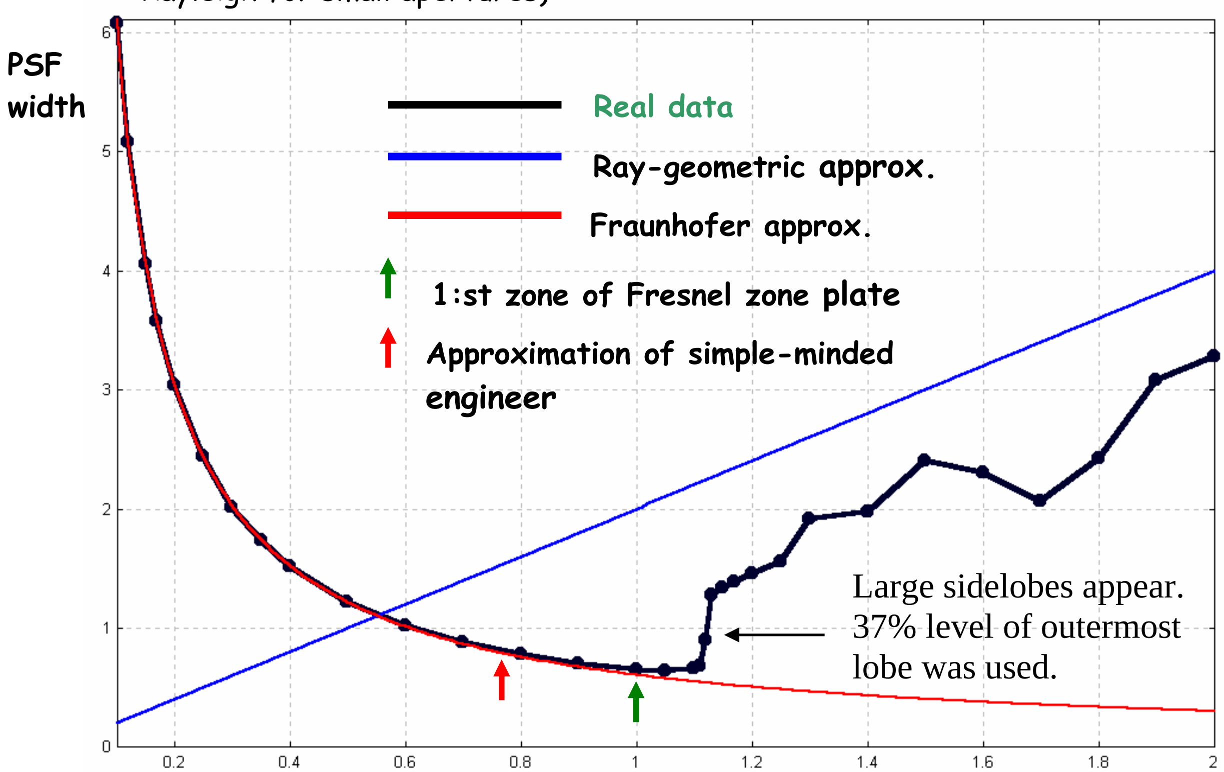


PSF for hole radius $\sqrt{\lambda f}$



Radial coordinate in image plane (100 = $2\sqrt{\lambda f}$)

Full width of PSF at 37% intensity level (corresponds to resolution acc. to Rayleigh for small apertures)



Pinhole radius (1 corresponds to $\sqrt{\lambda f}$)

