



# **Lösningar** Skriftlig Tentamen **IE1204 Digital Design** **2021-10-27**

Examiner/Examinator: Carl-Mikael Zetterling

Responsible teacher/Ansvarig lärare: Carl-Mikael Zetterling, 08-790 4344

## **Swedish/Svenska:**

Tentamenstexten ska lämnas in när lösningarna lämnas in.

Inga tillåtna hjälpmedel utom linjal.

Examen består av två delar:

Del 1 har 16 uppgifter med max 1 poäng per uppgift som ska besvaras på "Answer Form".

Del 2 har 4 uppgifter med max 4 poäng per uppgift som ska besvaras på separat papper.

Lämna in båda delar samtidigt. Disponera tiden själv mellan delarna.

Uppgifterna är inte ordnade efter svårighetsgrad.

Om slutsumman av tentan har halvpöäng avrundas det uppåt.

X = 1 om studenten har minst 8 poäng på del 1

Y = 1 om studenten har minst 16 poäng totalt

P = 1 om studenter får godkänt på tentamen

Fx = 1 om studenten kan godkännas efter en extra uppgift

X	Y	P	F <sub>X</sub>
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Totalt max 32 poäng på tentamen.

För godkänt krävs

**(minst 8 poäng på del 1) OCH (minst 16 poäng totalt)**

Fx om något villkor ej är uppfyllt med en poängs marginal.

**Betygskalan** för tentamen förutsatt att studenten har minst 8 poäng från del 1.

0-15	16-18	19-21	22-24	25-27	28-31	32
F	E	D	C	B	A	A+

Resultat meddelas inom tre veckor.

# **Extended Solutions** Written Exam

## IE1204 Digital Design

### 2021-10-27

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#### **English:**

The exam text should be handed in after the exam.

No aids allowed except ruler.

The exam consists of two parts:

Part 1 has 16 exercises for max 1 point per exercise to be answered on the “Answer Form”.

Part 2 has 4 exercises for max 4 points per exercise, to be answered on a separate paper.

Hand in both parts at the same time. Plan the time yourself between the parts.

The exercises are not in order of difficulty.

If the total sum of the exam has half points this will be rounded up.

X = 1 if student has at least 8 points from part 1

Y = 1 if student has at least 16 points in total

P = 1 if student passes exam

Fx = 1 if a student can pass after an extra task

X	Y	P	Fx
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Total max of 32 points on the exam.

To pass the exam requires

**(at least 8 points from part 1) AND (at least 16 points in total)**

Fx if any condition is not fulfilled by one point's margin.

**Grades** are given as follows provided the student has at least 8 points from part 1.

0-15	16-18	19-21	22-24	25-27	28-31	32
F	E	D	C	B	A	A+

The result will be announced within three weeks.

# IE1204 Digital Design Answer Form 2021-2022

Full Name		Personal Number		Program							
Exam 2021-10-27		YYYYMMDD-XXXX		NN							
#	Answer with	Answer				Points					
1	Hexadecimal number	B58C				1					
2	8 bit two's complement binary number	0	1	1	1	1	0	0	0	0	1
3	8 bit two's complement binary number	1	1	0	1	0	1	1	1	0	1
4	Circuit number(s)	#2 and #5				1					
5	Boolean expression, Y =	$\overline{B} + A \cdot \overline{C}$				1					
6	Boolean expression, Y =	$\overline{B} \cdot \overline{D} + A \cdot D = (\overline{B} + D)(A + \overline{D})$				1					
7	MUX connections	$\overline{A \oplus B}$				1					
	Row CD = 00	$B$									
	Row CD = 01	$A + B$									
	Row CD = 10	$\overline{A \cdot B}$									
	Row CD = 11										
8	Timing diagram					1					
9	Timing diagram					1					
10	Maximum clock frequency =	4 GHz				1					
11	Next state $Q_D Q_C Q_B Q_A =$	1111				1					
12	16 bit two's complement Product A x B MSB	1	1	1	1	1	1	1	0	1	
	LSB	0	1	0	1	0	0	1	1		
13	8 bit two's complement Quotient A / B	0	0	0	0	0	1	0	1	1	
	Remainder	0	0	0	0	0	1	0	1		
14	Decimal number	-3.25				1					
15	5 result bits (S4 S3 S2 S1 S0)	0	0	1	1	1				1	
16	ALUControl (2 bits)	(Subtract)				0	1				1
TOTAL POINTS		Examiner sign							16		

