Lecture 1

The history and concept of quantum entanglement

At the heart of quantum physics lie two new concepts: the superposition principle and quantum entanglement.

Superposition principle:

The superposition principle tells us that a particle can be in the superposition of several states, such as different polarizations (horizontal and vertical polarization for instance). A famous and ancient example is the Schrödinger cat that is in a superposition of dead and alive:

Entanglement:

In the famous Schrödinger cat story, a cat and a poison vial are placed in a box. At some random time, the poison vial opens and the cat dies. Because the box if entirely closed (no information can escape the box), we cannot say with any possible measurement whether the poison vial has opened or not, the cat is therefore in a superposition of dead and alive the same goes for the poison. We therefore have the following equation that now shows entanglement between the cat and the poison vial, measuring one of the two gives information on the other one:

Here making a measurement on the cat (checking whether it is dead or alive) tell us right away about the state of the poison vial (opened or closed) without any need to make a measurement on the poison vial.

The Einstein Podolsky Rosen (EPR) paper

Einstein, Podolsky and Rosen introduced the concept of entanglement in 1935 in an attempt to demonstrate that quantum mechanics is not a complete theory, this includes a discussion of what is reality and what is a complete physical theory.

They suggested a gedanken experiment in the second half of the article. A particle decays into two particles of equal mass:

Measuring the momentum of one particle gives information on the momentum of the other particle instantaneously. This of course was a problem for Einstein at it seems to violate relativity (nothing can travel faster than light), and also the uncertainty principle as two independent measurements can be done on the same system.

The first name 'spuckhafte Fernwirkung' was coined by Einstein, the term 'Entanglement' was coined by Niels Bohr, an interesting discussion between Einstein and Bohr took place on this.

In 1951, David Bohm turned this concept into a spin ½ particles problem, something easier to measure in the lab.

Bohm's concept

Dissociation of a spin zero two-atom molecule where each atom has spin ½. After dissociation, the atoms travel in opposite directions, making it possible to label the atoms (1) and (2):

Because the process must conserve angular momentum, there are only two possible outcomes:

- Particle (1) has spin UP and particle (2) has spin DOWN
- Particle (1) has spin DOWN and particle (2) has spin UP

Each of these two cases are equally likelky, the wave function can be written:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2\right]$$

Simpler notation:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_1\downarrow_2\rangle + |\downarrow_1\uparrow_2\rangle\right]$$

Paradox: The atoms travel in opposite directions, after some time they can no longer interact directly. But still, a measurement on one particle reveals to the experimenter what a measurement on the other particle would give, wherever the other particle might be. At first glance, it seems to contradict relativity, however no information is sent faster than light.

When measurements are carried out on one of the two particles, the outcome will be random: half of the time spin UP and half of the time spin DOWN. However, when both measurements are compared, something striking emerges: the results are always opposite: if measurement on particle 1 gives UP, the measurement on particle 2 give DOWN and vice versa. This is the case whatever the distance between the two particles and the knowledge we gain is instantaneous. Nothing to worry about since no information travels faster than light here: we can't use this to communicate.

We can also see what makes a solid, mathematical definition for an entangled state: it is not facorizable.

Note 1

Entanglement is not limited to two particles.

N particles can be entangled, for 3 particles, there are for instance the GHZ states named after Greenberger, Horne and Zeilinger:

$$|GHZ\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

Here a measurement on one particle reveals the outcome of measurements on the other two particles.

Note 2

While spin is often used in entanglement experiments (photon spin ~ polarization), continuous variables can also be entangled, such as position and linear momentum (as in the EPR paper). Or between two time bins (time-bin entanglement).

Note 3

Correlations are crucial. For the state

$$\frac{1}{\sqrt{2}}\left(|\uparrow_1 \downarrow_2\rangle + |\downarrow_1 \uparrow_2\rangle\right)$$

Measurement on individual particles give random results. However, measurements on pairs give correlated measurements. Correlations are therefore crucial in quantum entanglement studies. This raises a number of technological questions: how one measure correlations in the lab? We can look at two detectors and say that we have a correlation even if the two detectors give detection signals within a given time interval.

Note 4

Hidden variables have been proposed to explain these correlations. The hidden variables would be determined when pairs are created. 'God doesn't play dice'. Fully disproving hidden variables remains to be done in the lab. One question is how fast could hidden variables travel. This opens the room for loopholes.

Note 5

Different particles can be entangled. For instance, an electron spin can be entangled with a photon polarization. In this case measuring the photon polarization would give information on the electron spin and vice versa.

Note 6

Two particles (or more) can share several types of entanglement. For instance, polarization and time-bin entanglement. This is called hyper entanglement.

Note 7

To measure and produce the type of entanglement we are discussing here, one needs to be able to generate and detect light at the single photon level: we need single photon detectors as well as sources of single of pairs of photons. The generation of single photons Is an active field of research, a true single photon source on-demand, that would generate one and only one photon whenever required remains elusive but systems based on nanoscale quantum systems such as quantum dots are continuously improving.

Quick overview of the experimental setup that will be used in the laboratory.