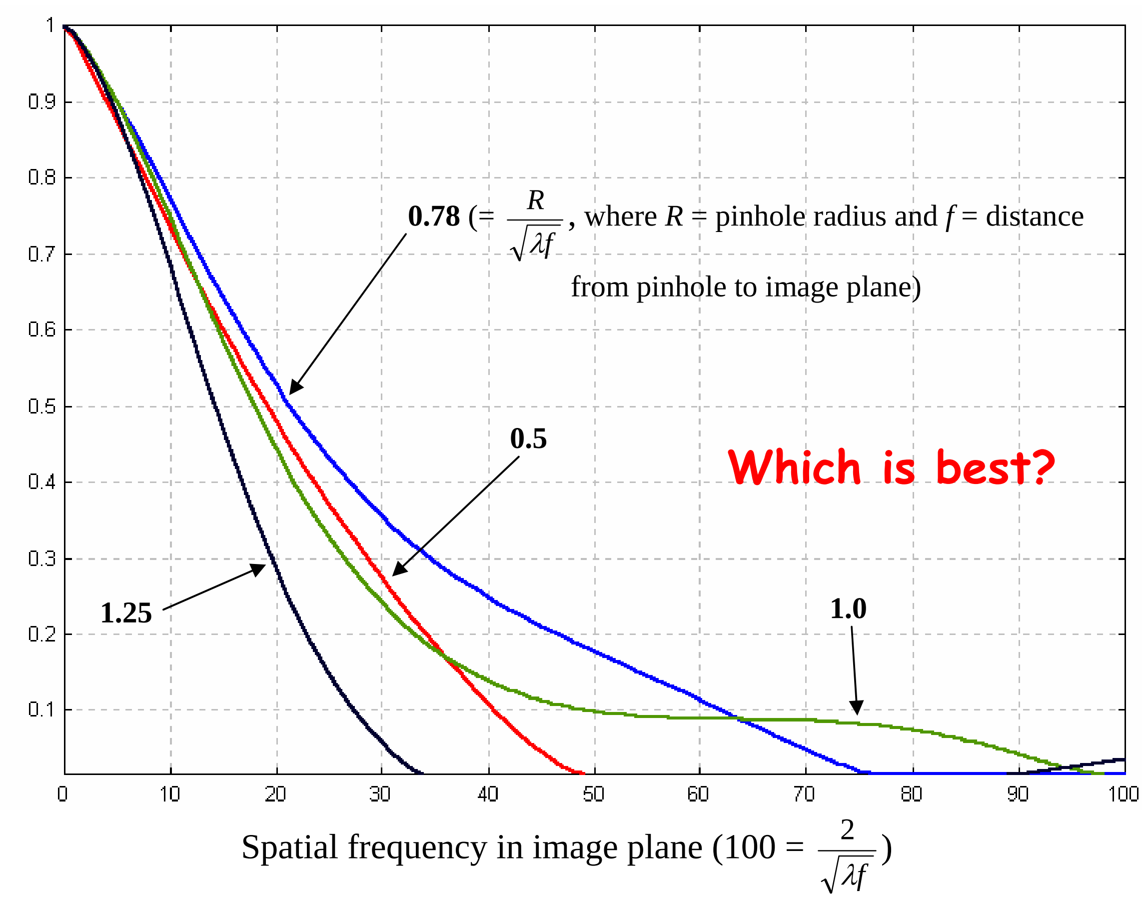
These data agree well with published experimental results:
M. Young, Applied Optics, 10, 2763 (1971)

End of discussion?

Not, quite. Let's look at MTF

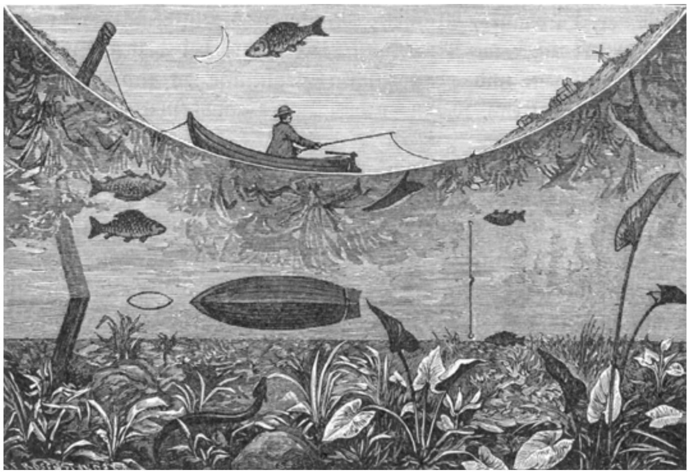
We obtain MTF by taking the Fourier transform of the PSF (Computers do this in a jiffy!)

Pinhole camera MTF



Let's look at some simulations using the above MTF curves!