# Del 1/Part 1, 1 point per exercise, fill in on “Answer Form”

## 1 Number Conversion

**Swedish:** Talet A är ett 16 bitars binärt tal.  
Konvertera till ett hexadecimal tal.

**English:** The number A is an 16-bit binary number.  
Convert to a hexadecimal number.

$$A = 1011\ 0101\ 1000\ 1100_2$$

Divide into groups of four bits, and convert each group to hexadecimal → B58C

## 2 Addition

**Swedish:** Konvertera A och B från decimala till 8 bitars två-komplement kodade tal.  
Beräkna A + B (binärt) och svara med ett 8 bitars två-komplement kodat tal.  
Tänk på att du kan kontrollera dina beräkningar med decimala tal.

**English:** Convert A and B from decimal to 8-bit binary (two’s complement) numbers.  
Calculate A + B (binary) and answer with an 8-bit binary (two’s complement) number.  
You can check your calculations using decimal numbers.

$$\begin{array}{r} A = 85_{10} \quad \quad 01010101 \\ B = 35_{10} \quad + \quad 00100011 \\ \hline A + B \quad \quad = 01111000 = 120_{10} \end{array}$$

## 3 Subtraction

**Swedish:** Konvertera A och B från decimala till 8 bitars två-komplement kodade tal.  
Beräkna A - B (binärt) och svara med ett 8 bitars två-komplement kodat tal.  
Tänk på att du kan kontrollera dina beräkningar med decimala tal.

**English:** Convert A and B from decimal to 8-bit binary (two’s complement) numbers.  
Calculate A - B (binary) and answer with an 8-bit binary (two’s complement) number.  
You can check your calculations using decimal numbers.

$$\begin{array}{r} A = 36_{10} \quad \quad \quad \quad 00100100 \\ B = 78_{10} \quad \quad 01001110 \\ \hline -B = -78_{10} \quad \quad \quad + 10110010 \\ A - B \quad \quad \quad \quad = 11010110 = -42_{10} \end{array}$$

## 4 CMOS

**Swedish:** Bestäm vilken/vilka av CMOS-grindnäten nedan som har den logiska funktionen Y.  
Det kan finnas mer än ett korrekt svar.

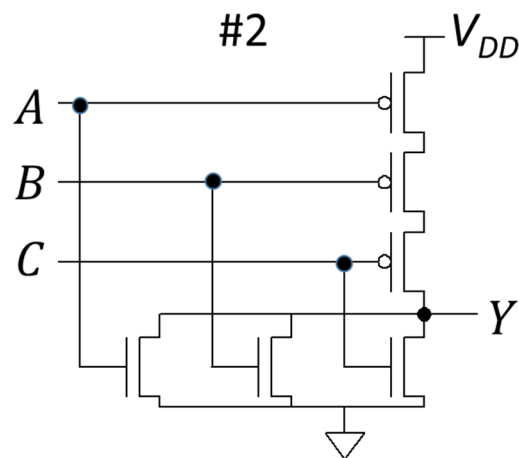
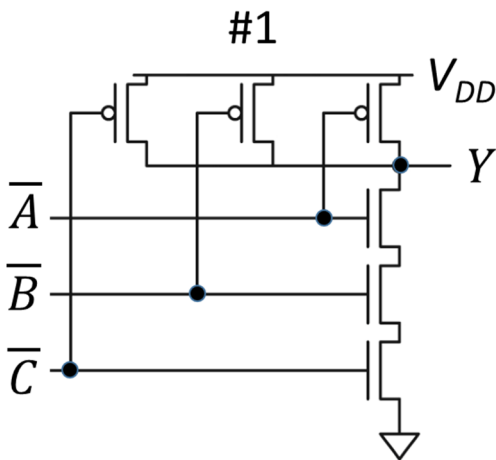
**English:** Determine which of the CMOS-circuits below have the logic function Y.  
There may be more than one correct answer.

