Lecture 4

The no-cloning theorem and polarization of light

<u>The no-cloning theorem</u>

Published in Nature in 1982 (Wooters , Nature 299, 802 (1982)).

Is there a physical limit to how perfect a copy can be made?

The answer is yes: in quantum physics, a perfect copy is impossible. Assume we have a 'quantum cloning machine': a copy operator U_{copy} that acts on a 'blank' state and makes a copy of a state:

$$U_{copy}\left(|V\rangle_1 |blank\rangle_2\right) = |V\rangle_1 |V\rangle_2$$
$$U_{copy}\left(|H\rangle_1 |blank\rangle_2\right) = |H\rangle_1 |H\rangle_2$$

This must also work for any superposition:

$$U_{copy}\left(\left[a|H\rangle_1 + b|V\rangle_1\right] |blank\rangle_2\right) = U_{copy}\, a\left(|H\rangle_1 |blank\rangle_2\right) + U_{copy}\, b\left(|V\rangle_1 |blank\rangle_2\right)$$
$$= a|H\rangle_1 |H\rangle_2 + b|V\rangle_1 |V\rangle_2$$

But what should we really get for a cloning process?

Sol:

$$\left(a|H\rangle_1 + b|V\rangle_1\right)\left(a|H\rangle_2 + b|V\rangle_2\right) = a^2 |H\rangle_1 |H\rangle_2 + ab|H\rangle_1 |V\rangle_2 + ba|V\rangle_1 |H\rangle_2 + b^2 |V\rangle_1 |V\rangle_2$$

This is the outcome we expect from our cloning m

We have a discrepancy between what a cloning machine would give for a superposition and what cloning should give: the cloning machine misses the cross terms.

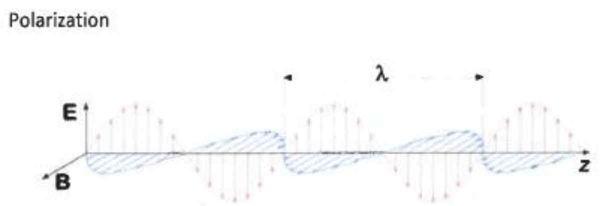
Cloning only works for a=1 and b=0 (and vice versa), but not for all the other cases! **Quantum cloning is impossible.** This is good news for quantum cryptography: an eavesdropper cannot intercept a message, clone it, send a copy to the intended receiver and keep a copy without introducing errors. We will come back to quantum cryptography later in the course.

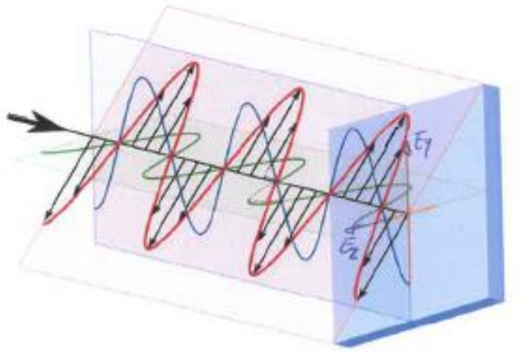
While quantum cloning is impossible, quantum teleportation is possible (will be discussed in a coming lecture). In quantum teleportation, the original state is destroyed and a copy of that state is made, therefore not violating the no cloning theorem.

Polarization of light

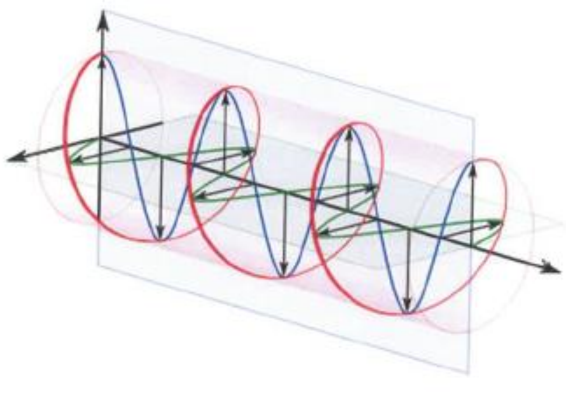
We can decompose the electric field of a light field traveling along the x direction into Ey and Ez components. Depending on the phase between the two components, light is linearly, circularly op elliptically polarized, this is true for the electromagnetic field associated with an intense light beam as well as for a single photon.

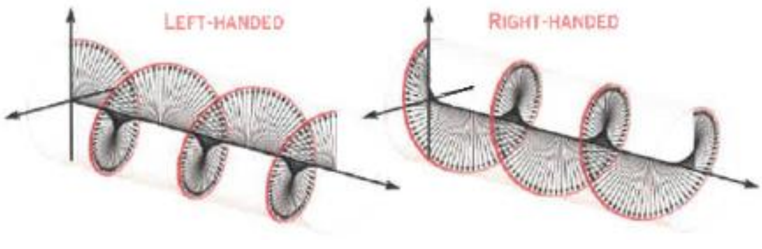
If Ey and Ez are in phase, we have linearly polarized light:





If Ey and Ez are out of phase by 90 degrees, we have circularly polarized light which can be left handed and right handed (+ or – 90 degrees phase difference):





The case between these two extremes gives elliptically polarized light.

