

# IE1204 Exam 20200114 Answers

## Part 1

1

$$A = 54_{10} = 00110110_2 = 36_{16}$$

$$B = 31_{10} = 00011111_2 = 1F_{16}$$

$$A + B = 01010101_2 = 55_{16} = 85_{10}$$

2

$$C = 01011101_2 = 93_{10} = 5D_{16}$$

$$D = 10100011_2 = -93_{10} = A3_{16}$$

$$-D = 01011101_2 = 93_{10} = 5D_{16}$$

$$C - D = 10111010_2 = BA_{16} = 186_{10} = -70_{10}$$

Calculation is correct if you realize that an overflow occurs since the result is greater than 127

3

$$\begin{aligned} Y &= \overline{(A \oplus B) + (C + D)} + \overline{B \oplus D} \cdot \overline{A \cdot C} = \overline{A \oplus B} \cdot \overline{C + D} + \overline{B \oplus D} \cdot (\overline{A} + \overline{C}) \\ &= (A \cdot B + \overline{A} \cdot \overline{B}) \cdot \overline{C} \cdot \overline{D} + (B \cdot D + \overline{B} \cdot \overline{D})(\overline{A} + \overline{C}) \\ &= A \cdot B \cdot \overline{C} \cdot \overline{D} + \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D} + \overline{A} \cdot B \cdot D + \overline{A} \cdot \overline{B} \cdot \overline{D} + B \cdot \overline{C} \cdot D + \overline{B} \cdot \overline{C} \cdot \overline{D} \end{aligned}$$

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

Format:

C, D

	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	1	1	0	0
10	1	0	0	0

$\overline{A}B\overline{D} + \overline{A}BD + A\overline{C}\overline{D} + ABC\overline{D}$

Format:

C, D

	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	1	1	0	0
10	1	0	0	0

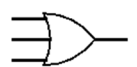
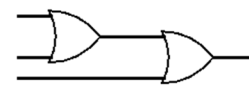
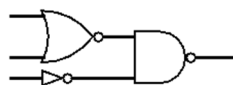
$(B + \overline{D})(A + \overline{B} + D)(\overline{A} + \overline{C})$

Several simplified solutions can be made

4

$$Y1 = \overline{A + B} = \overline{A} \cdot \overline{B}$$

$$Y = \overline{Y1 \cdot C} = \overline{\overline{A} \cdot \overline{B} \cdot \overline{C}} = A + B + C$$



Or use bubble pushing

Part 2

5

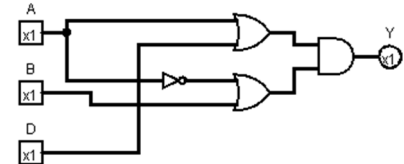
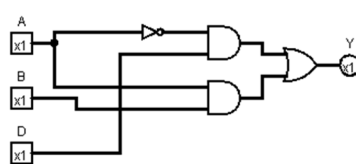
Output:

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$AB\bar{A}CD\bar{A}\bar{B}D\bar{A}B\bar{C}D$

$$Y = A \cdot B + \bar{A} \cdot C \cdot D + \bar{A} \cdot \bar{B} \cdot D + \bar{A} \cdot B \cdot \bar{C} \cdot D$$

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1



Format:

C, D

	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	1	1	1	1
10	0	0	0	0

A, B

$\bar{A}D + AB$

Format:

C, D

	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	1	1	1	1
10	0	0	0	0

A, B

$(A + D)(\bar{A} + B)$

6

Format:

C, D

	00	01	11	10
00	x	0	1	x
01	0	1	1	x
11	1	0	x	1
10	1	0	0	1

A, B

$\bar{A}C + \bar{A}BD + A\bar{D}$

Format:

C, D

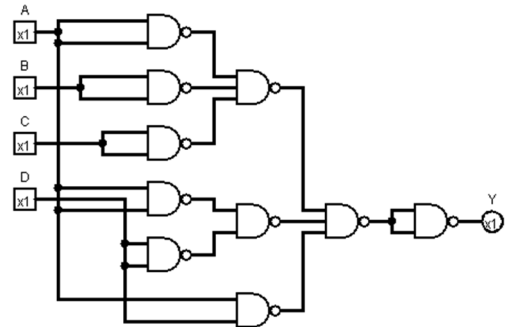
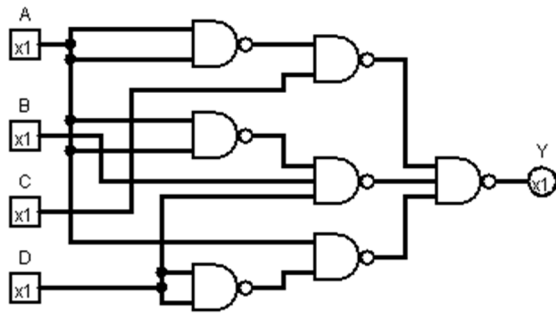
	00	01	11	10
00	x	0	1	x
01	0	1	1	x
11	1	0	x	1
10	1	0	0	1

A, B

$(A + B + C)(A + D)(\bar{A} + \bar{D})$

SOP is better for only NAND gates, use Bubble-pushing

One point was deducted if the expression was not simplest possible.  $\bar{B} \cdot \bar{D}$  should not be included.

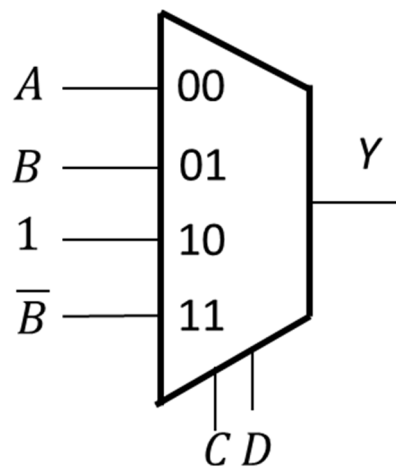


7

Format:  ▼

		C, D			
		00	01	11	10
A, B	00	0	0	1	1
	01	0	1	0	1
	11	1	1	0	1
	10	1	0	1	1

$\bar{B}C + C\bar{D} + B\bar{C}D + A\bar{D}$



(Boolean expression not needed for full points)

8

Q3	Q2	Q1	Q0	A	B	C	D	E	F	G
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	1	1	1	0
1	1	0	1	0	1	1	1	1	0	1
1	1	1	0	1	0	0	1	1	1	1
1	1	1	1	1	0	0	0	1	1	1

Three segments of seven (A-G) for full points

Output:

Format:

**Q1, Q0**

	00	01	11	10
00	1	0	1	1
01	0	1	1	1
11	1	0	1	1
10	1	1	0	1

$\overline{Q2} \overline{Q0} + \overline{Q3} Q1 + \overline{Q3} Q2 Q0$   
 $+ Q2 Q1 + Q3 \overline{Q2} \overline{Q1} + Q3 \overline{Q0}$

Output:

Format:

**Q1, Q0**

	00	01	11	10
00	1	0	1	1
01	0	1	1	1
11	1	0	1	1
10	1	1	0	1

$(Q3 + Q2 + Q1 + \overline{Q0})(Q3 + \overline{Q2}$   
 $+ Q1 + Q0)(\overline{Q3} + Q2 + \overline{Q1}$   
 $+ \overline{Q0})(\overline{Q3} + \overline{Q2} + Q1 + Q0)$

Output:

Format:

**Q1, Q0**

	00	01	11	10
00	1	1	1	1
01	1	0	1	0
11	0	1	0	0
10	1	1	0	1

$(Q3 + \overline{Q2} + Q1 + \overline{Q0})(\overline{Q2} + \overline{Q1}$   
 $+ Q0)(\overline{Q3} + \overline{Q1} + \overline{Q0})(\overline{Q3}$   
 $+ \overline{Q2} + Q0)$

