Written exam
IE1206 Embedded Electronics
IF1330 Electrical principles
Monday 3/6 2019 14.00-18.00

General Information

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Responsible teacher at exam: Per-Erik Hellström 08-790 43 25

All sheets that are handed in need **your name and personal number** written on them.
Mark every sheet with the **problem it deals** with.
You cannot have more than one problem per sheet.

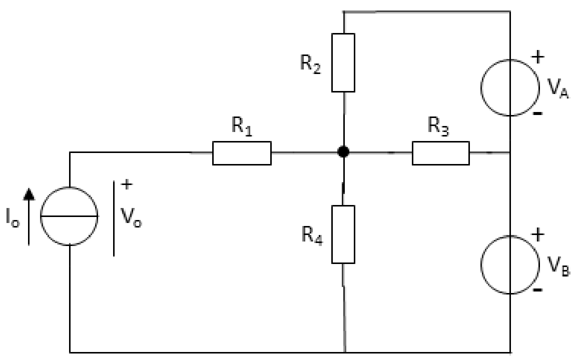
Aids: Calculator

The exam consists of 8 problems (5 points each) distributed over the 4 modules in the course:
Module 1: problem 1 and 2
Module 2: problem 3 and 4
Module 3: problem 5 and 6
Module 4: problem 7 and 8

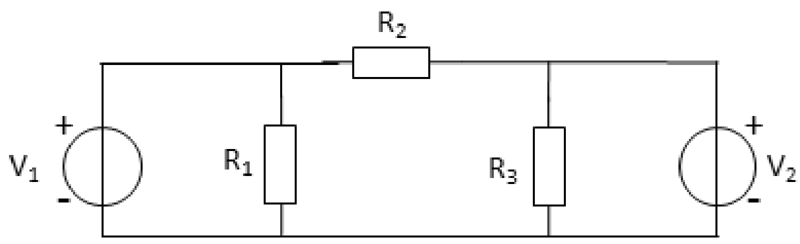
To **pass the exam** requires at least **2 points** from each module and preliminary **20 points** in total.
Grades are given as follows:

Points	<20	20-23	24-27	28-31	32-35	36-40
Grades	F	E	D	C	B	A

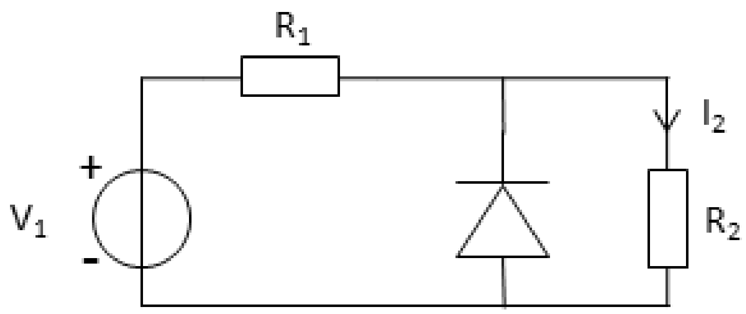
1. Determine the voltage V_o over the current source.
   R_1=1 kΩ, R_2=3 kΩ, R_3=2 kΩ, R_4=6 kΩ, I_o=1 mA, V_A=1 V, V_B=5 V.



2. V_1=2 V, V_2=4 V, R_1=2 kΩ, R_2= 8 kΩ, R_3= 4 kΩ.
   (A) Determine the power consumed in R_2.
   (B) What is the total power delivered by the voltage sources to the resistors?



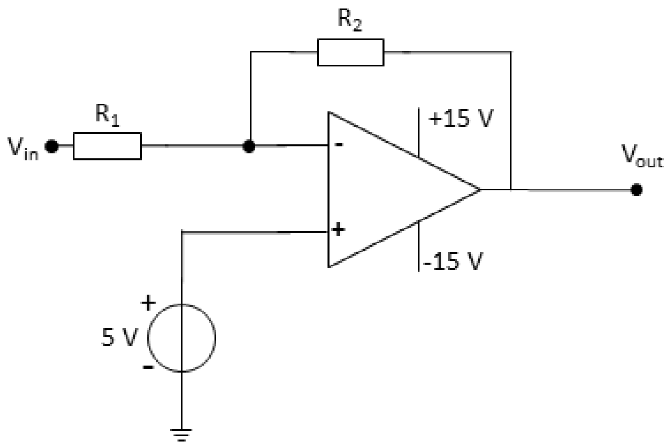
3. Determine the current I_2. V_1=4 V, R_1=10 kΩ and R_2=30 kΩ.



