

Exam Solutions IE1204 Digital Design 2018-10-22 (corrected 4)

Part 1

1.

Solution:

(a) (2p)

Z = (1) 0111 0111, overflow

(b) (2p)

W = 0000 1100 1101 0110 = 0x0CD6 (-106 x -31 = 3286)

Not correct: (1000-0011-1101-0110): 83D6

The signs must be removed before the bitwise multiplication is performed

2. (2p)

Solution: $Y = \overline{A + B}$

A B Y

0 0 1

0 1 0

1 0 0

1 1 0

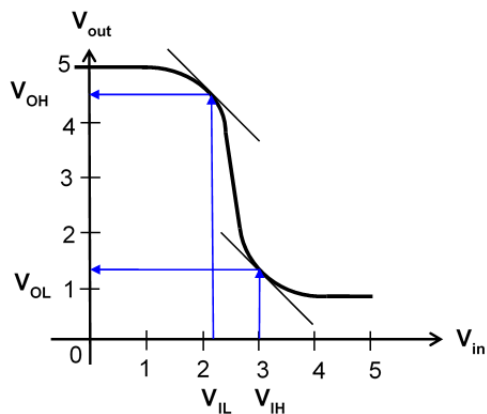
3. (2p)

Solution: f

a	b	c	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

4. (2p)

Solution: **YES it can be an inverter**



$V_{IL} = 2.2; V_{IH} = 3; V_{OL} = 1.2; V_{OH} = 4.5; NM_L = V_{IL} - V_{OL} = 1; NM_H = V_{OH} - V_{IH} = 1.5$

Part 2

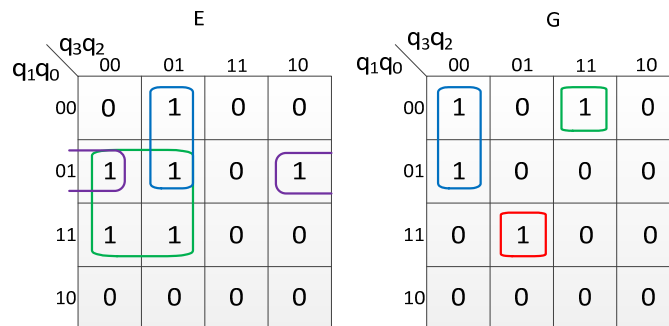
5.

Solution:

(a) (1p)

BCD				A	B	C	D	E	F	G
q_3	q_2	q_1	q_0							
0	0	0	0	0	0	0	0	0	0	1
0	0	0	1	1	0	0	1	1	1	1
0	0	1	0	0	0	1	0	0	1	0
0	0	1	1	0	0	0	0	1	1	0
0	1	0	0	1	0	0	1	1	0	0
0	1	0	1	0	1	0	0	1	0	0
0	1	1	0	0	1	0	0	0	0	0
0	1	1	1	0	0	0	1	1	1	1
1	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	1	0	0
1	0	1	0	0	0	0	1	0	0	0
1	0	1	1	1	1	0	0	0	0	0
1	1	0	0	0	1	1	0	0	0	1
1	1	0	1	1	0	0	0	0	1	0
1	1	1	0	0	1	1	0	0	0	0
1	1	1	1	0	1	1	1	0	0	0

(b) (2p)

