Photonic Quantum Sensing (N00N states)

LIGO Experiment

Detecting gravitational waves requires extremely high sensitivity of the measurement. The detection is based on a Michelson Interferometer (see Fig. 1). The power recycling mirror allows to have 700W of power inside the Michelson which increases the sensitivity. In addition, the Fabry-Pérot cavity with round trips of around 280 increases the effective length of the arms to 1120km. This allows the LIGO experiment to measure path-length difference of 10^-22.

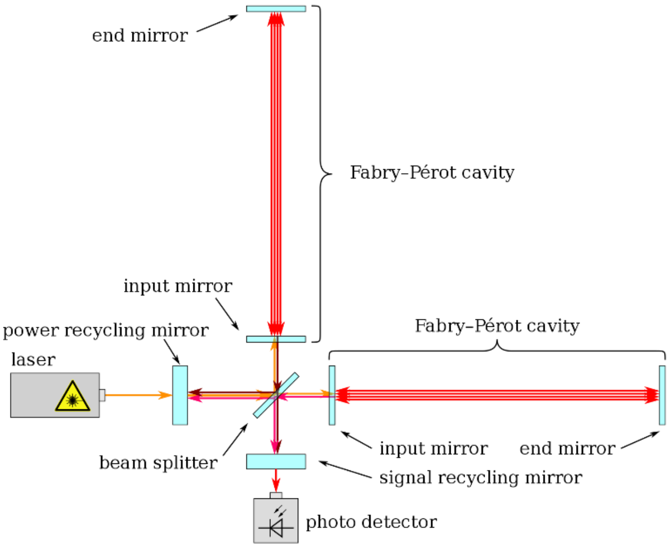

Figure 1: Simplified LIGO experiment [from wikipedia].

Classical Interferometer

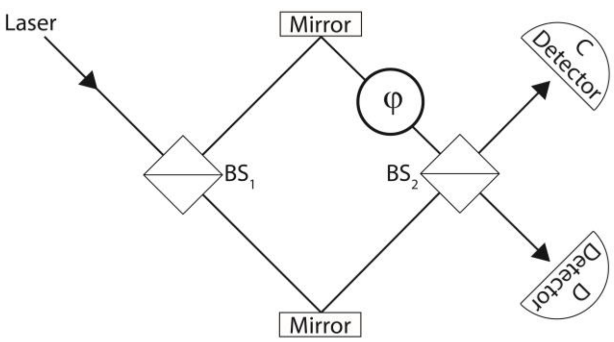

Figure 2: Mach-Zehnder Interferometer.

Example of a classical Interferometer. The laser sends light with intensity I_A and the detectors measure the intensity I_C and I_D, respectively. Depending on the Phase difference the interference at BS_2 will change and the laser light will exit different output ports of the BS_2.

$$I_C = I_A \, sin^2(\frac{\varphi}{2})$$

$$I_D = I_A \, cos^2(\frac{\varphi}{2})$$

$$M(\varphi) = I_D - I_C = I_A \, cos(\varphi)$$

Figure3 shows the intensity as a function of phase difference in a classical interferometer.

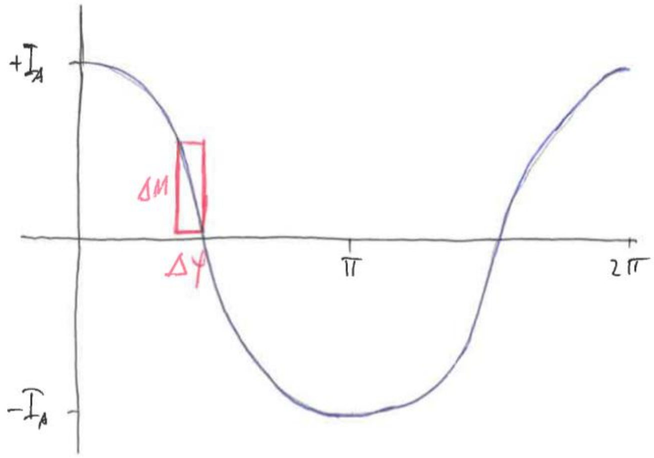

Figure 3: Intensity output at one detector of a classical Mach-Zehnder interferometer.

