Lecture 6

Bell basis and Quantum teleportation

Bell Basis

For two particle entanglement, there are four possible Bell states defining a basis:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}\left(|H_1H_2\rangle + |V_1V_2\rangle\right)$$
$$|\phi^-\rangle = \frac{1}{\sqrt{2}}\left(|H_1H_2\rangle - |V_1V_2\rangle\right)$$

positive correlations
→ same result for both particles

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|H_1V_2\rangle + |V_1H_2\rangle\right)$$
$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|H_1V_2\rangle - |V_1H_2\rangle\right)$$

negative correlations
→ opposite results.

Generalization:

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left[|HH\rangle + e^{i\theta}|VV\rangle\right]$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|HV\rangle + e^{i\theta}|VH\rangle\right]$$

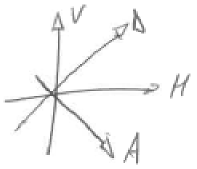
Bell states in different bases

For positive correlations, we have:

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left[|HH\rangle + e^{i\theta}|VV\rangle\right]$$

with $\theta = 0$ or π we get φ^+ or φ^-.

What if we are another basis? like the diagonal-antidiagonal D and A:



We express $|H\rangle$ and $|V\rangle$ in the $|A\rangle$ and $|D\rangle$ basis.

$$|H\rangle = \frac{1}{\sqrt{2}}\left(|D\rangle + |A\rangle\right)$$

$$|V\rangle = \frac{1}{\sqrt{2}}\left(|D\rangle - |A\rangle\right)$$

We can write $|HH\rangle$ and $|VV\rangle$ in terms of $|A\rangle$ and $|D\rangle$

$$\begin{aligned} |HH\rangle &= |H\rangle |H\rangle \\ &= \frac{1}{2}\left(|D\rangle + |A\rangle\right)\left(|D\rangle + |A\rangle\right) \\ &= \frac{1}{2}\left(|DD\rangle + |AA\rangle + |AD\rangle + |DA\rangle\right) \end{aligned}$$

$$\begin{aligned} |VV\rangle &= |V\rangle |V\rangle \\ &= \frac{1}{2}\left(|D\rangle - |A\rangle\right)\left(|D\rangle - |A\rangle\right) \\ &= \frac{1}{2}\left(|DD\rangle - |DA\rangle - |AD\rangle + |AA\rangle\right) \end{aligned}$$

We can rewrite the general state $|\varphi\rangle = \frac{1}{\sqrt{2}}\left[|HH\rangle + e^{i\theta}|VV\rangle\right]$ in the DA basis.

$$\begin{aligned} |\varphi\rangle &= \frac{1}{2\sqrt{2}}\left[|DD\rangle + |AA\rangle + |AD\rangle + |DA\rangle + e^{i\theta}\left(|DD\rangle - |DA\rangle - |AD\rangle + |AA\rangle\right)\right] \\ &= \frac{1}{2\sqrt{2}}\left[\left(1 + e^{i\theta}\right)\left(|DD\rangle + |AA\rangle\right) + \left(1 - e^{i\theta}\right)\left(|DA\rangle + |AD\rangle\right)\right] \end{aligned}$$

We see that θ has a physical meaning!

For example, for θ=0 we measure high counts for positive correlations (DD and AA) but for $\theta = \pi$ ($\varphi^+ = HH + VV$) we measure high count rates for positive correlations ($DD + AA$).

But for $\theta = \pi$ we measure high count rates for negative cor[illegible] HV and VH.

$\phi^- = HH - VV$

To go from the HV to DA basis we rotate the half wave plate by 22.5 degrees

* If we want to produce the first Bell state:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right)$$

$$|\phi\rangle = \frac{1}{2\sqrt{2}}\left[\left(1+e^{i\theta}\right)\left(|DD\rangle + |AA\rangle\right) + \left(1-e^{i\theta}\right)\left(|DA\rangle + |AD\rangle\right)\right] \quad (1)$$

Let $\theta = 0$

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left[|DD\rangle + |AA\rangle\right]$$

- To produce this state, we adjust the system until DD and AA ~~count~~ correlations are max and DA and AD are min.

* To generate the second Bell state:

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}\left[|HH\rangle - |VV\rangle\right] \qquad (\text{here } \theta = \pi)$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}\left(|AD\rangle + |DA\rangle\right) \text{ from } (1)$$

- To produce this state, we adjust the system until DA and AD are max and DD and AA are min.