Output:

Format:

**Q1, Q0**

	00	01	11	10
00	1	1	1	0
01	1	1	1	1
11	0	1	0	0
10	1	1	1	1

$\overline{Q3} \overline{Q1} + \overline{Q3} Q0 + \overline{Q1} Q0$   
 $+ \overline{Q3} Q2 + Q3 \overline{Q2}$

Output:

Format:

**Q1, Q0**

	00	01	11	10
00	1	1	1	0
01	1	1	1	1
11	0	1	0	0
10	1	1	1	1

$(Q3 + Q2 + \overline{Q1} + Q0)(\overline{Q3} + \overline{Q2}$   
 $+ Q0)(\overline{Q3} + \overline{Q2} + \overline{Q1})$

Output:

Format:

**Q1, Q0**

	00	01	11	10
00	1	0	1	1
01	0	1	0	1
11	1	1	0	1
10	1	1	1	0

$\overline{Q3} \overline{Q2} \overline{Q0} + \overline{Q2} Q1 Q0$   
 $+ Q2 \overline{Q1} Q0 + Q2 Q1 \overline{Q0}$   
 $+ Q3 \overline{Q1}$

Output:

Format:

**Q1, Q0**

	00	01	11	10
00	1	0	1	1
01	0	1	0	1
11	1	1	0	1
10	1	1	1	0

$(Q3 + Q2 + Q1 + \overline{Q0})(Q3 + \overline{Q2}$   
 $+ Q1 + Q0)(\overline{Q2} + \overline{Q1}$   
 $+ \overline{Q0})(\overline{Q3} + Q2 + \overline{Q1} + Q0)$

Output:

Format:

**Q1, Q0**

	00	01	11	10
00	1	0	0	1
01	0	0	0	1
11	1	1	1	1
10	1	0	1	1

$\overline{Q2} \overline{Q0} + Q1 \overline{Q0} + Q3 Q1$   
 $+ Q3 Q2$

Output:

Format:

**Q1, Q0**

	00	01	11	10
00	1	0	0	1
01	0	0	0	1
11	1	1	1	1
10	1	0	1	1

$(Q_3 + \overline{Q_0})(Q_2 + Q_1 + \overline{Q_0})(Q_3 + \overline{Q_2} + Q_1)$

Output:

Format:

**Q1, Q0**

	00	01	11	10
00	1	0	0	0
01	1	1	0	1
11	1	0	1	1
10	1	1	1	1

$\overline{Q_1} \overline{Q_0} + \overline{Q_3} Q_2 \overline{Q_1} + Q_2 \overline{Q_0} + Q_3 \overline{Q_2} + Q_3 Q_1$

Output:

Format:

**Q1, Q0**

	00	01	11	10
00	1	0	0	0
01	1	1	0	1
11	1	0	1	1
10	1	1	1	1

$(Q_3 + Q_2 + \overline{Q_0})(Q_3 + Q_2 + \overline{Q_1})(Q_3 + \overline{Q_1} + \overline{Q_0})(Q_3 + \overline{Q_2} + Q_1 + \overline{Q_0})$

Output:

Format:

**Q1, Q0**

	00	01	11	10
00	0	0	1	1
01	1	1	0	1
11	0	1	1	1
10	1	1	1	1

$\overline{Q_2} Q_1 + Q_1 \overline{Q_0} + \overline{Q_3} Q_2 \overline{Q_1} + Q_3 \overline{Q_2} + Q_3 Q_0$

Output:

Format:

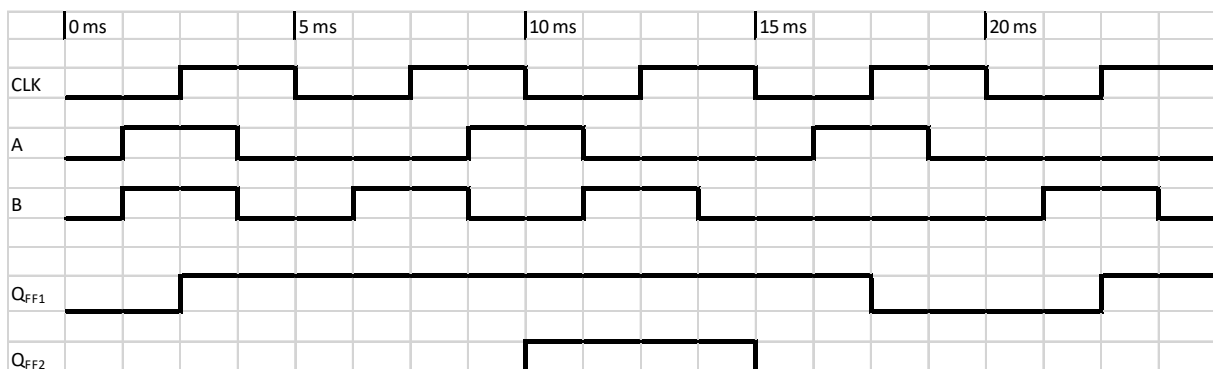
**Q1, Q0**

	00	01	11	10
00	0	0	1	1
01	1	1	0	1
11	0	1	1	1
10	1	1	1	1

$(Q_3 + Q_2 + Q_1)(Q_3 + \overline{Q_2} + \overline{Q_1} + \overline{Q_0})(Q_3 + \overline{Q_2} + Q_1 + Q_0)$

### Part 3

9

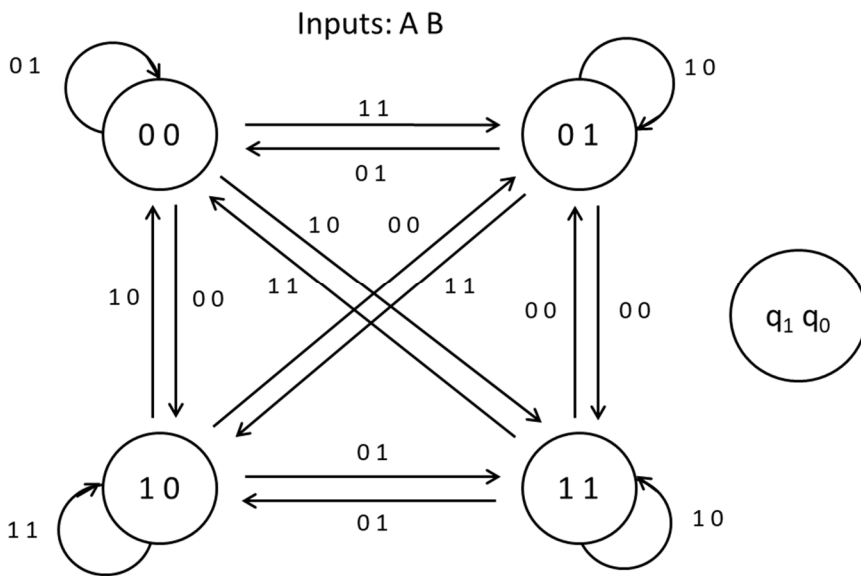


10

q1+	A B =			
	00	01	11	10
q1q0 = 00	1	0	0	1
01	1	0	1	0
11	0	1	0	1
10	0	1	1	0

q0+	A B =			
	00	01	11	10
q1q0 = 00	0	0	1	1
01	1	0	0	1
11	1	0	0	1
10	1	1	0	0

Present state		Next state							
		A B = 00		A B = 01		A B = 11		A B = 10	
q1	q0	q1+	q0+	q1+	q0+	q1+	q0+	q1+	q0+
0	0	1	0	0	0	0	1	1	1
0	1	1	1	0	0	1	0	0	1
1	1	0	1	1	0	0	0	1	1
1	0	0	1	1	1	1	0	0	0