<u>Classical phase uncertainty</u>

$$\frac{\Delta M}{\Delta\varphi} = \frac{\delta M}{\delta\varphi} = I_A \, sin(\varphi)$$

$$\longrightarrow \quad \Delta\varphi = \frac{\Delta M}{\delta M/\delta\varphi} = \frac{\Delta M}{I_A \, sin(\varphi)}$$

<u>Shot noise limit</u>

$$\Delta n \Delta\varphi \geq 1$$

$$\Delta\varphi = \frac{1}{\Delta n}$$

Δn for a laser? → coherent light → Poissonian distribution (see lecture 2) → $\Delta n = \sqrt{n}$

Shot noise limit, is a quantum noise effect, related to the discreteness of photons and electrons. (Remark: The standard quantum limit for high-frequency intensity noise of a laser is the shot noise limit.)

Squeezed light

Figure 4 is a polar coordinate light state representation, where the phase is given by the angle and the photon number by the distance to the origin. As one can see from the graph there is a specific state of light, called squeezed state, which can have reduced uncertainties for n or φ on the expense of the uncertainty of the other. Such states can be created by using χ^3 non-linearity crystals or lasers (e.g. optical parametric oscillators). A lower degree of squeezing in bright amplitude-squeezed light can under some circumstances be obtained with frequency doubling. The Kerr nonlinearity in optical fibers also allows the generation of amplitude-squeezed light. Semiconductor lasers can generate amplitude-squeezed light when operated with a carefully stabilized pump current. The non linear interaction reduces the noise in one of the quadratures of the electric field. For example if you take frequency doubling as a nonlinear effect, this only occurs if your initial laser has 2 photons at a certain time bin. So the fluctuation in amplitude is reduced since the doubling does not work if only 1 photon was present and the probability that 4 initial photon were present to generate 2 frequency doubled photons is very small (Poisson distribution).

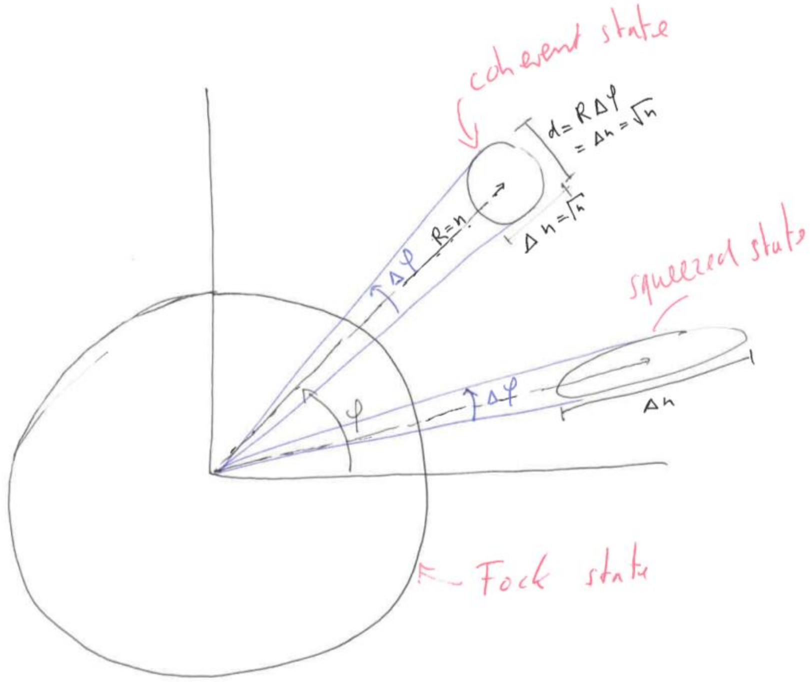

Figure 4: Illustration of 3 different light states based on their uncertainty. (note that is not a very precise quantum mechanical depiction which normally used the quadrature operators, since phase is not an observable)

Now consider a squeezed state with $\Delta n = n$:

$$\Delta\varphi = \frac{1}{n}$$

➔ Heisenberg limit!

(remark: comes from the Margolus-Levitin bound and not the Heisenberg principle)

Rayleigh criteria

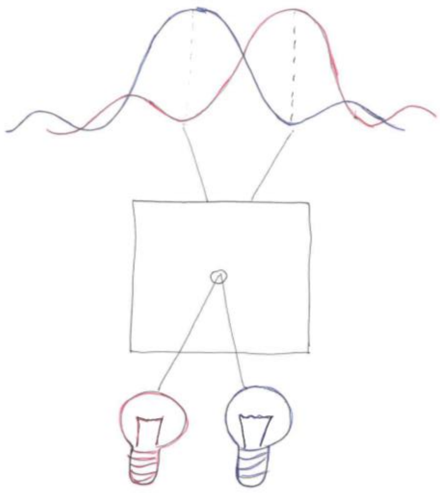

Figure 5: Rayleigh criteria shown in a single slit experiment.