Simulated imaging with pinhole cameras having R = 1 and 0.78 (in units of $\sqrt{\lambda f}$)



Original
(Fish perspective)

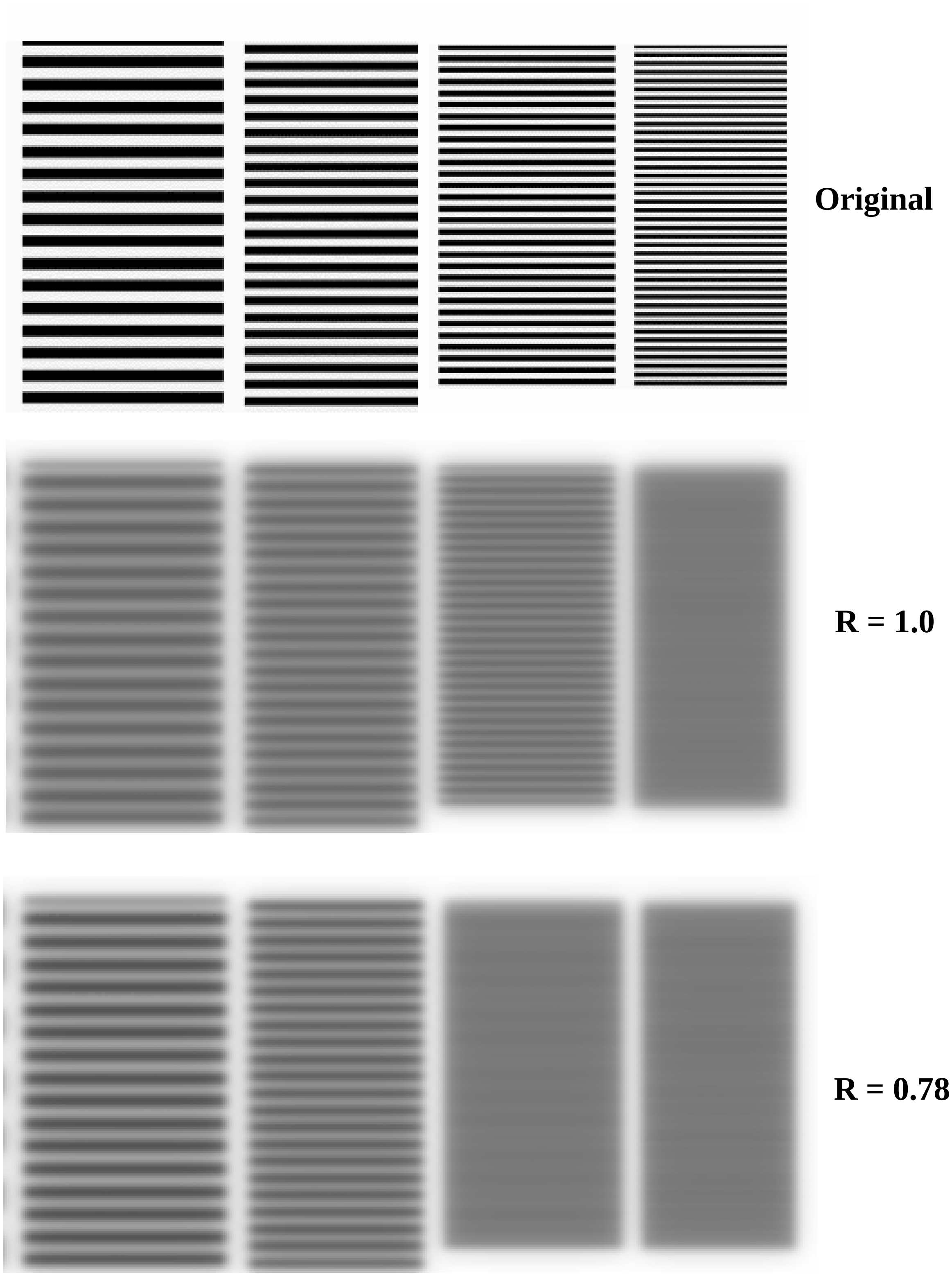
R = 1.0

R = 0.78

The difference between the images with R = 0.78 and 1.0 is rather subtle, but most people will agree that the former looks sharper than the latter. (Try different viewing distances if you find it difficult to see a difference.)

