SF2822 Applied nonlinear optimization, final exam
Thursday June 3 2021 8.00–13.00

Instructor: Shen Peng, tel. 0762261558.

Examiner: Anders Forsgren, tel. 08-790 71 27.

Allowed tools: Pen/pencil, ruler and eraser. *Note!* Calculator is not allowed.

Solution methods: Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Motivate your conclusions carefully. If you use methods other than what have been taught in the course, you must explain thoroughly.

Note! Personal number must be written on the title page. Write only one question per sheet. Number the pages and write your name on each page.

22 points are sufficient for a passing grade. For 20-21 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam are announced.

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1. Consider a nonlinear programming problem (NLP) defined by

$$(NLP) \quad \begin{array}{ll} \min & e^{x_1} + x_1x_2 + x_2^2 - 2x_2x_3 + x_3^2 \\ \text{subject to} & -x_1^2 - x_2^2 - x_3^2 + 10 \geq 0, \\ & a^Tx - 2 \geq 0, \end{array}$$

where $a \in I\!R^3$ is a given constant. Let $\widetilde{x} = (0\ 0\ 1)^T$.

(a) Find a such that $\widetilde{x}$ fulfils the first order necessary optimality conditions to (NLP). .... (5p)

(b) For the value on a you found in (1a), decide whether $\widetilde{x}$ is a local optimal solution to (NLP). .... (5p)

2. Derive the expression for the symmetric rank-1 update, C_k, in a quasi-Newton update $B_{k+1} = B_k + C_k$. .... (10p)

3. Consider the quadratic programming problem (QP) defined as

$$(QP) \quad \begin{array}{ll} \min & x_1^2 + x_1x_2 + \frac{3}{2}x_2^2 + 2x_3^2 - 4x_1 - 7x_2 + 4x_3 \\ \text{subject to} & -x_1 - x_2 - x_3 \geq -2, \\ & -x_1 + x_3 \geq -1, \\ & x_1 \geq 0, \\ & x_3 \geq 0. \end{array}$$

Solve (QP) using an active-set strategy, with the initial point $x^{(0)} = (0\ 0\ 0)^T$ and the constraint $x_1 \geq 0$ and $x_3 \geq 0$ in the working set. The equality-constrained quadratic programs that arise need not be solved in a systematic way. However, the values of the generated iterates $x^{(k)}$ and corresponding Lagrange multipliers $\lambda^{(k)}$ should be calculated. .... (10p)

4. Consider the nonlinear programming problem (P) defined as

$$(P) \quad \begin{array}{ll} \min & \frac{1}{2}(x_1-2)^2 + \frac{1}{2}(x_2-3)^2 \\ \text{subject to} & 1 - \frac{1}{2}x_1^2 - \frac{1}{2}x_2^2 \geq 0. \end{array}$$

Let $x^{(0)} = (0\ 1)^T$ and $\lambda^{(0)} = 2$. (As the textbook we define the Lagrange function as $\mathcal{L}(x,\lambda) = f(x) - \lambda^T g(x)$.)

(a) Assume that one wants to solve (P) by using sequential quadratic programming. Perform one iteration by sequential quadratic programming for solving (P) for the given $x^{(0)}$ and $\lambda^{(0)}$, i.e., calculate $x^{(1)}$ and $\lambda^{(1)}$. You may solve the subproblem in an arbitrary way that need not be systematic, and you do not need to perform any linesearch. . . . . (5p)

(b) Assume that one wants to solve (P) by using a primal-dual interior-point method without introducing slack variables. Assume that μ initially is chosen to 1 and the given initial point $x^{(0)}$ and $\lambda^{(0)}$ is used. Formulate the arising linear system of equations to be solved and calculate $x^{(1)}$ and $\lambda^{(1)}$ without considering merit function. . . . . (5p)

5. Consider the semidefinite programming problem

$$(PSDP) \quad \begin{array}{ll} \min & \text{trace}(MX) \\ \text{subject to} & \text{trace}(X) = 1, \\ & X = X^T \succeq 0, \end{array}$$

where M is a given symmetrical $n \times n$-matrix.

(a) Formulate the dual problem $(DSDP)$ corresponding to $(PSDP)$. . . . . (3p)

(b) Express the optimal value of $(DSDP)$ as simple as possible. . . . . (3p)

(c) Show that $(PSDP)$ has an optimal solution of rank one, i.e. an optimal solution on the form xx^T, where x is a n-dimensional vector. . . . . (4p)

Hint: If A is a $n \times n$-matrix and x is a n-vector, then $\text{trace}(Axx^T) = x^T Ax$.

Good luck!