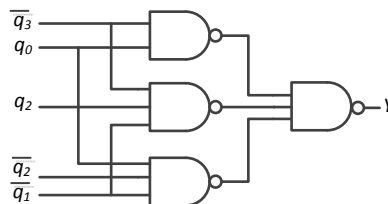


$$E = \bar{q}_3 q_0 + \bar{q}_3 q_2 \bar{q}_1 + \bar{q}_2 \bar{q}_1 q_0$$

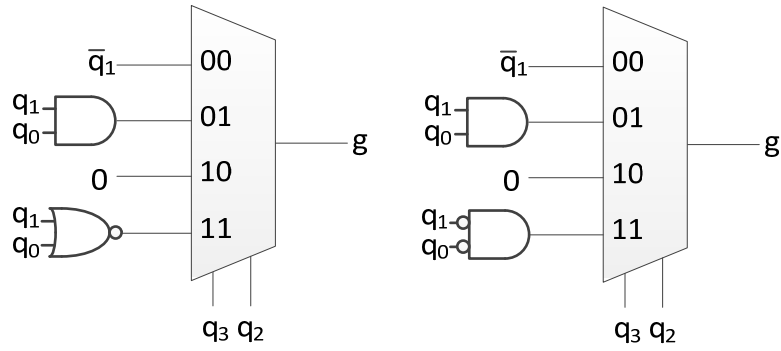
$$G = \bar{q}_3 \bar{q}_2 \bar{q}_1 + \bar{q}_3 q_2 q_1 q_0 + q_3 q_2 \bar{q}_1 \bar{q}_0$$

(c) (1p)

$$E = \bar{q}_3 q_0 + \bar{q}_3 q_2 \bar{q}_1 + \bar{q}_2 \bar{q}_1 q_0 = \overline{\overline{\bar{q}_3 q_0 + \bar{q}_3 q_2 \bar{q}_1 + \bar{q}_2 \bar{q}_1 q_0}} = \overline{\bar{q}_3 q_0 \cdot \bar{q}_3 q_2 \bar{q}_1 \cdot \bar{q}_2 \bar{q}_1 q_0}$$



(d) (1p)



6.

Solution:

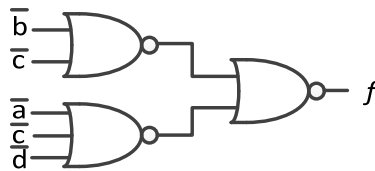
(a) (1p)

$$f = (\bar{b} + \bar{c})(\bar{a} + \bar{c} + \bar{d})$$

cd	ab			
	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	0	0	0
10	1	0	0	1

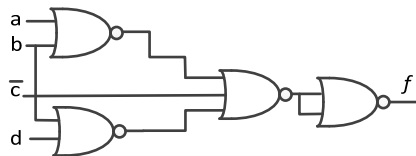
(b) (1p)

$$f = (\bar{b} + \bar{c})(\bar{a} + \bar{c} + \bar{d}) = \overline{\overline{(\bar{b} + \bar{c}) + (\bar{a} + \bar{c} + \bar{d})}}$$



Grouping ones:

$$f = \bar{c} + \bar{a}\bar{b} + \bar{b}\bar{d} = \bar{c} + \overline{\overline{\bar{a}\bar{b} + \bar{b}\bar{d}}} = \bar{c} + \overline{\overline{(a+b) + (b+d)}} =$$



7.

Solution:

(a) (1p) before:

	AB			
CD	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	1	1	1	1
10	0	0	1	0

(b) after:

	AB			
CD	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	1	1	1	1
10	0	0	1	0

(b) (1p)

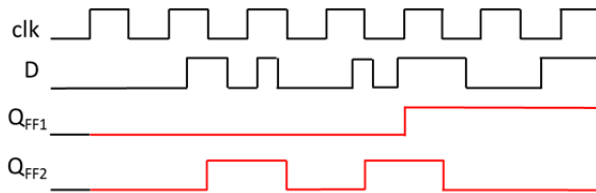
$Y = A \cdot B + C \cdot D + A \cdot \bar{C} + A \cdot D$ (the gate for $A \cdot \bar{B} \cdot \bar{C}$ is no longer needed)
Both terms needed for full points.

(c) (1p) for the circuit

Part 3

8.