11

Present state			Next state			Output
c	b	a	c+	b+	a+	y
0	0	0	X	X	X	X
0	0	1	0	1	0	0
0	1	0	0	1	1	1
0	1	1	1	0	0	1
1	0	0	1	0	1	0
1	0	1	1	1	0	1
1	1	0	1	1	1	0
1	1	1	0	0	1	1

Output:

Format:

**b, a**

		00	01	11	10
<b>c</b>	0	x	0	1	1
	1	0	1	1	0

$\bar{c}b + ca$

Output:

Format:

**b, a**

		00	01	11	10
<b>c</b>	0	x	0	1	0
	1	1	1	0	1

$\bar{c}ba + c\bar{b} + c\bar{a}$

Output:

Format:

**b, a**

		00	01	11	10
<b>c</b>	0	x	1	0	1
	1	0	1	0	1

$\bar{b}a + b\bar{a}$

Output:

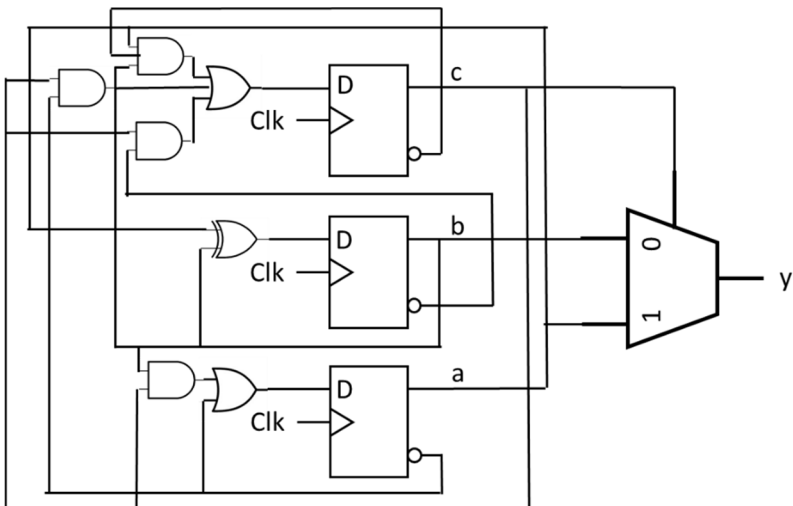
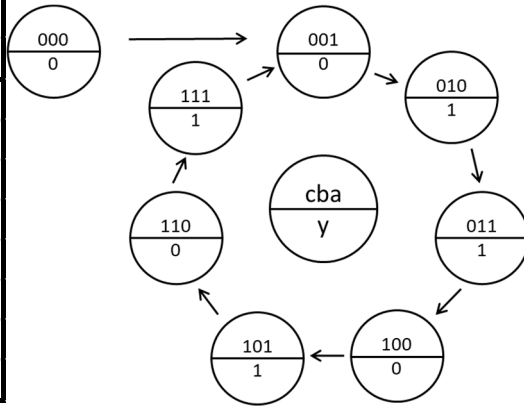
Format:

**b, a**

		00	01	11	10
<b>c</b>	0	x	0	0	1
	1	1	0	1	1

$\bar{a} + cb$

Present state			Next state			Output
c	b	a	c+	b+	a+	y
0	0	0	0	0	1	0
0	0	1	0	1	0	0
0	1	0	0	1	1	1
0	1	1	1	0	0	1
1	0	0	1	0	1	0
1	0	1	1	1	0	1
1	1	0	1	1	1	0
1	1	1	0	0	1	1