$$Y = \overline{\overline{\overline{A} \cdot \overline{B}}} + C = \overline{A \cdot B \cdot C} = \overline{A + B + C}$$

(De Morgan's theorem), 3 input NOR

#1  $Y = \overline{\overline{A} \cdot \overline{B} \cdot \overline{C}} = A + B + C$  3 input OR

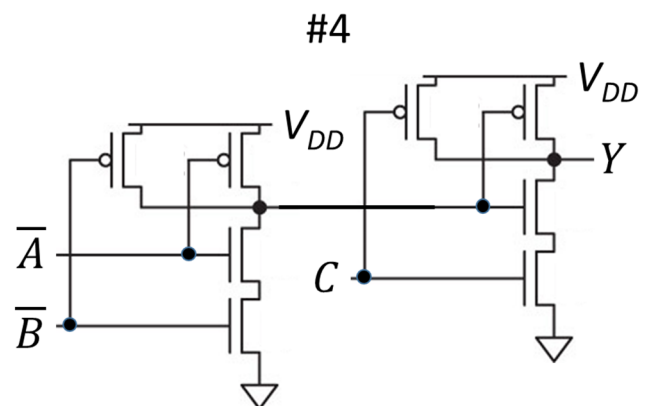
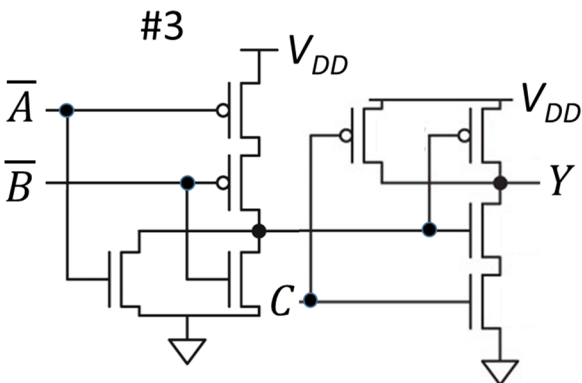
#2  $Y = \overline{A + B + C} = \overline{A} \cdot \overline{B} \cdot \overline{C}$  3 input NOR



#3  $Y = \overline{\overline{\overline{A + B} \cdot C}} = \overline{\overline{A + B} + C} = \overline{A + B} + C$

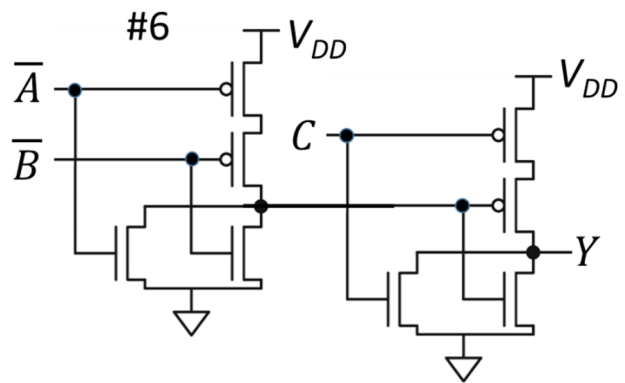
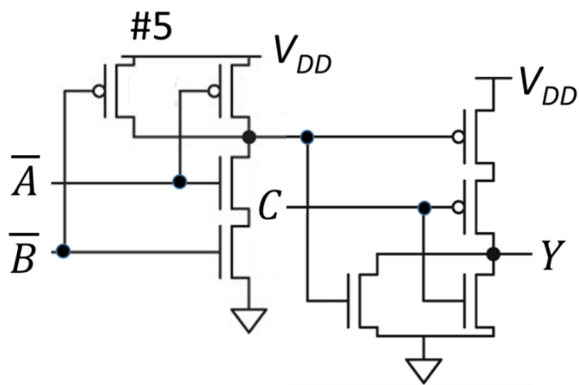
#4  $Y = \overline{\overline{\overline{A} \cdot \overline{B} \cdot C}} = \overline{A \cdot B + C}$