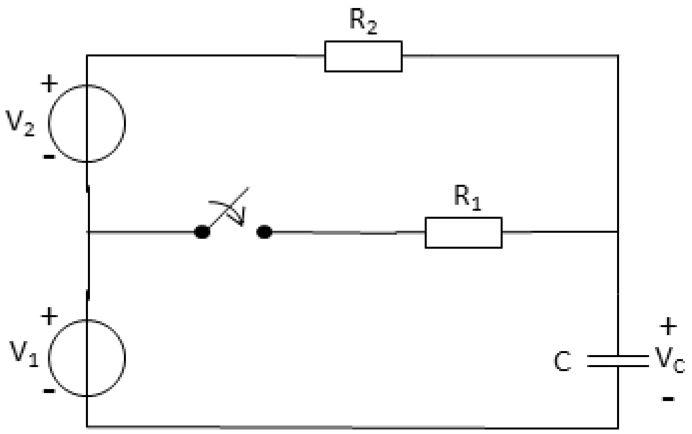
4. Assume that the operational amplifier is ideal.

   (A) Express V_{out} as function of V_{in}, R_1 and R_2 when the operational amplifier works in the linear region.

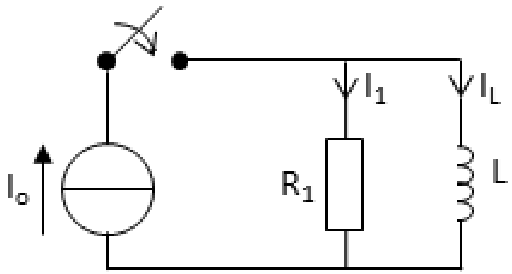
   (B) Plot V_{out} versus V_{in} for -15 V < V_{in} < +15 V when $\frac{R_2}{R_1} = 2$.



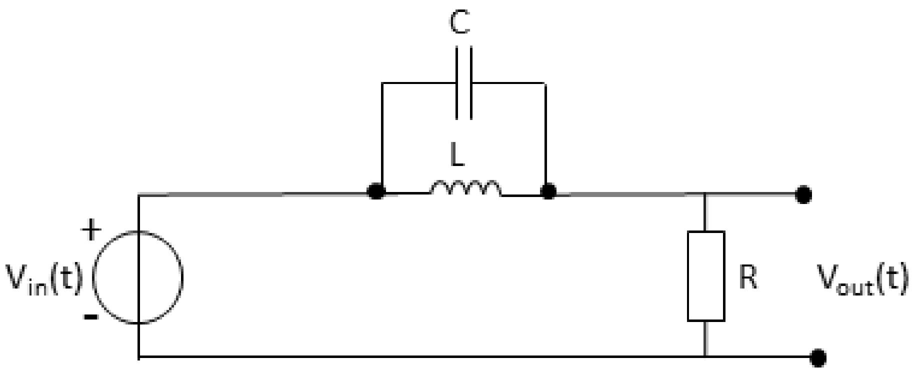
5. The switch has been open for a long time. At t=0 s the switch closes. Determine the voltage V_c over the capacitor at t=6 µs. V_1=1 V, V_2=6 V, R_1=10 kΩ, R_2=15 kΩ and C=1 nF.



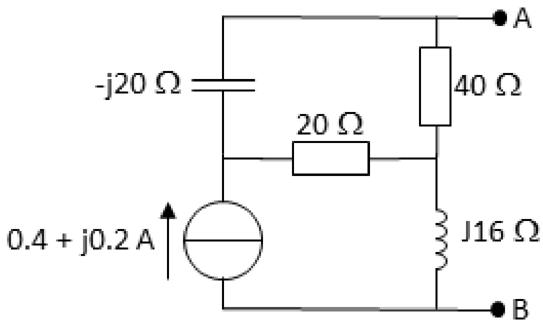
6. The switch has been open for a long time. At t=0 s the switch closes.
   (A) Derive an expression for the current I_1 as a function I_o, R_1, L and t.
   (B) Plot $I_1(t)$ for -3 μs < t < 3 μs when R_1=1 kΩ, L=1 mH, I_o=1 mA.



7. $v_{in}(t)$ is a steady-state cosine voltage source with amplitude A, angular frequency ω and phase angle ϕ.
   (A) Determine v_{out} when the angular frequency $\omega = \frac{1}{\sqrt{LC}}$.
   (B) What type of filter function does the circuit perform? Motivate your answer.



