Written exam
IE1206 Embedded Electronics
IF1330 Electrical principles
Monday 3/6 2019 14.00-18.00

General Information
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Responsible teacher at exam: Per-Erik Hellström 08-790 43 25

All sheets that are handed in need your name and personal number written on them.
Mark every sheet with the problem it deals with.
You cannot have more than one problem per sheet.

Aids: Calculator
The exam consists of 8 problems (5 points each) distributed over the 4 modules in the course:
Module 1: problem 1 and 2
Module 2: problem 3 and 4
Module 3: problem 5 and 6
Module 4: problem 7 and 8

To pass the exam requires at least 2 points from each module and preliminary 20 points in total.
Grades are given as follows:

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1. Determine the voltage $V_o$ over the current source.
   $R_1=1 \, k\Omega$, $R_2=3 \, k\Omega$, $R_3=2 \, k\Omega$, $R_4=6 \, k\Omega$, $I_0=1 \, mA$, $V_A=1 \, V$, $V_B=5 \, V$.

2. $V_1=2 \, V$, $V_2=4 \, V$, $R_1=2 \, k\Omega$, $R_2=8 \, k\Omega$, $R_3=4 \, k\Omega$.
   (A) Determine the power consumed in $R_2$.
   (B) What is the total power delivered by the voltage sources to the resistors?

3. Determine the current $I_2$. $V_1=4 \, V$, $R_1=10 \, k\Omega$ and $R_2=30 \, k\Omega$. 
4. Assume that the operational amplifier is ideal.

(A) Express $V_{\text{out}}$ as function of $V_{\text{in}}$, $R_1$ and $R_2$ when the operational amplifier works in the linear region.

(B) Plot $V_{\text{out}}$ versus $V_{\text{in}}$ for $-15 \text{ V} < V_{\text{in}} < +15 \text{ V}$ when $\frac{R_2}{R_1} = 2$.

5. The switch has been open for a long time. At $t=0 \text{ s}$ the switch closes. Determine the voltage $V_c$ over the capacitor at $t=6 \mu\text{s}$. $V_1=1 \text{ V}$, $V_2=6 \text{ V}$, $R_1=10 \text{ k}\Omega$, $R_2=15 \text{ k}\Omega$ and $C=1 \text{ nF}$. 
6. The switch has been open for a long time. At $t=0$ s the switch closes.
   (A) Derive an expression for the current $I_1$ as a function $I_o$, $R_1$, $L$ and $t$.
   (B) Plot $I_1(t)$ for $-3 \mu s < t < 3 \mu s$ when $R_1=1$ k$\Omega$, $L=1$ mH, $I_o=1$ mA.

   ![Image of a circuit diagram with switches and components]

7. $v_{in}(t)$ is a steady-state cosine voltage source with amplitude $A$, angular frequency $\omega$ and phase angle $\phi$.
   (A) Determine $v_{out}$ when the angular frequency $\omega = \frac{1}{\sqrt{LC}}$.
   (B) What type of filter function does the circuit perform? Motivate your answer.

   ![Image of a circuit diagram with a capacitor, inductor, and resistor]

8. Determine the Thévenin equivalent circuit seen at A-B.
   Draw a schematic of the Thévenin equivalent circuit and express all parameters in the time domain when $\omega=1000$ rad/s.

   ![Image of a Thévenin equivalent circuit with components labeled]

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KCL in node 1: \[ I_0 + \frac{V_A + V_B - V_1}{R_2} + \frac{V_B - V_1}{R_3} + \frac{O - V_1}{R_4} = 0 \]

\[ \Rightarrow V_1 \left[ \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right] = I_0 + \frac{V_A + V_B}{R_2} + \frac{V_B}{R_3} \]

\[ \Rightarrow V_1 \left[ \frac{1}{2} + \frac{1}{2} + \frac{1}{6} \right] = 1 + \frac{6}{3} + \frac{5}{2} \Rightarrow V_1 \left[ \frac{5}{2} \right] = 6 + \frac{13}{2} \]

\[ \Rightarrow V_1 = \frac{33}{6} = 5.5 \text{ V} \]

\[ V_1 + I_0 R_4 = V_0 \Rightarrow V_0 = 5.5 + 1.7 = 7.2 \text{ V} \]

\[ \text{Answer: } V_0 = 7.2 \text{ V} \]
$I_2$ is the current through $R_2$

$P_2 = I_2 \cdot V_2 = R_2 \cdot I_2^2 = \left\{ I_2 = \frac{V_2 - V_1}{R_2} \right\} = \left( \frac{V_2 - V_1}{R_2} \right)^2 = \left\{ \begin{align*}
V_2 &= 4V \\
V_1 &= 2V \\
R_2 &= 8 \cdot 10^3 \Omega = \frac{2^2}{8 \cdot 10^3} = 0.5 \text{ mW}
\end{align*} \right.$

B. Power in resistors = Power in sources.

Power in $R_1$: $P_1 = \frac{V_1^2}{R_1} = \frac{2^2}{2} = 2 \text{ mW}$

Power in $R_3$: $P_3 = \frac{V_2^2}{R_3} = \frac{4^2}{4} = 4 \text{ mW}$

Total power in sources: $P_1 + P_2 + P_3 = 0.5 + 2 + 4 = 6.5 \text{ mW}$
4 A  \[ \frac{V_{in} - 5}{R_1} + \frac{V_{out} - 5}{R_2} = 0 \] \[ \Rightarrow \frac{V_{out}}{R_2} = \frac{5}{R_2} \cdot \frac{5}{R_1} - \frac{V_{in}}{R_1} \]

\[ \Rightarrow V_{out} = 5 + \frac{R_2}{R_1} \cdot 5 - \frac{R_2}{R_1} V_{in} \]

