SF2812 Applied linear optimization, final exam
Tuesday June 2 2020 8.00–13.00

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Allowed tools: Pen/pencil, ruler and eraser. *Note!* Calculator is not allowed.

Solution methods: Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Motivate your conclusions carefully. If you use methods other than what have been taught in the course, you must explain carefully.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each page.

22 points are sufficient for a passing grade. For 20-21 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam are announced.

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1. Consider the problem of minimizing $c^T x$ over a polyhedron P. Prove the following:

(a) A feasible solution x is optimal if and only if $c^T d \geq 0$ for every feasible direction d at x. ....(5p)

(b) A feasible solution x is the unique optimal solution if and only if $c^T d > 0$ for every nonzero feasible direction d at x. ....(5p)

2. Consider the following IP:

$$\begin{array}{ll} \underset{x}{\text{minimize}} & 7x_1 + x_2 \\ \text{subject to} & -3x_1 + 2x_2 \leq 5, \\ & 5x_1 + 8x_2 \geq 20, \\ & 20x_1 + x_2 \leq 80, \\ & x \geq 0, \; x \in \mathbb{Z}^2. \end{array}$$

(a) Solve the relaxed ($x \in I\!R^2$) problem and obtain the optimal solution. You need not use a systematic method to solve the linear program. It may for example be solved graphically. ....(3p)

(b) Using branch-and-bound, obtain the optimal integer solution. You need not use a systematic method to solve the linear programs that arise. They may for example be solved graphically. ....(7p)

3. Consider the following LP:

$$\begin{array}{ll} \underset{x \in I\!R^2}{\text{minimize}} & x_1 + 2x_2 \\ \text{subject to} & 2x_1 + 3x_2 \geq 5, \\ & 9x_1 + x_2 \leq 10, \\ & x \geq 0. \end{array}$$

(a) For a fixed positive barrier parameter μ, formulate the primal-dual system of nonlinear equations corresponding to the problem above. Use the fact that the problem is small to give explicit expressions for the solution $x(\mu)$, $y(\mu)$ and $s(\mu)$ to the system of nonlinear equations. .... (7p)

Hint: You may for example find an explicit expression for $y(\mu)$ and then express $x(\mu)$ and $s(\mu)$ in terms of this $y(\mu)$.

(b) Calculate optimal solutions to (LP) and the corresponding dual problem by letting $\mu \to 0$ in the expressions for $x(\mu)$, $y(\mu)$ and $s(\mu)$ that you derived in Question 3a. Verify optimality. .... (3p)

4. Consider the stochastic program (P) given by

$$(P) \quad \begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & Ax = b, \\ & T(\omega)x = h(\omega), \\ & x \geq 0, \end{array}$$

where ω is a stochastic variable and $T(\omega)x = h(\omega)$ is to be interpreted as an "informal" stochastic constraint. Assume that ω takes on a finite number of values $\omega_1, \ldots, \omega_N$ with corresponding probabilities $p_1, \ldots, p_N$. Let T_i denote $T(\omega_i)$ and let h_i denote $h(\omega_i)$.

(a) Explain how the deterministically equivalent problem

$$\begin{array}{ll} \text{minimize} & c^T x + \displaystyle\sum_{i=1}^{N} p_i q_i^T y_i \\ \text{subject to} & Ax = b, \\ & T_i x + W y_i = h_i, \quad i = 1, \ldots, N, \\ & x \geq 0, \\ & y_i \geq 0, \quad i = 1, \ldots, N, \end{array}$$

arises. (We assume, for simplicity, "fix compensation", i.e., W does not depend on i.) .... (6p)

(b) Define *VSS* in terms of suitable optimization problems. .... (2p)

(c) Define *EVPI* in terms of suitable optimization problems. .... (2p)

5. Consider a cutting-stock problem with the following data:

$$W = 12, \quad m = 3, \quad w_1 = 2, \quad w_2 = 6, \quad w_3 = 10, \quad b = \begin{pmatrix} 20 & 15 & 10 \end{pmatrix}^T.$$

Notation and problem statement are in accordance to the textbook.

Given are rolls of width W, referred to as *W-rolls* below, containing the raw material. Smaller rolls of m different widths are to be cut out of the W-rolls, where each such smaller roll i has width w_i, $i = 1, \ldots, m$, and the demand for each such smaller roll i is given by b_i, $i = 1, \ldots, m$.

The aim is to cut the W-rolls so that a minimum number of W-rolls are used. This is done by forming *cut patterns*, where a cut pattern is a specification of how many of each smaller roll i that are included in this particular cutting of a W-roll. A cut pattern is represented by a nonnegative integer vector $(a_1, a_2, \ldots, a_m)$, where a_i specifies how many of the smaller roll i, $i = 1, \ldots, m$, that are included in the particular cut pattern.

(a) Solve the the LP-relaxed problem associated with the above problem. Start with the basic feasible solution associated with the three "pure" cut patterns $(6\ 0\ 0)^T$, $(0\ 2\ 0)^T$ and $(0\ 0\ 1)^T$. The subproblems that arise may be solved in any way, that need not be systematic. .... (8p)

(b) Determine a "near-optimal" solution to the original problem. Give a bound on the maximum deviation from the optimal value of the original problem... (2p)

Good luck!