And some more simulations – this time of line patterns

(Note: Compared with the images on previous page, these images are magnified to show details more clearly)

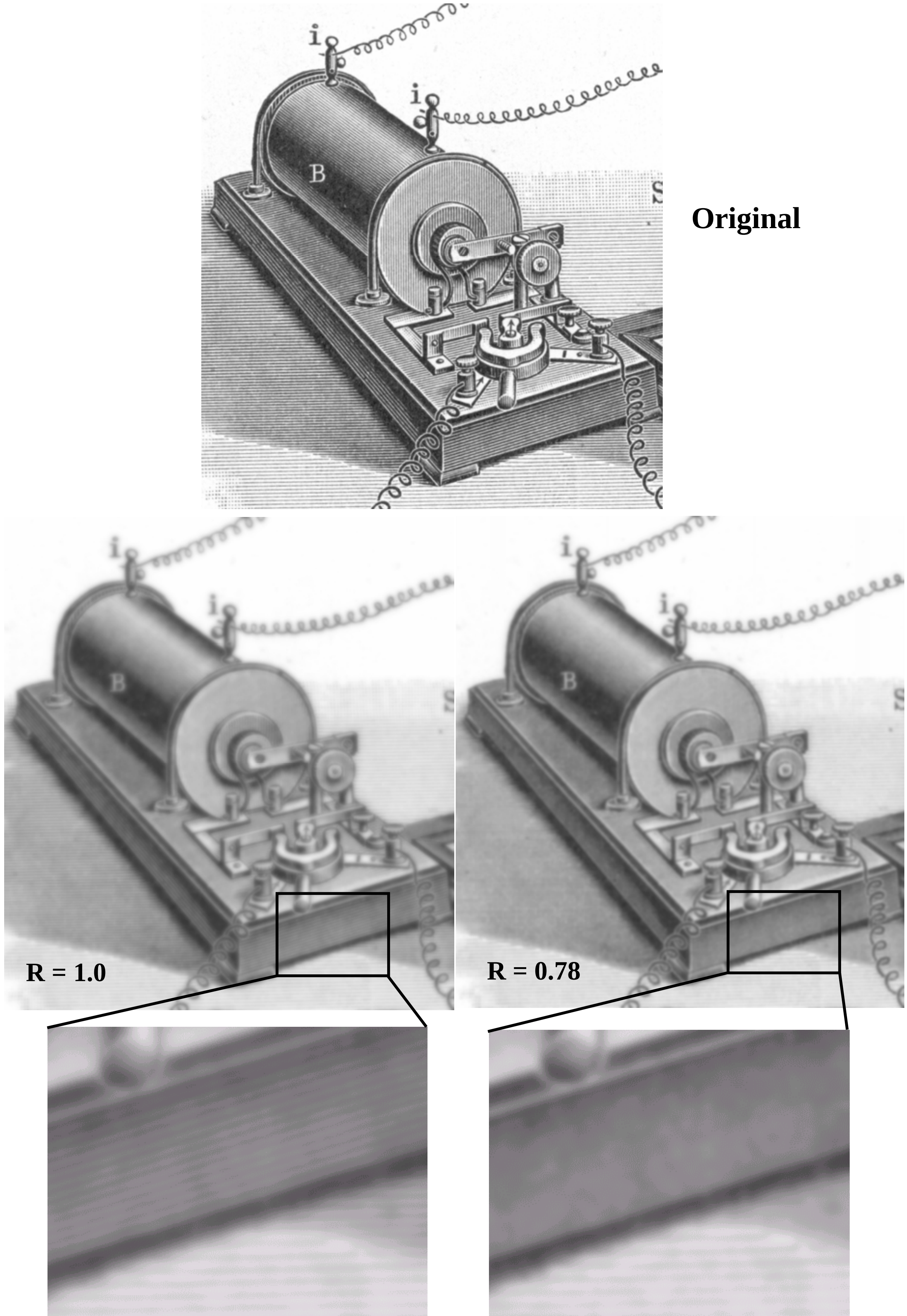


The images of line patterns are quite interesting and revealing. There is no doubt at all that the resolution is higher for R = 1.0 than for 0.78. The third pattern from the left is clearly visible for R = 1.0, but invisible for R = 0.78. Why then, did the "fish perspective" image with R = 0.78 look sharper than the one with R = 1.0?