8. Determine the Thévenin equivalent circuit seen at A-B.
   Draw a schematic of the Thévenin equivalent circuit and express all parameters in the time domain when ω=1000 rad/s.



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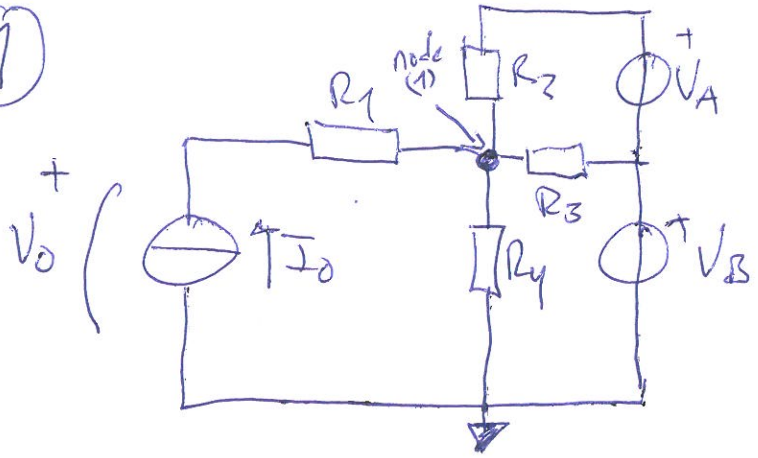


$R_1 = 1\,k\Omega$ $V_A = 1V$

$R_2 = 3\,k\Omega$ $V_B = 5V$

$R_3 = 2\,k\Omega$ $I_0 = 1mA$

$R_4 = 6\,k\Omega$

KCL in node 1: $I_0 + \frac{V_A+V_B-V_1}{R_2} + \frac{V_B-V_1}{R_3} + \frac{0-V_1}{R_4} = 0$

$\Rightarrow V_1\left[\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right] = I_0 + \frac{V_A+V_B}{R_2} + \frac{V_B}{R_3}$

$\Rightarrow V_1\left[\frac{1}{3}+\frac{1}{2}+\frac{1}{6}\right] = 1 + \frac{6}{3} + \frac{5}{2} \Rightarrow V_1[2+3+1] = 6+12+15$

$\Rightarrow V_1 = \frac{33}{6} = 5{,}5\,V$

$V_1 + I_0R_1 = V_0 \Rightarrow V_0 = 5{,}5 + 1\cdot 1 = 6{,}5\,V$

Answer: $V_0 = 6{,}5\,V$

I_2 ~~is~~ ~~the~~ current trough R_2

2

A

$$P_2 = I_2 \cdot V_2 = R_2 I_2^2 = \left\{ I_2 = \frac{V_2 - V_1}{R_2} \right\} = \frac{(V_2 - V_1)^2}{R_2} =$$

$$= \left\{ \begin{array}{l} V_2 = 4V \\ V_1 = 2V \quad R_2 = 8k\Omega \end{array} \right\} = \frac{2^2}{8 \cdot 10^3} = 0{,}5\ mW$$

B Power in resistors = Power in sources.