Solution: (2p)

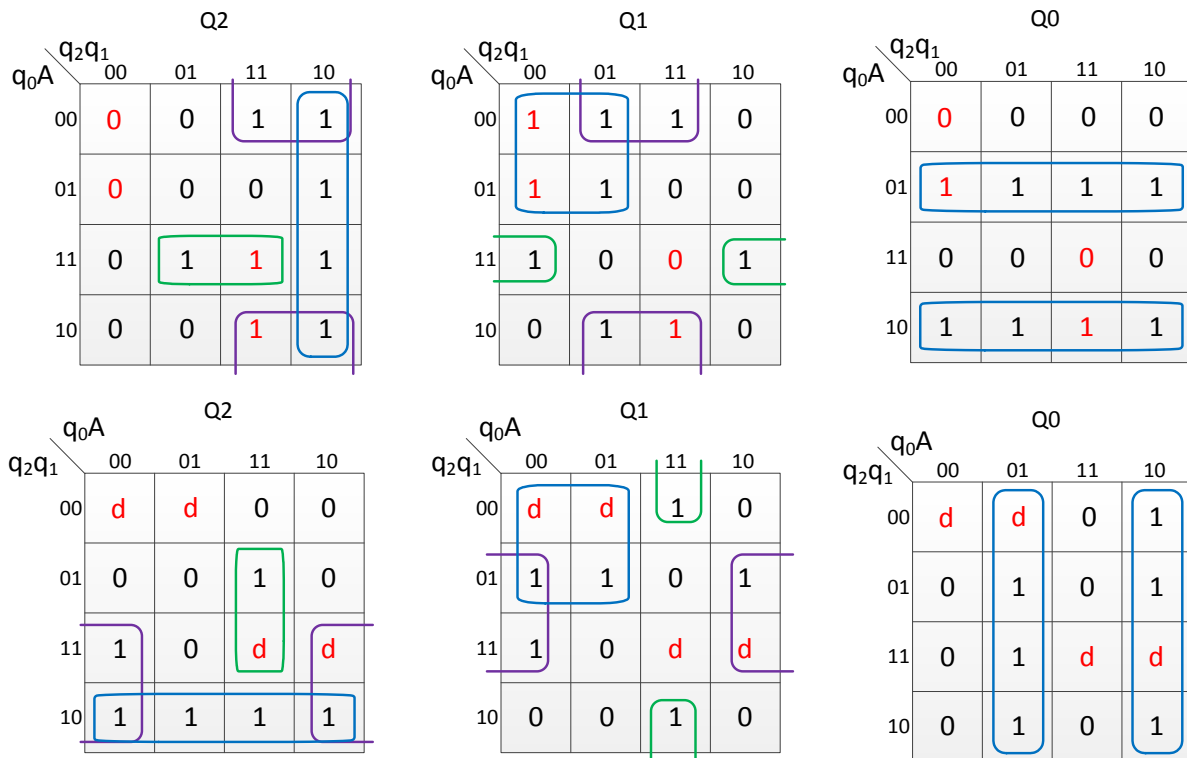


9.

(1p)

Present State			A=0			A=1		
q2	q1	q0	Q2	Q1	Q0	Q2	Q1	Q0
0	0	0	-	-	-	-	-	-
0	0	1	0	0	1	0	1	0
0	1	0	0	1	0	0	1	1
0	1	1	0	1	1	1	0	0
1	0	0	1	0	0	1	0	1
1	0	1	1	0	1	1	1	0
1	1	0	1	1	0	0	0	1
1	1	1	-	-	-	-	-	-

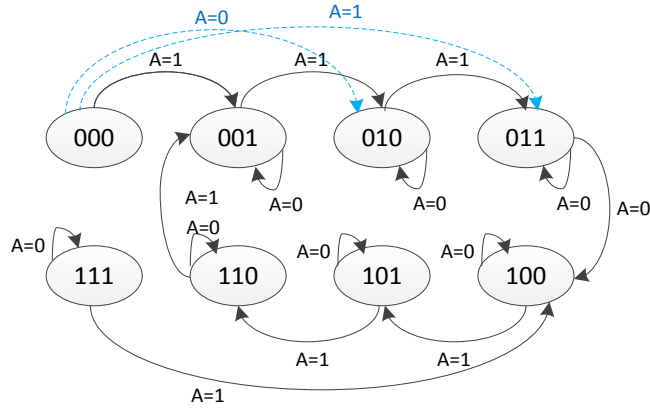
(2p)