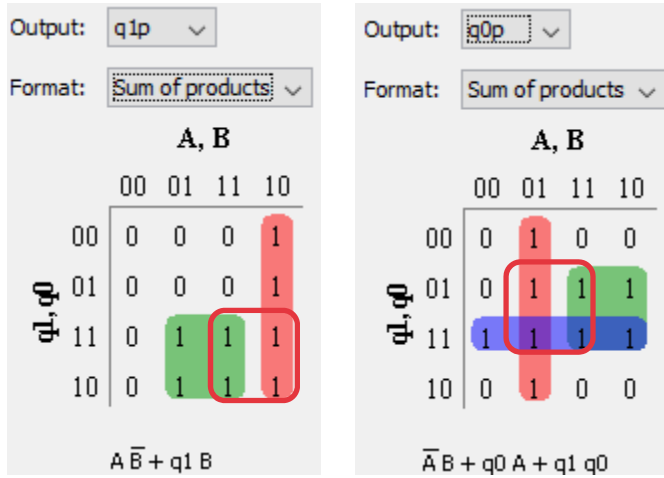


(not necessary to draw the FSM)

12

$$q_1^+ = q_1 \cdot B + A \cdot \bar{B}$$

$$q_0^+ = q_0 \cdot A + \bar{A} \cdot B + q_1 \cdot q_0$$



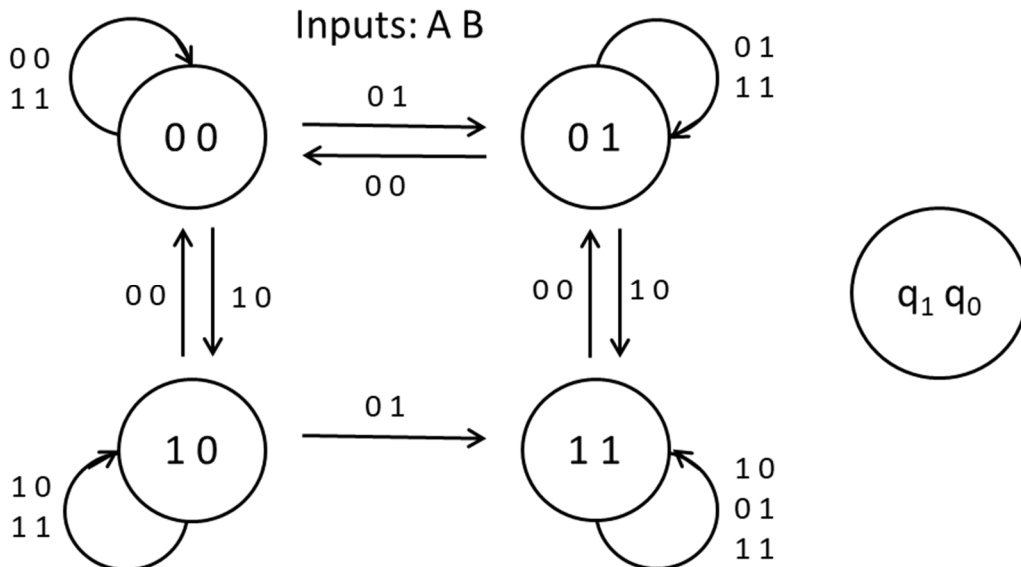
For glitch free expressions add one term each:

$$q_1^+ = q_1 \cdot B + A \cdot \bar{B} + q_1 \cdot A$$

$$q_0^+ = q_0 \cdot A + \bar{A} \cdot B + q_1 \cdot q_0 + q_0 \cdot B$$

Present state		Next state							
		A B = 00		A B = 01		A B = 11		A B = 10	
q1	q0	q1+	q0+	q1+	q0+	q1+	q0+	q1+	q0+
0	0	0	0	0	1	0	0	1	0
0	1	0	0	0	1	0	1	1	1
1	1	0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	0	1	0

No forbidden states





**Part 4**

**13**

Add 4 1-bit full adders in each row, and add 4 rows.  
 In total 56 1-bit full adders and 64 AND gates.  
 The output will have 16 bits.

