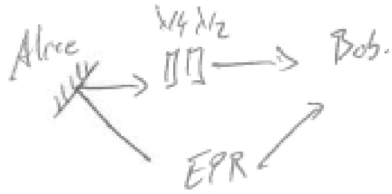
Lecture 9

Quantum dense coding

Can we make use of entanglement to increase the bandwidth of a communication channel?

Bennett introduced in 1992 the concept of quantum dense coding to communicate 4 bits while sending only one photon from Alice to Bob, this requires the photon traveling from Alice to Bob to be entangled with a second photon that travels to Bob.



Paradox: we can only measure the polarization of one photon (=1 bit of information) but we can exchange 4 bits of information.

Our EPR source generates

$$|H_A H_B\rangle + |V_A V_B\rangle$$

Alice can perform any of 4 transformations on her photon:

$$I(|H_A H_B\rangle + |V_A V_B\rangle) = |H_A H_B\rangle + |V_A V_B\rangle$$ - do nothing.

$$X(|H_A H_B\rangle + |V_A V_B\rangle) = |V_A H_B\rangle + |H_A V_B\rangle$$ - rotate polarization by 90° ($\frac{\lambda}{2}$ waveplate)

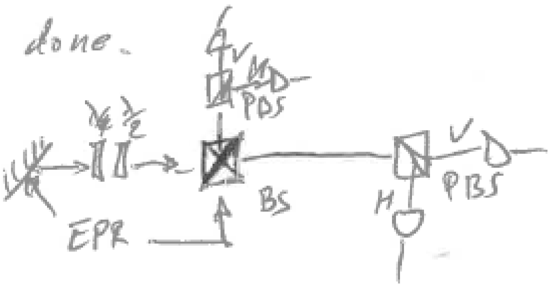
$$Y(|H_A H_B\rangle + |V_A V_B\rangle) = |V_A H_B\rangle - |H_A V_B\rangle$$ - rotate by 90° and phase shift.

$$Z(|H_A H_B\rangle + |V_A V_B\rangle) = |H_A H_B\rangle - |V_A V_B\rangle$$ - only phase shift.

Alice does no measurements, she only performs a transformation and sends her photon to Bob. Bob then measures in which of the four possible Bell states the entangled pair is. Bob then finds out which of the four transformations Alice applied, this gives 4 possible values and hence 2 bits. But only one photon travelled from Alice to Bob!

But can it be done?

Bob needs to identify which of the four Bell states he gets in one shot (one single measurement on only one photon pair). While possible in principle, it hasn't yet been done.



The photons leave the beam splitter in the same output (HOM) or in each output.

Only

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right)$$

Is anti-symmetric, only this one gives photons in each port. What counts here is the symmetry of the state that has to be taken into account with the beam splitter transformation.

For the 3 other states, both photons come out in the same port.

$|\Psi^+\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle + |VH\rangle\right)$ can be distinguished from the others on a PBS.

Φ^+ and Φ^- give the same outcome and cannot be distinguished, we can only guess. This enables 1.5 bits per photon to be transferred from A to B.

Note that to measure photon number (that is to distinguish one from two photons) we can stack detectors:

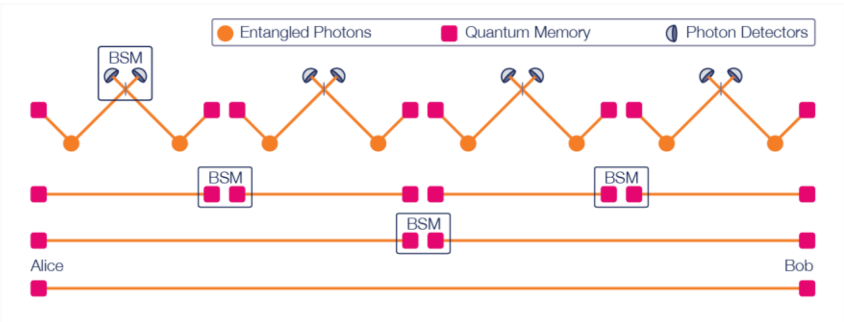
Quantum repeaters

Even with the best optical fibers and generation of entanglement at the wavelength with minimum losses (1550 nm), it is not possible to have distances larger than a few hundreds of km between Alice and Bob. To cover longer distances, **quantum repeaters** are required.