Figure 5 shows a single slit experiment with two sources which one wants to distinguish. The smallest distance to resolve both is given by the Rayleigh criteria: $\Delta x = 1.22\,\frac{\lambda f}{D}$, with f focal length and D the diameter of the aperture.

The objects are resolved when the first minima of one object lies on the maxima of the other" → Diffraction limit

Important for lithography and microscopy.

How do we reduce the wavelength of the photons even further?

de Broglie wavelength

Initial idea was to attribute a wavelength to matter (wave-particle dualism). One can also attribute a photon a de Broglie wavelength which is in fact for a single photon just its normal wavelength. However, when having for example a Bose condensate or a quantum state of several photons where all photons are indistinguishable one can use the de Broglie formalism to get an effective wavelength. De Broglie wavelength: $\lambda_{de\ Broglie} = \frac{h}{p}$ where h is the Planck constant and p the momentum. A single photon also carries momentum p. Two photons carry the momentum 2p and n photon np.

➔ $\lambda_{effective\ n-photon} = \frac{h}{n\,p}$

Thus a N00N state (see lecture notes 5) has an effective wavelength of λ/n.

Super Resolution

Classically speaking super resolution means higher resolution than the one imposed by the diffraction limit. In interferometers, phase super resolution means that the interference oscillation occurs over a phase smaller than one cycle using classical light. Fig 6 depicts the interferometer oscillations for a classical field (black) and for N00N state n=2 with and without losses. As one can see the visibility is reduced for the case with losses and thus the sensitivity is reduced.

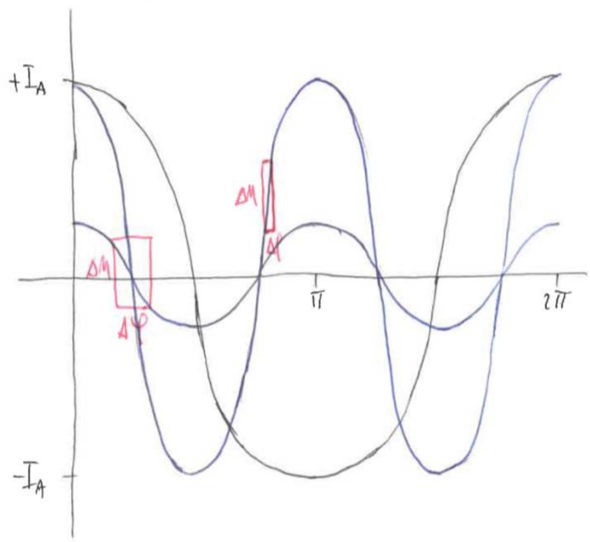

Figure 6: Output of one detector of a Mach-Zehnder Interferometer. Black classical field and black N00N state n=2. With losses the amplitude is reduced.

Super Sensitivity

Phase super sensitivity means that there is a smaller phase uncertainty than achievable with classical resources. Unlike phase super-resolution, phase super sensitivity cannot be determined solely from the fringe pattern but also depends on its visibility *V*.

$\Delta\varphi_{N00N} = \frac{1}{\sqrt{n\,M}}$, where *M* is the detected photons. In an ideal case (all photons detected) this leads to the Heisenberg limit $\Delta\varphi_{N00N} = \frac{1}{n}$.

N00N states are maximally uncertainty states as required by the Heisenberg limit. But photon losses are a problem.

$$\Delta\varphi = \frac{\Delta M}{\delta M/\delta\varphi} = \frac{\Delta M}{I_A\, n\, sin(n\,\varphi)}$$

when there are no losses and thus I_A = n and M = n this leads to the Heisenberg limit 1/n.

Without losses and an ideal interferometer, the visibility would be unity. In general, the visibility of the interference pattern is given by:

$$V = \frac{M_{max} - M_{min}}{M_{max} + M_{min}}$$

and $M = I\,(1 + V\cos(N\varphi)$. Thus to achieve super sensitivity with N00N states the visibility needs to be $V \geq \frac{1}{\sqrt{n}}$ for total efficiency of the experiment of unity. In real world applications the limit is $V = \frac{1}{\sqrt{\varepsilon_p(\varepsilon_i\varepsilon_d)^{n-1}\,n}}$, where ε_p is the generation efficiency, ε_i the transmission efficiency and ε_d the detection efficiency.