$$\begin{aligned}
 Q2 &= q_2\bar{q}_1 + q_2\bar{A} + q_1q_0A \\
 Q1 &= q_1\bar{A} + \bar{q}_2\bar{q}_0 + \bar{q}_1q_0A \\
 \text{OR } Q1 &= q_1\bar{A} + \bar{q}_2\bar{q}_0A + \bar{q}_1q_0A \\
 \text{OR } Q1 &= q_1\bar{A} + \bar{q}_2q_1\bar{q}_0 + \bar{q}_1q_0A \\
 Q0 &= q_0\bar{A} + \bar{q}_0A
 \end{aligned}$$

Present State			A=0			A=1		
q2	q1	q0	Q2	Q1	Q0	Q2	Q1	Q0
0	0	0	0	1	0	0	1	1
0	0	1	0	0	1	0	1	0
0	1	0	0	1	0	0	1	1
0	1	1	0	1	1	1	0	0
1	0	0	1	0	0	1	0	1
1	0	1	1	0	1	1	1	0
1	1	0	1	1	0	0	0	1
1	1	1	1	1	1	1	0	0

(1p)



10.

(a) (1p) $T_c \geq t_{pcq} + t_{pdAND} + t_{pdAND} + t_{pdAND} + t_{pdXOR} + t_{setup} = 2 + 3 + 3 + 3 + 4 + 3 = 18 \text{ ns}$

(b) (1p) $t_{hold} < t_{ccq} + t_{cdXOR} : 2 < 1 + 2$

(c) (1p)

$$Q_0 = En \oplus q_0 = En \cdot \bar{q}_0 + \bar{En} \cdot q_0$$

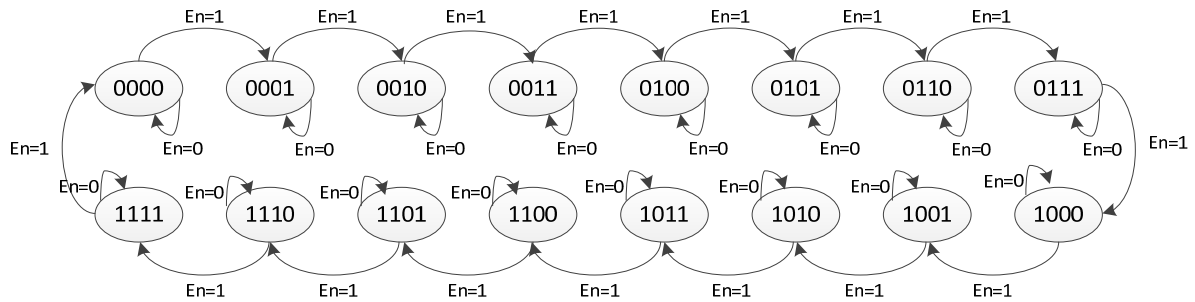
$$Q_1 = (En \cdot q_0) \oplus q_1 = (\bar{En} \cdot q_0)q_1 + (En \cdot q_0)\bar{q}_1$$

$$Q_2 = (En \cdot q_0 \cdot q_1) \oplus q_2$$

$$Q_3 = (En \cdot q_0 \cdot q_1 \cdot q_2) \oplus q_3$$

Present state				Enable=0				Enable=1			
q3	q2	q1	q0	Q3	Q2	Q1	Q0	Q3	Q2	Q1	Q0
0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	0	0	1	0	1
0	1	0	1	0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	0	0	1	1	1
0	1	1	1	0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	0	1	0	0	1
1	0	0	1	1	0	0	1	1	0	1	0
1	0	1	0	1	0	1	0	1	0	1	1
1	0	1	1	1	0	1	1	1	1	0	0
1	1	0	0	1	1	0	0	1	1	0	1
1	1	0	1	1	1	0	1	1	1	1	0
1	1	1	0	1	1	1	0	1	1	1	1
1	1	1	1	1	1	1	1	0	0	0	0