**14**

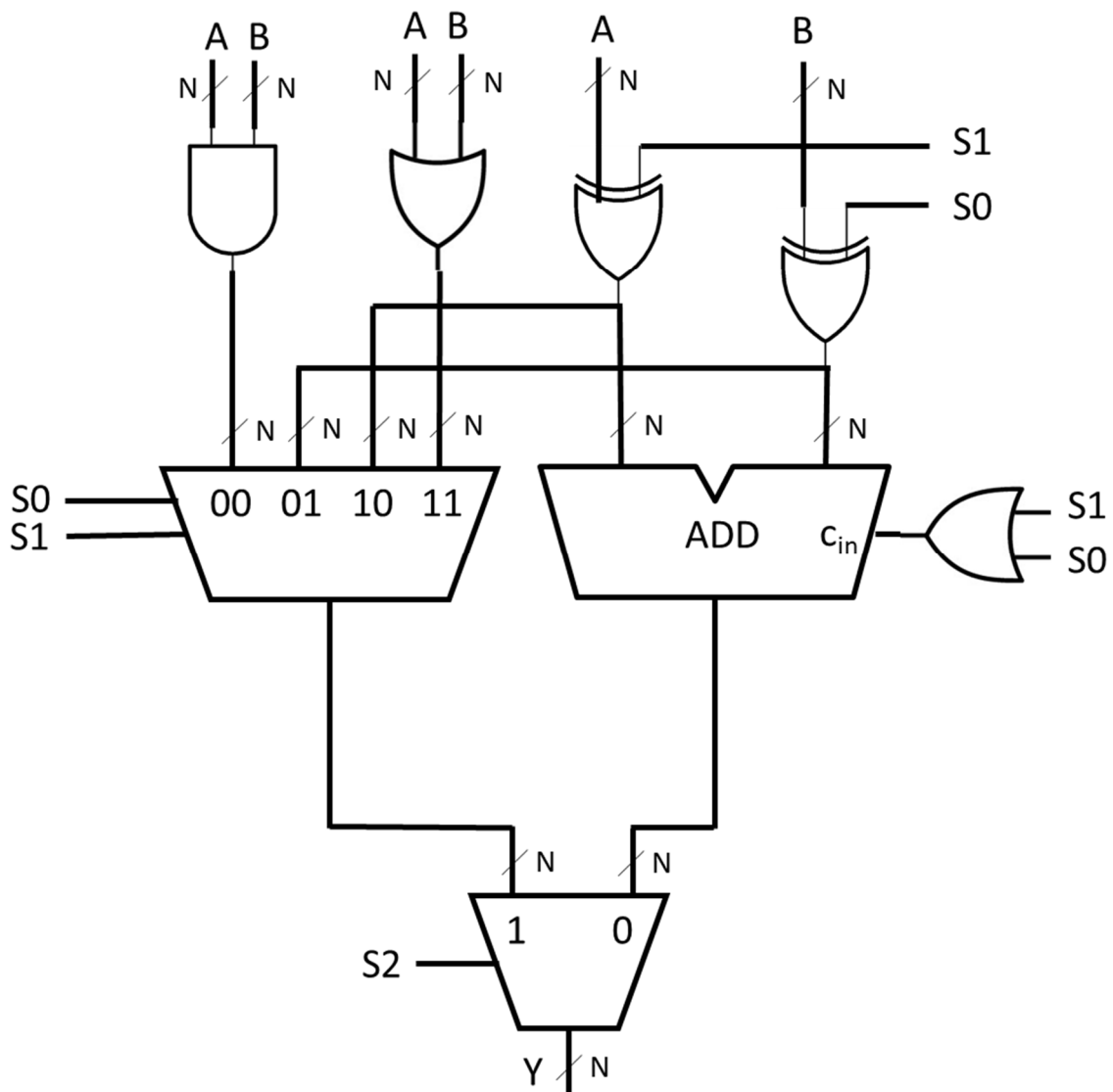
$E = 01100110_2 = 102_{10}$

$F = 00010001_2 = 17_{10}$

$P = E \times F = 0000\ 0110\ 1100\ 0110_2 = 6 \times 256 + 12 \times 16 + 6 = 1734_{10} = 102 \times 17$

$Q = E / F = 00000110_2 = 6_{10} = 102 / 17$

**15**



Other solutions are also possible

