Estimation theory
Magnus Jansson
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Homework # 5

Numbers below refer to problems in Kay's book Fundamentals of Statistical Signal Processing: Estimation Theory

1. Consider the data model

$$x(n) = A\cos(2\pi f_0 n) + w(n); \quad n = 0, 1, \ldots, N-1$$

   where the frequency f_0 is known. Here, the noise $w(n)$ is iid $\mathcal{N}(0, \sigma^2)$. Further assume the prior $A \sim \mathcal{N}(0, \sigma_A^2)$ and independent of the noise.

   (a) Derive the (Bayesian) MMSE estimator of the amplitude A.

   (b) What is the minimum Bayesian MSE?

   (c) Discuss qualitatively the influence of the prior.

2. Problem 10.10

3. Assume we have the posterior density function

$$p(\theta|x) = \frac{\alpha}{\sqrt{2\pi}} \exp\{(\theta - x)^2/2\} + \frac{1-\alpha}{\sqrt{2\pi}} \exp\{(\theta - 2x)^2/2\}$$

   that is, a Gaussian mixture. Now assume $\alpha = 1/2$, plot the PDF and derive the MMSE as well as the MAP estimators. Then, let $\alpha = 1/4$ and repeat the exercise. Comment on the results.

4. Problem 11.4

5. Problem 11.11