Rigid Body Dynamics (SG2150)
Exam, 2019-10-21, 14.00-18.00

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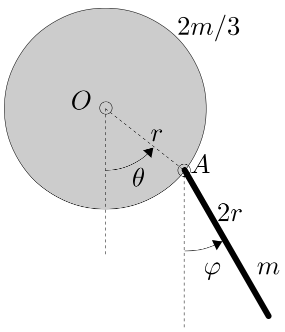
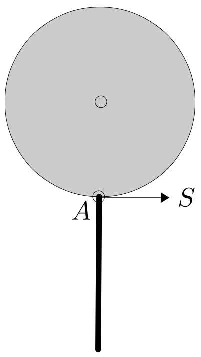
Each problem gives a maximum of 3 points, so that the total maximum is 18. Grading: 1–3 F; 4–5 FX; 6: E; 7–9 D; 10–12 C; 13–15 B; 16–18 A.

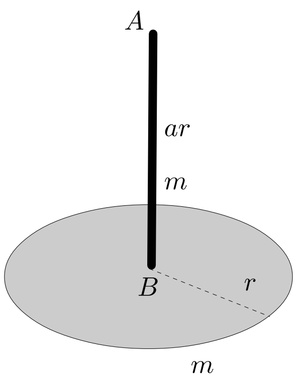
Allowed equipment: Handbook of mathematics and physics. One one-sided A4 page with your own compilation of formulae.



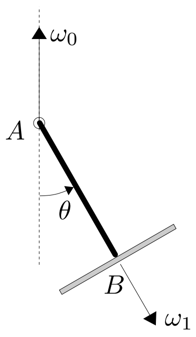
Problem 1. A thin homogeneous circular disc of mass $2m/3$ and radius r can rotate freely in a vertical plane about the fixed horizontal axis through O at the centre of the disc. A thin homogeneous rod of mass m and length $2r$ can rotate freely in a vertical plane about a parallel axis through the point A, which is on the perimeter of the disc and at one end point of the rod. Find a stable equilibrium solution for the system, and compute the angular frequencies of small oscillations about that equilibrium.



Problem 2. The system of Problem 1 is resting with $\theta = 0$, $\varphi = 0$. An impulse S to the right is applied at the point A. Compute the angular velocities of the two bodies immediately after the impulse is applied.



Problem 3. To the centre B of a homogeneous circular disc of mass m and radius r, one end point of a thin homogeneous rod of mass m and length ar is rigidly attached, where a is a parameter. The rod is perpendicular to the disc. Compute all three principal moments of inertia about the other end point A for the composite rigid body. Show that the value of a can be chosen to make all principal moments of inertia have equal values.



Problem 4. The body of Problem 3 is suspended from a smooth ball joint at the point A. The motion is such that the angular velocity of the body consists of one part about the upward direction with constant component ω_0, and a second part about the symmetry axis $\boldsymbol{r}_{AB}$ with constant component ω_1. The angle θ between the symmetry axis and the downward direction is also constant. Show that this motion is consistent with the equations of motion provided a certain relation between ω_0, ω_1, and θ is fulfilled. Also comment on how this relation simplifies for the particular value of a discussed in Problem 3.

Problem 5. The Hamilton function of a particular mechanical system with one degree of freedom is

$$H(q,p) = \frac{p^2}{2m}.$$

Write down Hamilton's equations for this system. Given the initial conditions

$$q(0) = q_0, \quad p(0) = p_0,$$

solve the equations to find $q(t)$, $p(t)$. Try to give an example of a concrete system having this Hamilton function.

Problem 6. When studying satellite motion in polar coordinates r and θ, one finds the kinetic and potential energy to be

$$T = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right), \quad V = -\frac{mGM}{r}.$$

Find two non-trivial expressions in $(r, \theta, \dot{r}, \dot{\theta})$ that are constant during the motion. Further, if it is known that the minimal and maximal values of $r(t)$ are

$$r_{\min} = r_0, \quad r_{\max} = 2r_0,$$

find the (constant) values of the two expressions.