Power in R_1: $P_1 = \frac{V_1^2}{R_1} = \frac{2^2}{2} = 2\ mW$

Power in R_3: $P_3 = \frac{V_2^2}{R_3} = \frac{4^2}{4} = 4\ mW$

Total power in sources is $P_1 + P_2 + P_3 = \cancel{6{,}5}\ 0{,}5 + 2 + 4 =$

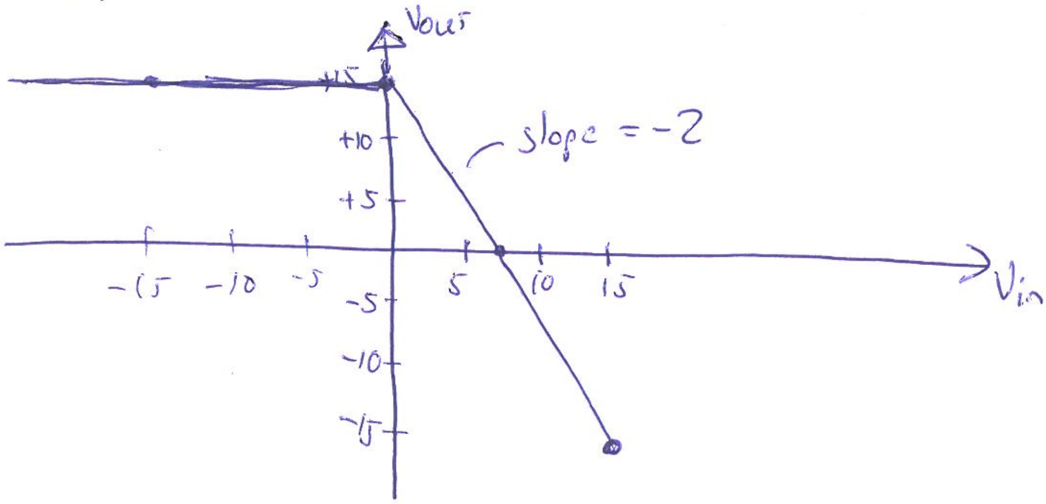
$= 6{,}5\ mW$

4 A

$$\frac{V_{in}-5}{R_1}+\frac{V_{OUT}-5}{R_2}=0 \Rightarrow \frac{V_{OUT}}{R_2}=\frac{5}{R_2}+\frac{5}{R_1}-\frac{V_{in}}{R_1}$$

$$\Rightarrow V_{OUT}=5+\frac{R_2}{R_1}\cdot 5-\frac{R_2}{R_1}V_{in}$$

B $\frac{R_2}{R_1}=2 \Rightarrow V_{OUT}=15-2V_{in}$



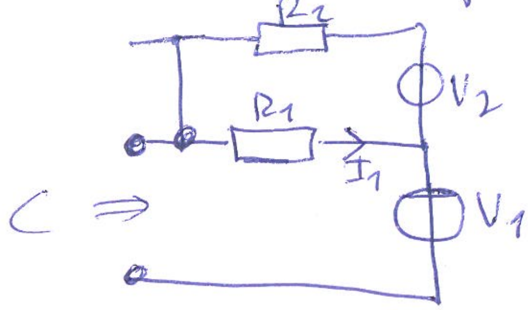
5. $V_c(t) = V_c(\infty) + [V_c(0) - V_c(\infty)] e^{-\frac{t-t_0}{\tau}}$

$t_0 = 0s$

at $t<0s$: The capaciter is chaged to $V_c = V_1 + V_2$

$\Rightarrow V_c(0) = V_1 + V_2 = 1 + 6 = 7V$

at $t>0s$: The capactior sees the circuit:



$V_{TH} = V_1 + I_1 \cdot R_1 = V_1 + \frac{V_2}{R_1+R_2} \cdot R_1 =$

$= 1 + 6 \cdot \frac{10}{25} = 3{,}4\,V$

$R_{TH} = R_1 // R_2 = \frac{10 \cdot 15}{10+15} = 6\,k\Omega$

$V_c(\infty) = V_{TH} = 3{,}4\,V$

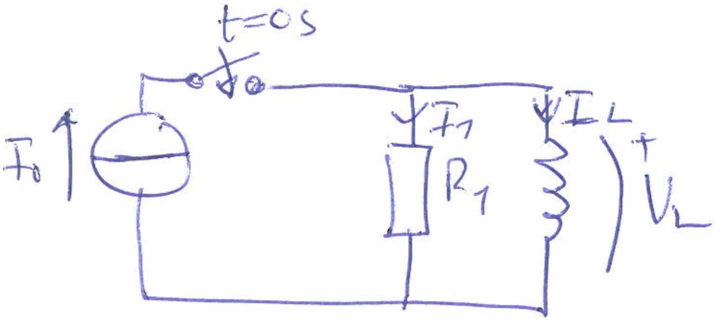
$\tau = R_{TH} \cdot C = 6 \cdot 10^3 \cdot 1 \cdot 10^{-9} = 6\,\mu s.$

$\Rightarrow V_c(t) = 3{,}4 + [7 - 3{,}4] e^{-\frac{t}{6\mu s}} = 3{,}4 + 3{,}6 \cdot e^{-\frac{t}{6\mu s}}$

$V_c(6\mu s) = 3{,}4 + 3{,}6\, e^{-1} = 4{,}7\,V.$

Answer: $V_c(t = 6\mu s) = 4{,}7\,V$

6. A $I_1 = \frac{V_L}{R_1}$ $V_L = L \frac{dI_L}{dt}$



$$I_L = I_L(\infty) + [I_L(0) - I_L(\infty)] e^{-\frac{t-t_0}{\tau}}$$

$$\left(\begin{array}{l} t_0 = 0s \\ \tau = \frac{L}{R_1} = \frac{10^{-3}}{10^3} = 1\mu s \end{array} \right)$$