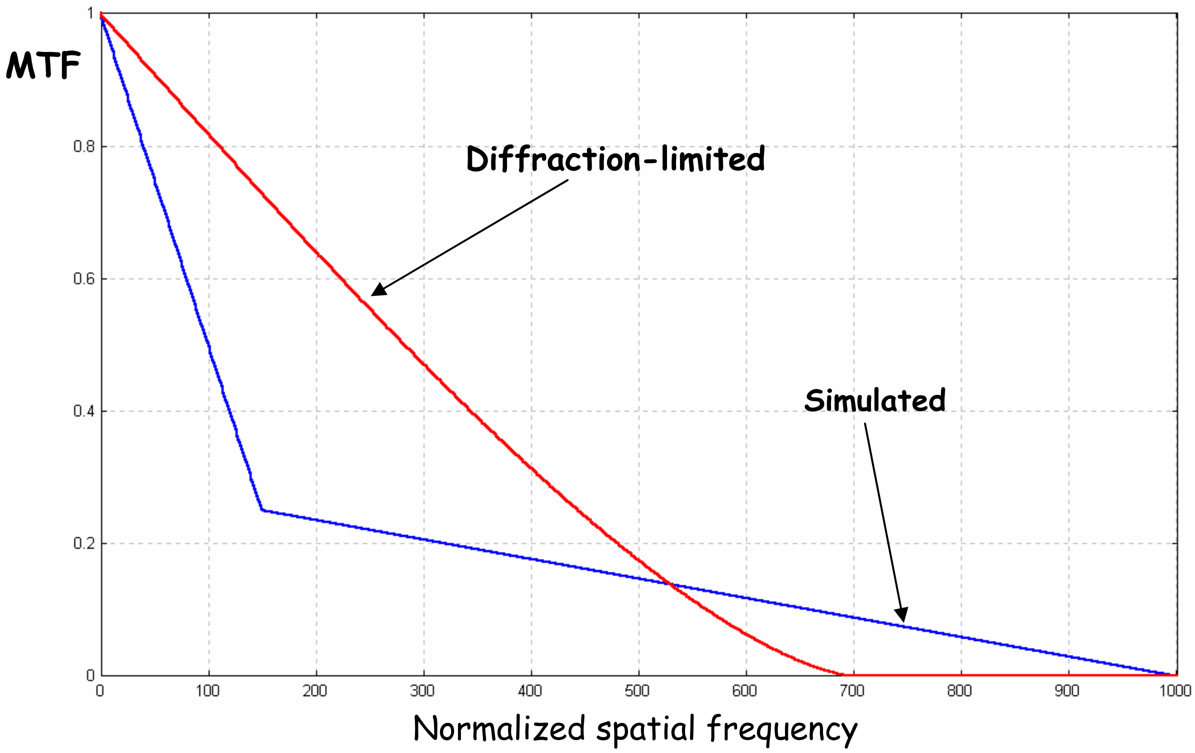
The reason is found by looking again at the R = 0.78 line pattern images on the previous page. Even though the third pattern from the left is missing, the first two patterns have a higher contrast compared with the R = 1.0 image. This result agrees quite well with the MTF data previously shown. We can see that the MTF curve for R = 1.0 extends to higher spatial frequencies (denser line patterns) than the 0.78 curve. On the other hand, the 0.78 curve has clearly higher values (meaning higher image contrast) for the low and medium spatial frequencies.

The conclusion must therefore be that resolution numbers don't tell you everything. In fact they can be misleading as we have just seen. In order to produce a sharp-looking image, the imaging system must have high MTF values at low and medium-high frequencies. MTF curves that drop rapidly as the spatial frequency increases will produce fuzzy-looking images, even if they extend out to high spatial frequencies. This is a well-known fact that has been experimentally verified several times. See, for example K. Biedermann, Photographische Korrespondenz, 103, p. 5-14, 26-31, 42-48 (1967), or Stroebel et al., Basic Photographic Materials and Processes, Focal Press 1990, p. 171

On next page are some additional simulations of pinhole camera performance for R = 0.78 and 1.0. Again, most people will agree that the R = 0.78 image is the sharper one. Below the two images are magnifications showing beyond doubt that the R = 1.0 image actually has the higher resolution. (The slanting line pattern is visible only in the R = 1.0 image.)