Existing communication systems make use of amplifiers, this is fine for classical information but is not compatible with quantum information because of the no-cloning theorem.

With quantum repeaters, the long communication line is broken up in shorter segments with acceptable losses. In every segment we place a source of entangled photon pairs (see figure below) one photon is sent to a quantum memory that can store a single photon (and its quantum state) for a long enough time, the other photon is sent to a Bell State Measurement apparatus (BSM in the figure below). When a Bell state measurement is successful we therefore have entanglement between two quantum memories. We can repeat this process until all links have shared entanglement and we have then entanglement all the way from Alice to Bob. This is essentially parallel processing: we establish links in parallel within the line, an important component is a quantum memory that can store reliably a single photon for long periods of time (we would like to have milliseconds or more).

The photon in the quantum memory is stored until the neighbouring link is ready, this reults in longer links with entanglement stored in the quantum memories.

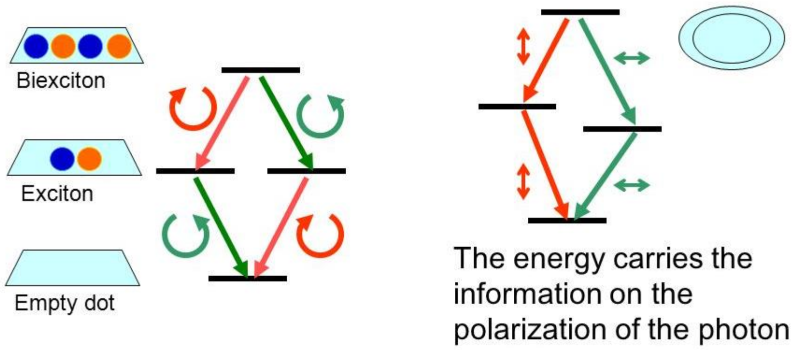


Other sources of entangled photons

While parametric down conversion as used in the laboratory provides entanglement, it has limitations:

- The efficiency is very low: the probability that a laser photon is down converted into two entangled photons is less than one in a million, this is not energetically efficient and will prevent large scale use.
- The statistics of the entanglement generation is Poissonian, we can't generate an entangled photon pair at a precise time.

There is a solution: quantum dots, small pieces of semiconducting material whose energy levels are obtained by solving Schrödinger's equation. In quantum dots, we can excite an electron to a higher energy level and form an exciton. We can excite a second photon to a higher energy level and then have a bi-exciton. This bi exciton can then recombine and give us two photons in a cascade and these two photons can be entangled in polarization. By adjusting the size and composition of the quantum dot, we can tailor the emission wavelength and aim for particular values such as 1550 nm for optical fiber applications.



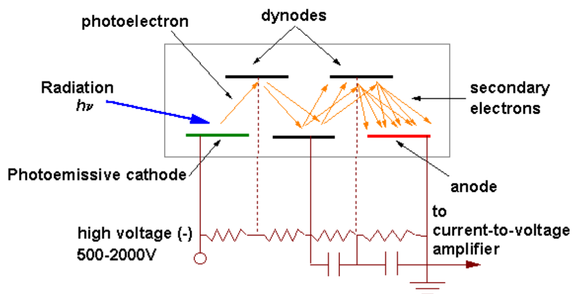
(in circular polarization basis:)

$$\Psi = | R_X L_{XX} > + | L_X R_{XX} >$$

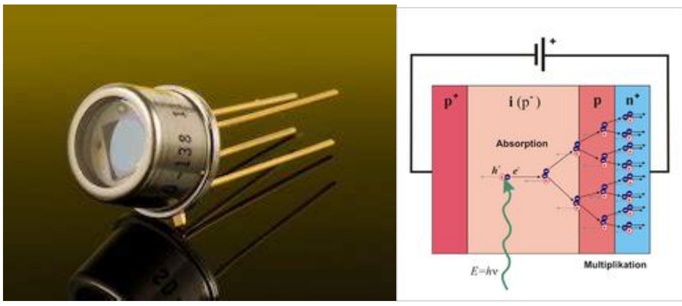
Single photon detectors

A near infrared photon carries ~ 10^{-19} Joules, a typical electrical pulse used for computer communication carries 10^{10} more energy. This means that to detect a single photon and process the detection event we need an amplification factor of 10. It is under debate whether the human eye is able to detect single photons, it might be able to detect small numbers of photons but with a time resolution that makes it irrelevant for most experiments in physics.