$I_L(0) = 0 \text{ A}$

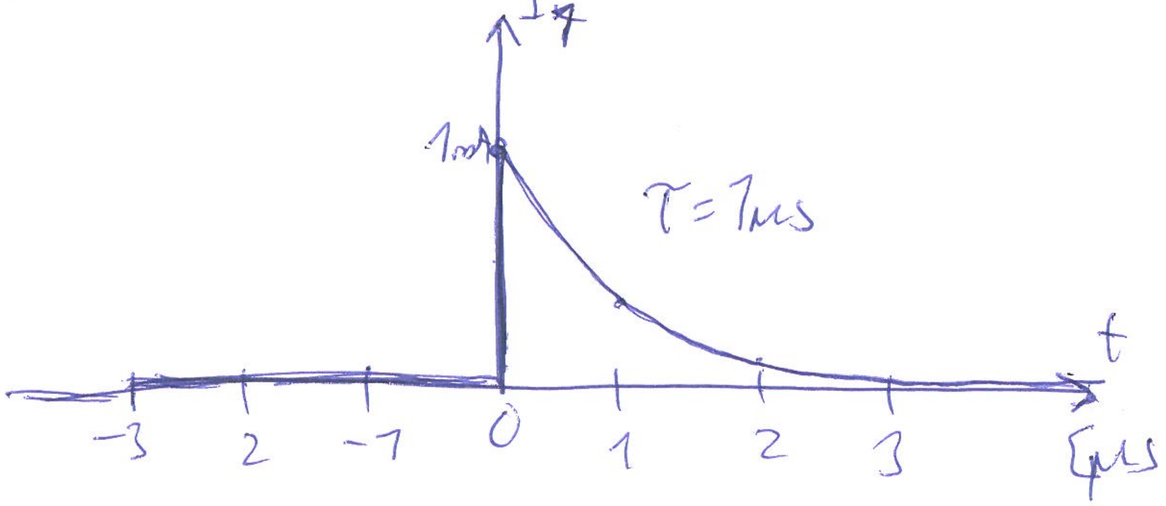
$I_L(\infty) = I_0 \Rightarrow I_L = I_0 \left(1 - e^{-\frac{t}{\tau}}\right)$ $\tau = \frac{L}{R_1}$

$$\Rightarrow V_L = L \cdot \frac{dI_L}{dt} = L\left(-I_0 e^{-\frac{t}{\tau}} \cdot \left(-\frac{1}{\tau}\right)\right) = \frac{L I_0}{\tau} e^{-\frac{t}{\tau}} = \left\{\tau = \frac{L}{R_1}\right\} =$$

$$= R_1 I_0 e^{-\frac{t}{\tau}}$$

$$\Rightarrow \boxed{I_1(t) = \frac{V_L}{R_1} = I_0 e^{-\frac{t}{\tau}}}$$

B $\tau = 1\mu s$ $I_0 = 1mA$



7.

A $\hat{V}_{out} = \hat{V}_{in} \dfrac{R}{Z_1 + R}$ where $Z_1 = j\omega L // \dfrac{1}{j\omega C}$

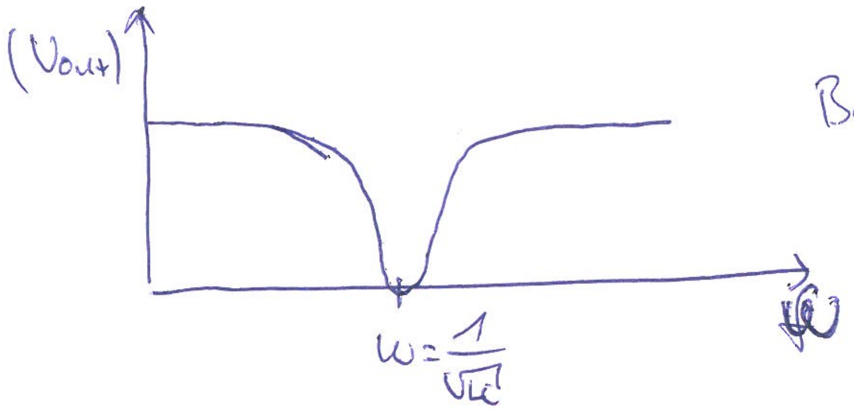
$$Z_1 = \frac{j\omega L \cdot \frac{1}{j\omega C}}{\frac{1}{j\omega C} + j\omega L} = \frac{j\omega L}{1-\omega^2 LC} \qquad Z_1 \to \infty \text{ when } \omega \to \frac{1}{\sqrt{LC}}$$