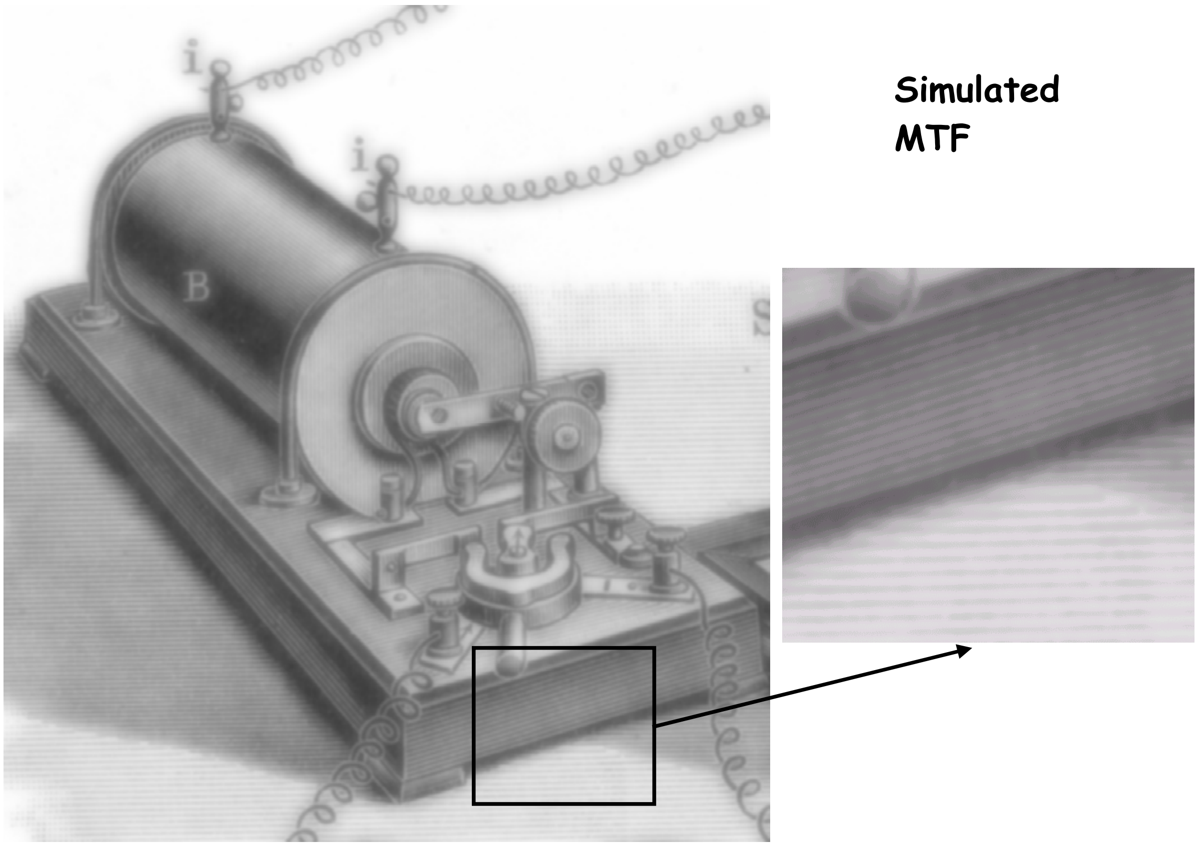

i
i
B
S
Original
R = 1.0
R = 0.78

Another example of the influence of the shape of MTF

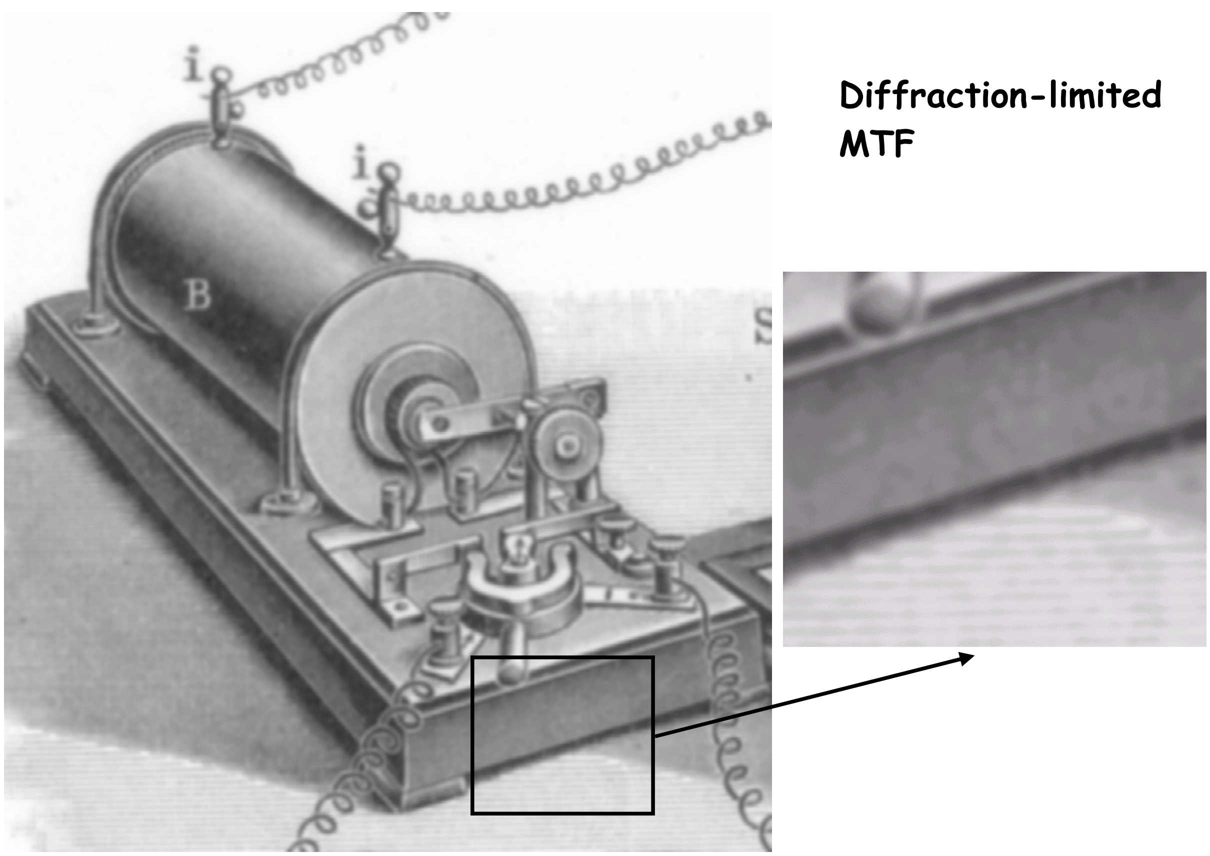


The red MTF curve above represents a diffraction-limited optical system, and the blue curve a simulated optical system. The simulated system is purely hypothetical, but it is based on a non-negative PSF, i.e. it is physically feasible. These two curves display the same general characteristics as the R = 0.78 and 1.0 curves on page 8, but in a more pronounced way. Thus, the simulated curve drops rapidly at first, but then levels off and extends out to high frequencies. The diffraction-limited curve drops less rapidly, and almost uniformly up to the highest frequency.

On the next page are some imaging results obtained for the diffraction-limited and simulated cases above. As expected, the results resemble those on page 12, but they are perhaps a bit more pronounced.