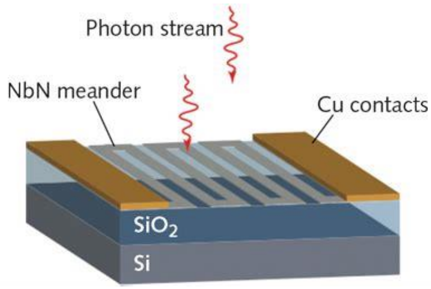
The photomultiplier tube has been used since the 1930s to detect single photons, it is based on the photoelectric effect where an incoming photon knock off an electron that is then accelerated towards another metal plate to release more electrons, this process is repeated several times to produce a large current.



Limitations with the detection efficiency of the photomultiplier tube were addressed with avalanche photodiodes that are monolithic semiconductor devices. Detection efficiencies in excess of 60% can be achieved in the near-visible range but time resolution and noise levels are not as good as one might wish for.



The most recent type of single photon detector to be developed is the superconducting single photon detector where superconducting nanostructures can switch from the superconducting state to the resistive state with the absorption of a single photon, this results in very high detection efficiencies in excess of 90%, in very good time resolution better than 20 ps and in very law noise levels.



Time resolution: uncertainty in the timing of the output electrical pulse for a fixed photon absorption time.

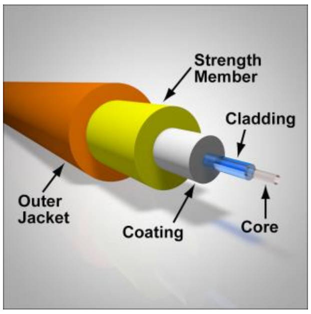
Dark noise: detection pulses generated by the detector in the absence of incident photons. On the order of hundreds of photons per second for a usual avalanche photodiode. Can be far less than one event per second for state of the art superconducting single photons.

Dead time: time interval following a detection event where the detector is not able to detect an incoming photon.

Quantum efficiency: probability for a photon to generate a detection event.

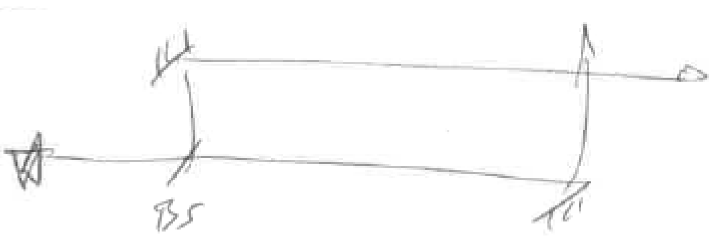
Optical fibers

In the 1960s it was suggested to make fibers with glass to guide light via total internal reflection. Over the years the purity of the glass was drastically increased and we now have very pure optical fibers where the transmission losses are very low, of the order of 0.02 dB/km, this means that half of the light intensity is lost after 150 km for a wavelength of 1550 nm. These fibers carry the data traffic for the internet and could also be used for quantum communication in the future.

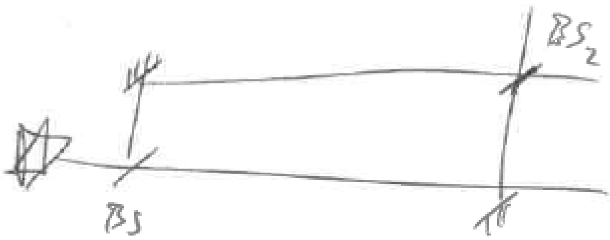


Wheeler's delayed choice

In experiments, photons can manifest properties of particle or a wave, but not both at the same time. Which property is observed depends on the type of experiment carried out. John Wheeler came up with experiments addressing the question when does the photon decide it is going to travel as a wave or as a particle? In the experiment below, we test the particle nature of light: photons from the source on the left will take either of both arms.



In this second experiment below, we will observe intereferences: we are testing the wave nature of light, the photons travel along both paths and interfere with themselves at the second beam splitter.