4 B  \[ \frac{R_2}{R_1} = 2 \] \[ \Rightarrow V_{out} = 15 - 2V_{in} \]
\[ V_c(t) = V_c(\infty) + \left[ V_c(0) - V_c(\infty) \right] e^{-\frac{t-t_0}{C}} \]

at \( t < 0 \text{ s} \): The capacitor is charged to \( V_c = V_1 + V_2 \)
\[ \Rightarrow V_c(0) = V_1 + V_2 = 1 + 6 = 7 \text{ V} \]

at \( t > 0 \text{ s} \): The capacitor sees the circuit:

\[ C \Rightarrow \quad V_{TH} = V_1 + I_1 \cdot R_1 = V_1 + \frac{V_2}{R_1 \cdot R_2} = \]
\[ = 1 + 6 \cdot \frac{10}{25} = 3.4 \text{ V} \]
\[ R_{TH} = \frac{R_1}{R_2} = \frac{10 \cdot 1.5}{10 + 1.5} = 0.625 \]

\[ V_c(\infty) = V_{TH} = 3.4 \text{ V} \]
\[ C = R_{TH} \cdot C = 6 \cdot 10^3 \cdot 1 \cdot 10^{-9} = 6 \mu \text{F} \]

\[ \Rightarrow V_c(t) = 3.4 + [7 - 3.4] e^{-\frac{t}{6\mu \text{s}}} = 3.4 + 3.6 e^{-\frac{t}{6\mu \text{s}}} \]
\[ V_c(6 \mu \text{s}) = 3.4 + 3.6 e^{-\frac{6}{6\mu \text{s}}} = 4.7 \text{ V} \]

Answer: \( V_c(t=6 \mu \text{s}) = 4.7 \text{ V} \)
6. \( A \)

\[ I_1 = \frac{V_L}{R_1} \quad V_L = L \frac{dI_1}{dt} \]

\[ I_L = I_L(\infty) + [I_L(0) - I_L(\infty)]e^{-\frac{t-t_0}{\tau}} \quad \begin{cases} t_0 = 0.5 \\ \tau = \frac{L}{R_1} = \frac{10^{-3}}{10^{-3}} = 1 \mu s \end{cases} \]

\( I_L(0) = 0 \ A \)

\( I_L(\infty) = I_0 \Rightarrow I_L = I_0 \left(1 - e^{-\frac{t}{\tau}}\right) \quad \tau = \frac{L}{R_1} \)

\[ V_L = L \frac{dI_L}{dt} = L \left(-I_0 e^{-\frac{t}{\tau}} \cdot (-1)\right) = \frac{L I_0}{\tau} e^{-\frac{t}{\tau}} = \left(\tau = \frac{1}{\tau_1}\right) \]

\[ = R_1 I_0 e^{-\frac{t}{\tau}} \]

\[ I_1(t) = \frac{V_L}{R_1} = I_0 e^{\frac{t}{\tau}} \]

\( B \)

\[ \tau = 1 \mu s \quad I_0 = 1 \text{mA} \]
\( V_{out} = \hat{V}_{in} \frac{R}{Z_1 + R} \) where \( Z_1 = \frac{jwL}{jw} \frac{1}{jwc} + jwL \)

\[ Z_1 = \frac{jwL}{1 - w^2LC} \]

\( Z_1 \to \infty \) when \( w \to \frac{1}{\sqrt{LC}} \)

\( \Rightarrow V_{out} \to 0 \)

**Answer:** \( V_{out}(t) = 0 \) at \( w = \frac{1}{\sqrt{LC}} \)

\( \omega \to 0 \Rightarrow jwL \to 0 \Rightarrow V_{out} = V_{in} \)

\( \omega \to \infty \Rightarrow \frac{1}{jwc} \to 0 \Rightarrow V_{out} = V_{in} \)

**Band-reject filter.**
8. **Determine** $Z_{TH}$: Zero current source and find $Z_{AB} = Z_{TH}$

$Z_{TH} = Z_1 + j16$ where

$Z_1 = \frac{Y_0}{(20-j20)} = \frac{40 \cdot 20\ (1-j)}{60-j20} = \frac{40 \ 1-j}{3+j}$

$= \frac{40 \ (1-j)(3+j)}{9+1} = 4 \ (3+j-3j-j) = 16 - 8j \ \Omega$

$Z_{TH} = 16 - 8j + j16 = 16 + 8j \ \Omega$

**Find** $V_{TH}$

$V_{TH} = V_{AB}$ when open between A-B

$I_B = 0H + 0I_2j \ \ A$

$I_A = (-j20)I_A - y_0 I_A - 20(I_A - I_B) = 0$

$j20I_A - y_0 I_A - 20I_A + 20(0H + 0I_2j) = 0$

$-I_A (60 - 20j) + (8 + 4j) = 0 \Rightarrow I_A = \frac{8 + 6j}{60 - 20j}$

$= \frac{2 + j}{15 - 5j} = \frac{(2+j)(15+5j)}{15^2 + 5^2} = \frac{1}{250} (30 + 10j + 15j - 5) = \frac{25 + 25j}{250}$

$= 0.1 + 0.1j \ \ A$

Cont. →
\[ V_{TH} = U_0 (I_A + j 16 I_B) = U_0 (0.11 + 0.1 j) + 16 j (0.4 + 0.2 j) = 0.4 + 4 j + 6.4 j - 3.2 = 0.8 + 10.4 j \]

\[ \Rightarrow V_{TH} = 10.4 \angle 85.6^\circ \Rightarrow V_{TH} = 10.4 \cos (\omega t + 85.6^\circ) \]

\[ R = 16 \Omega \quad L = 8 mH \]