(d) (1p)



Part 4

11. (2p)

SRAM

Volatile

6 transistors

Larger

Slightly faster

No refresh

Easily integrated in CPU

DRAM

Volatile

1 transistor 1 capacitor

Smaller

Fast

Capacitor needs refresh

Needs extra mask layers for buried capacitor

12. (2p)

Solution:

$$Q = 0011 + Q(\text{prev})$$

$$0000 \rightarrow 0011 \rightarrow 0110 \rightarrow 1001 \rightarrow 1100$$

13.

Solution:

a) (1p) Seven gate delays. From pin 1, 18, 20 or 22 “B0”-“B3” to pin 14 “A = B”

b) (1p) NOT: 5, AND: 34, NAND: 4, NOR: 12, XOR: 8 (63)

$$5x2 + 34x6 + 18x2 + 4x4 + 5x2 + 12x4 + 9x2 + 8x8 = 406$$

Not gates: 5

2 transistors

AND (1-input): 5

4

AND (2-input): 12

6

AND (3-input): 12

8

AND (4-input): 4

10

AND (5-input): 1

12

NOR (2-input): 5

4

NOR (3-input): 5

6

NOR (4-input): 2

8

NAND (2-input): 2

4

(one OR with inverted inputs counted as NAND)

NAND (4-input): 1

8

NAND (5-input): 1

10

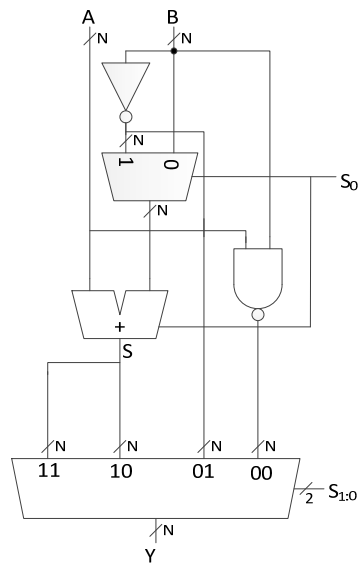
XOR (2-input): 8

8

$$5*2 + (5*4 + 12*6 + 12*8 + 4*10 + 1*12) + (5*4 + 5*6 + 2*8) + (2*4 + 1*8 + 1*10) + 8*8 = 406$$

14.

Solution: (2p)



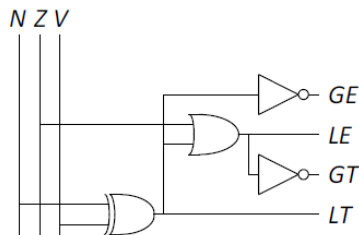
(The inverter can be replaced with an XOR between B and So)
 One point was deducted for solutions with more than one full adder.

15.

Solution: (2p)

(a) $GE = N \oplus V$
 $LE = Z + (N \oplus V)$
 $GT = \overline{LE} = \overline{Z(N \oplus V)}$
 $LT = \overline{GE} = N \oplus V$

(b)



GE

	NZ	00	01	11	10
V	0	1	1	d	0
	1	0	d	d	1

LE

	NZ	00	01	11	10
V	0	0	1	d	1
	1	1	d	d	0

GT

	NZ	00	01	11	10
V	0	1	0	d	0
	1	0	d	d	1

LT

	NZ	00	01	11	10
V	0	0	0	d	1
	1	1	d	d	0

Z and V cannot be “1” at the same time. Z and N cannot be “1” at the same time too, so they are “d” in the K-map.

Based on this K-map, the equations will be:

$$\begin{aligned}GE &= \bar{V}\bar{N} + VN = \overline{V \oplus N} \\LE &= V\bar{N} + \bar{V}N + Z = Z + (V \oplus N) \\GT &= \bar{V}\bar{N}\bar{Z} + VN \\LT &= V\bar{N} + \bar{V}N = V \oplus N\end{aligned}$$