John Wheeler proposed the delayed choice: place beam splitter BS2 after the photon crossed the first beam splitter so that the photon has to 'decide' whether it will behave as a particle or a wave after traveling through the first beam splitter.

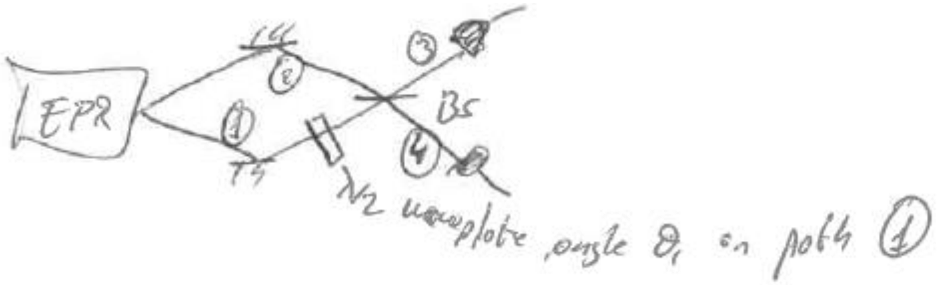
When the experiment is carried out and the second beam splitter is placed after the photon has crossed the first beamsplitter we see that the presence or absence of the second beam splitter always determines the wave or particle demonstration.

'The past has no existence except as recorded in the present'.

The quantum eraser

The Hong-Ou Mandel effect is based on the indistinguishability of the two incoming photons: they have the same polarization.

If we now control the polarization in one arm:



We now have:

$$|\theta\rangle_1 = |H\rangle_1 \cos\theta_1 + |V\rangle_1 \sin\theta_1$$

And the input at the beam splitter is now:

t of the BS is now:

$$|H_2\rangle|\theta\rangle_1 = \cos\theta_1 |H\rangle_2 |H\rangle_1 + \sin\theta_1 |H\rangle_2 |V\rangle_1$$

This implies that the output of the beam splitter will be:

output of the BS will be:

$$|\Psi_{out}(\theta)\rangle = \frac{i}{\sqrt{2}} \cos\theta_1 \left(\underbrace{|2H\rangle_3}_{\text{2 photons}} \underbrace{|0\rangle_4}_{\text{no photons}} + |0\rangle_3 |2H\rangle_4 \right) + \frac{1}{\sqrt{2}} \sin\theta_1 \left(|H\rangle_3 |V\rangle_4 - |V\rangle_3 |H\rangle_4 \right)$$

The terms containing the single photon states no longer cancel each other because of the different polarization.

In the extreme case,

$$\theta_1 = \frac{\pi}{2} \text{ we get: } |\Psi_{out}(\tfrac{\pi}{2})\rangle = \frac{1}{\sqrt{2}} \left(|H\rangle_3 |V\rangle_4 - |V\rangle_3 |H\rangle_4 \right)$$

Rotating one photon's polarization by 90 degrees is like 'marking' it: it gives a which path information, even if it is not known/measured by the experimenter, just like for the famous double slit experiment.

We now only act after the beam splitter where we place a polarizer with angle θ_3 on path (3).

$$|\theta_3\rangle = |H_3\rangle \cos\theta_3 + |V\rangle_3 \sin\theta_3$$

Only photons with polarization θ_3 are detected.

We get

$$|\psi, \theta_3\rangle = \frac{|\theta_3\rangle\langle\theta_3|\psi_{out}(\pi/2)\rangle}{\langle\psi_{out}(\frac{\pi}{2})|\theta_3\rangle\langle\theta_3|\psi_{out}(\frac{\pi}{2})\rangle^{1/2}} = |\theta_3\rangle\left(|V_4\rangle\cos\theta_3 - |H_4\rangle\sin\theta_3\right)$$

Where the photon of mode (4) has perpendicular polarization to the detected photon.

We place a similar polarizer at angle θ_4 on beam 4 and calculate the probability to get coincident detections at angles θ_3 and θ_4.

$$P_{coinc} = |\langle\theta_3|\theta_4|\psi_{out}(\pi/2)\rangle|^2 = \frac{1}{2}\sin^2(\theta_2 - \theta_1)$$

The dip in coincidence is recovered!

Polarizers (3) and (4) erase the information encoded on one of the beams by the waveplate. A question we can now ask ourselves is where does the Hong Ou Mandel effect take place?