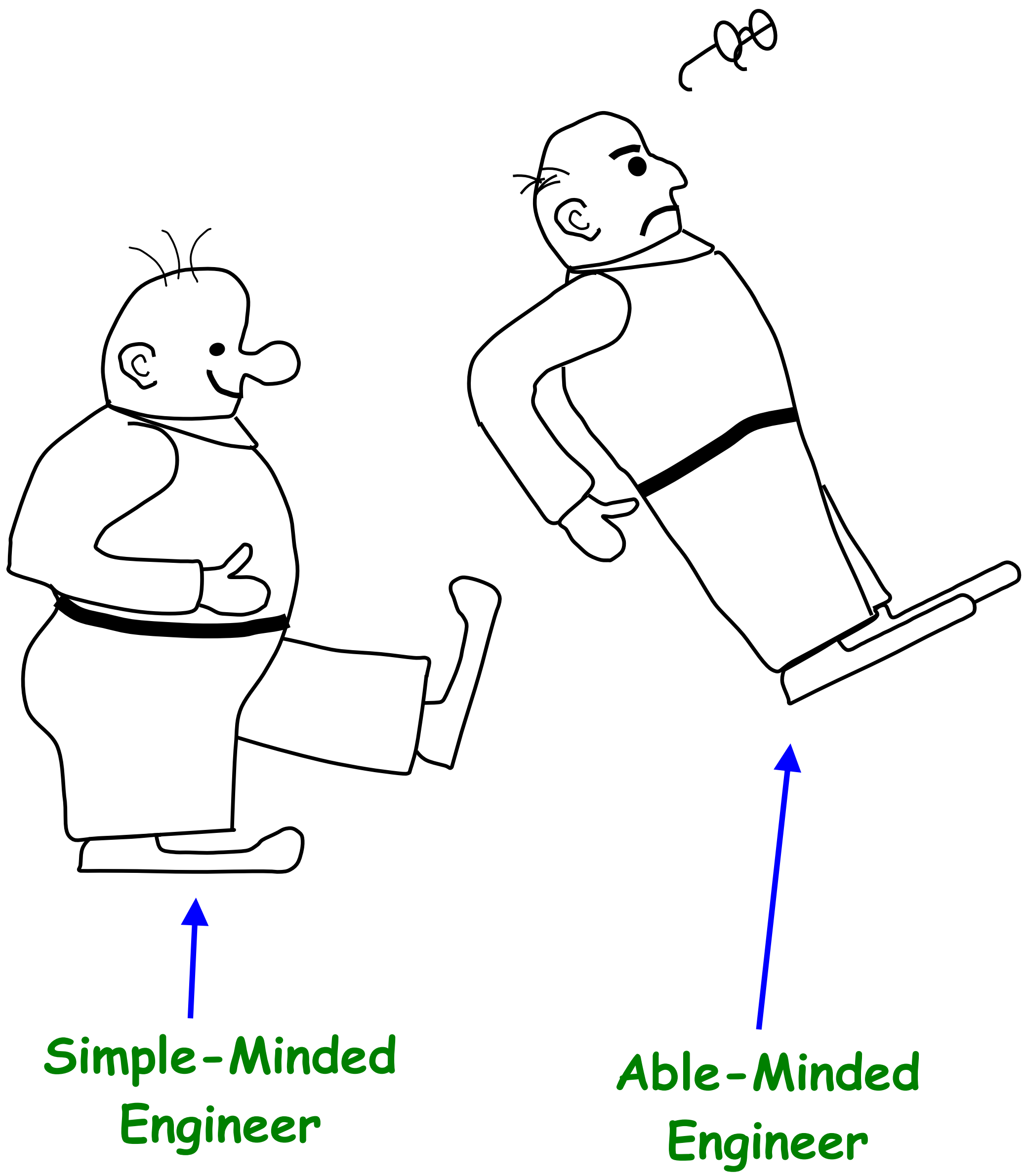
Simulated MTF



Diffraction-limited MTF

Now it's time for

Revenge!!



The "quick-and-dirty" estimate by the Simple-minded Engineer turned out to be better than the more "professional" guess by the Able-minded Engineer. So, what's the moral? Is it dangerous to know too much?

Of course not, knowledge is a wonderful thing!

The problem for the Able-minded Engineer was that he didn't know enough. If he had known about the importance of MTF behavior, and not just optimized for resolution, he would not have been beaten by the Simple-minded Engineer.

We should also recognize that the Simple-minded Engineer was very lucky when he made his guess. He cannot always expect to be so lucky.

Moral:

If you don't know enough, you may be beaten by the ignorant!!

Finally, another pinhole camera photograph from the mid 1970s. The author is sitting rigidly in a sofa, looking at a watch to time the exposure (which was very long!). The short time intervals needed to walk to and from the camera, to start and stop the exposure, were not registered on the film.