Estimation theory
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# Homework # 3

Numbers below refer to problems in Kay's book Fundamentals of Statistical Signal Processing: Estimation Theory

1. Assume we observe data $x(n)$ following the model

$$x(n) = \theta h(n) + w(n)$$

for $n = 0, 1, \ldots, N-1$. Here, $h(n)$ is a known signal while $w(n)$ is zero mean correlated noise. Denote the covariance matrix of the noise vector $[w(0)\ w(1) \ldots w(N-1)]^T$ by $\mathbf{C}$ and assume it is known. Determine the BLUE estimator for the amplitude $\theta$. What happens if the signal vector made from $h(n)$ is an eigenvector of $\mathbf{C}$? What is the minimum variance of $\hat{\theta}$?

2. Consider the problem of estimating the mean of a signal in correlated noise. That is, we have the model

$$x(n) = A + w(n)$$

for $n = 0,\ 1$. The noise $w(n)$ is zero mean with a covariance matrix

$$\mathbf{C} = \mathrm{E}\{\begin{bmatrix} w(0) \\ w(1) \end{bmatrix} \begin{bmatrix} w(0) & w(1) \end{bmatrix}\} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

Derive the BLUE for $A$ and its error variance. Next assume we do not know $\mathbf{C}$ exactly so that, in the design of the estimator, we use

$$\hat{\mathbf{C}} = \begin{bmatrix} 1 & 0.5+\delta \\ 0.5+\delta & 1 \end{bmatrix}$$

in place of the true $\mathbf{C}$. Discuss what happens with the estimator and its performance (bias and variance) for different $\delta$.

3. Problem 7.3

4. Problem 7.9

5. Problem 7.16