<u>For negative correlations:</u>

$$|HV\rangle = |H\rangle|V\rangle$$
$$= \left(\frac{1}{\sqrt{2}}(|D\rangle + |A\rangle)\right)\left(\frac{1}{\sqrt{2}}(|D\rangle - |A\rangle)\right)$$
$$= \frac{1}{2}\left[|DD\rangle - |AA\rangle + |AD\rangle - |DA\rangle\right]$$

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$|VH\rangle = |V\rangle|H\rangle$

$$= \frac{1}{2}\left[|DD\rangle - |AA\rangle - |AD\rangle + |DA\rangle\right]$$

We express the general state $|\psi\rangle$ in terms of D and A.

$$|\psi\rangle = \frac{1}{2\sqrt{2}}\left[(1+e^{i\theta})(|DD\rangle - |AA\rangle) + (1-e^{i\theta})(|AD\rangle - |DA\rangle)\right] \quad (2)$$

* To have the third Bell state $|\psi^+\rangle = \frac{1}{\sqrt{2}}\left[|HV\rangle + |VH\rangle\right]$ we let $\theta = 0$ in (2) and get

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|DD\rangle - |AA\rangle\right]$$

To get this state, we adjust the system until AA and DD are max and AD and DA are min.

* To produce the fourth Bell state:

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left[|HV\rangle - |VH\rangle\right] \quad \text{we let } \theta = \pi \text{ in (2)}$$

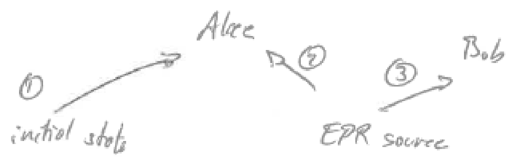
$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left[|AD\rangle - |DA\rangle\right]$$

To get this state we adjust the system until DA and AD are max and minimize AA and DD.

Quantum teleportation

First realized in 1997 by the Zeilinger group in Vienna.

Quantum teleportation relies on a pair of entangled photons:



$$\psi_{23} = \frac{1}{\sqrt{2}}\left(|H\rangle_2|V\rangle_3 - |V\rangle_2|H\rangle_3\right)$$

Ψ_1 is the arbitrary state we want to teleport to Bob:

$$\Psi_1 = a|H\rangle_1 + b|V\rangle_1$$

We let particles (1) and (2) interact on a beamsplitter to get:

$$\Psi_{12} = \frac{1}{\sqrt{2}}\left(|H\rangle_1 |V\rangle_2 - |V\rangle_1 |H\rangle_2\right)$$

Once particles 1 and 2 are in the state Ψ_{12}, particle (3) is projected into the initial state of particle 1.

→ particle (2) is in a state opposite to (1) and (3) is opposite to (2). Hence (3) is in the same state than particle (1).

<u>Note</u>: Ψ_1 is destroyed (no-cloning theorem)

$$
\begin{aligned}
|\Psi\rangle &= |\Psi_1\rangle \otimes |\Psi_{23}\rangle \\
&= \frac{a}{\sqrt{2}}\left(|V_1 V_2 H_3\rangle - |V_1 H_2 V_3\rangle\right) + \frac{b}{\sqrt{2}}\left(|H_1 V_2 H_3\rangle - |H_1 H_2 V_3\rangle\right) \\
&= \frac{1}{2}\Big[\left(|V_1 H_2\rangle - |H_1 V_2\rangle\right)\left(-a|V_3\rangle - b|H_3\rangle\right) \\
&\quad + \left(|V_1 H_2\rangle + |H_1 H_2\rangle\right)\left(-a|V_3\rangle + b|H_3\rangle\right) \\
&\quad + \left(|V_1 V_2\rangle - |H_1 H_2\rangle\right)\left(a|H_3\rangle + b|V_3\rangle\right) \\
&\quad + \left(|V_1 V_2\rangle + |H_1 H_2\rangle\right)\left(a|H_3\rangle - b|V_3\rangle\right)\Big]
\end{aligned}
$$

We see that after Alice measures on (1) and (2), she can tell Bob which of the four possible measurements she performed and send this information over a public channel. Bob can then complete the teleportation.