Waveplates

Waveplates are made of birefringent materials. A birefringent material has two refractive indexes: n_o and n_e, for extraordinary and ordinary, respectively, along two different orientations.

Calcite is a natural, birefringent crystal.

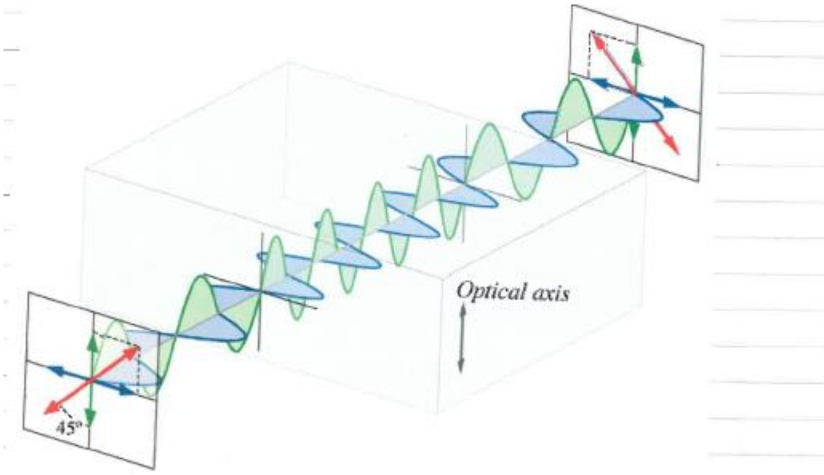
This means that light propagating along a given direction can propagate at different velocities depending on its polarization. We can acquire a phase difference between the two polarizations during propagation:

$$\Delta\phi = \frac{2\pi d}{\lambda}(n_e - n_o)$$

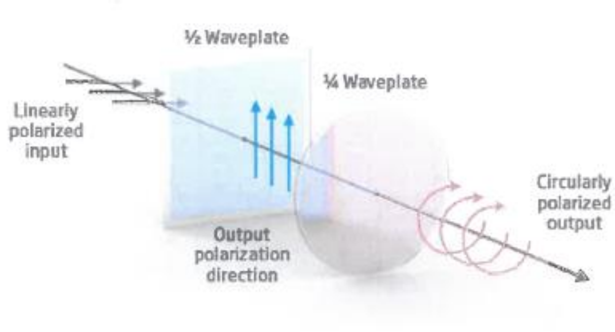
where d is the thickness and $\Delta\phi$ is the retardance.

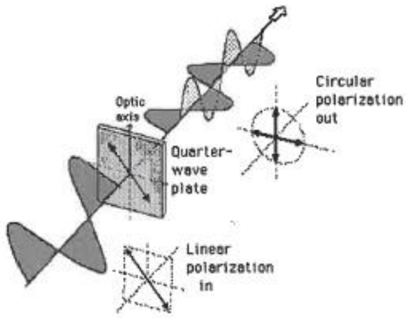
A given waveplate thickness will introduce a given retardance. There are two thicknesses of special interest: the half-waveplate and the quarter-waveplate:

- With a half-waveplate we can rotate linear polarization. When linearly polarized light propagates through half-waveplate its linear polarization rotates by 2α where α is the angle of the half-waveplate.
- With a quarter-waveplate we can turn circular polarization into linear and vice versa.

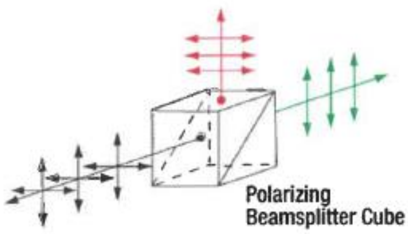


A quarter waveplate turns linear polarization into circular polarization if placed at the correct angle:



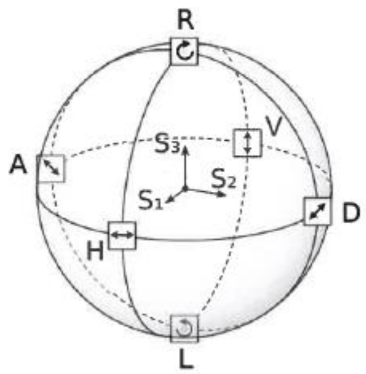


To study polarization, we use polarizers such as polarizing beam splitters (PBS) that transmit one particular polarization and reflect the perpendicular polarization:



The polarizing beam splitter is an excellent polarizer that offers very high selectivity: the output is highly polarized.

All possible polarizations can be mapped on the Poincare sphere:



Complete circular polarization is at the poles: right circular polarization on top, left circular polarization at the bottom. In the plane of the equator we have linear polarization. Between the equator and the poles, we have elliptical polarization.

To go from any polarization to any other polarization, one simple solution is to use a quarter waveplate to go down to the equator, followed by a half-waveplate to move along the equator and another quarter-waveplate to reach the target.