$\Rightarrow \hat{V}_{out} \to 0$ Answer: $V_{out}(t) = 0$ at $\omega = \dfrac{1}{\sqrt{LC}}$

B

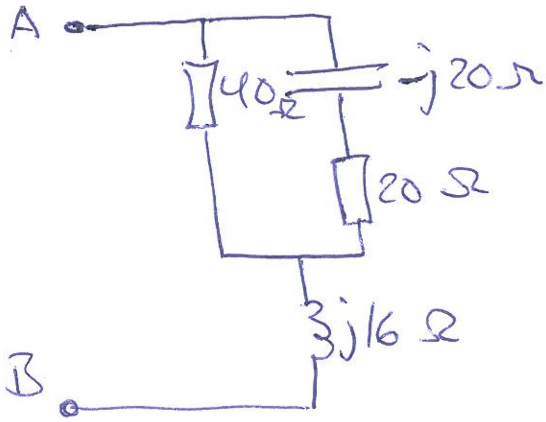
$\omega \to 0 \Rightarrow j\omega L \to 0 \Rightarrow V_{out} = V_{in}$

$\omega \to \infty \Rightarrow \dfrac{1}{j\omega C} \to 0 \Rightarrow V_{out} = V_{in}$



Band-reject filter.

8. <u>Determine Z_{TH}</u>: Zero current source and find $Z_{AB} = Z_{TH}$



$Z_{TH} = Z_1 + j16$ when

$Z_1 = 40 // (20 - j20) =$

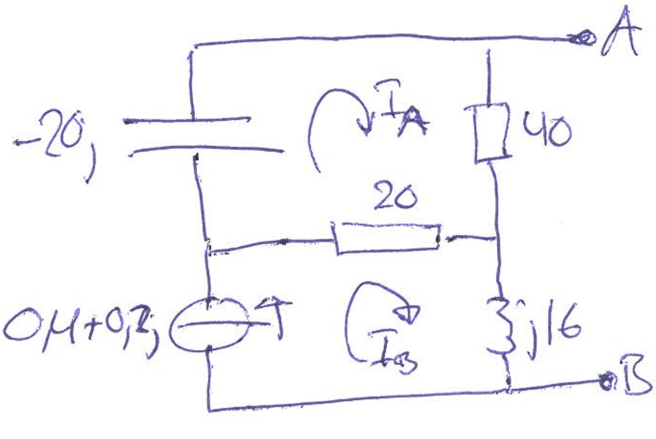
$= \frac{40 \cdot 20(1-j)}{60 - j20} = 40 \frac{1-j}{3-j} =$

$= 40 \frac{(1-j)(3+j)}{9+1} = 4(3+j-3j+1) = 16 - 8j\ \Omega$

$Z_{TH} = 16 - 8j + j16 = 16 + 8j.$

<u>Find V_{TH}</u>

$V_{TH} = V_{AB}$ when open between A-B



$I_B = 0{,}4 + 0{,}2j\ A$

$I_a: -(-j20)I_A - 40I_A - 20(I_A - I_B) = 0$

$j20I_A - 40I_A - 20I_A + 20(0{,}4 + 0{,}2j) = 0$

$-I_A(60 - 20j) + (8 + 4j) = 0 \Rightarrow I_A = \frac{8+4j}{60-20j} =$

$= \frac{2+j}{15-5j} = \frac{(2+j)(15+5j)}{15^2+5^2} = \frac{1}{250}(30 + 10j + 15j - 5) = \frac{25+25j}{250} =$

$= 0{,}1 + 0{,}1j\ A$

cont. →

$$V_{TH} = 40\, I_A + j16\, I_B = 40\,(0{,}1 + 0{,}1j) + 16j\,(0{,}4 + 0{,}2j) =$$

$$= 4 + 4j + 6{,}4j - 3{,}2 = 0{,}8 + 10{,}4j$$

$$\Rightarrow V_{TH} = 10{,}4\angle 85{,}6^\circ \Rightarrow V_{TH} = 10{,}4\cos(\omega t + 85{,}6^\circ)$$