OR  $Y = \overline{\overline{\overline{A + B} \cdot C}} = \overline{\overline{A \cdot B} + C} = \overline{A \cdot B} + C$  3 input NAND



#5  $Y = \overline{\overline{\overline{A \cdot B} + C}} = \overline{\overline{A \cdot B} \cdot \overline{C}} = \overline{A + B + C}$  3 input NOR

#6  $Y = \overline{\overline{\overline{A + B} + C}} = (\overline{A + B})\overline{C} = \overline{A} \cdot \overline{C} + \overline{B} \cdot \overline{C}$



Answer: #2 and #5

## 5 Boolean Algebra

**Swedish:** Ta fram enklast möjliga booleska uttryck.

**English:** Derive the simplest possible Boolean expression.

$$\begin{aligned}
 Y &= \bar{B} + A \cdot B(\overline{C + D}) + (A + B + C) \cdot D + A \cdot \bar{C} \cdot D + \bar{A} \cdot \bar{B} \cdot C \\
 &= \bar{B} + A \cdot B \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot D + A \cdot \bar{C} \cdot D + \bar{A} \cdot \bar{B} \cdot C \\
 &= \bar{B}(1 + \bar{A} \cdot \bar{C} \cdot D + \bar{A} \cdot C) + A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + A \cdot B \cdot \bar{C} \cdot \bar{D} + A \cdot \bar{C} \cdot D \\
 &= \bar{B} + A \cdot \bar{C} \cdot \bar{D}(B + \bar{B}) + A \cdot \bar{C} \cdot D \\
 &= \bar{B} + A \cdot \bar{C}(D + \bar{D}) = \bar{B} + A \cdot \bar{C}
 \end{aligned}$$

Can be verified with a K-Map:

$$Y = \boxed{\bar{B}} + \boxed{A \cdot B(\overline{C + D})} + \boxed{(A + B + C) \cdot D} + \boxed{A \cdot \bar{C} \cdot D} + \boxed{\bar{A} \cdot \bar{B} \cdot C} = \boxed{\bar{B}} + \boxed{A \cdot \bar{C}}$$

Y	CD 00	CD 01	CD 11	CD 10
AB 00	1	1	1	1
AB 01				
AB 11	1	1		
AB 10	1	1	1	1

Y	CD 00	CD 01	CD 11	CD 10
AB 00	1	1	1	1
AB 01				
AB 11	1	1		
AB 10	1	1	1	1

## 6 MUX to K-map

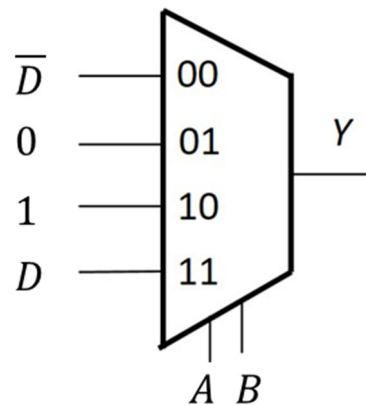
**Swedish:** Fyll i K-Map från MUX-kopplingen.

Ta fram enklast möjliga booleska uttryck för Y från K-map. Välj PoS eller SoP.

**English:** Fill in the K-Map from the MUX circuit.

Derive simplest possible Boolean expression from the K-map. Select PoS or SoP.

Y	CD 00	CD 01	CD 11	CD 10
AB 00	1	0	0	1
AB 01	0	0	0	0
AB 11	0	1	1	0
AB 10	1	1	1	1



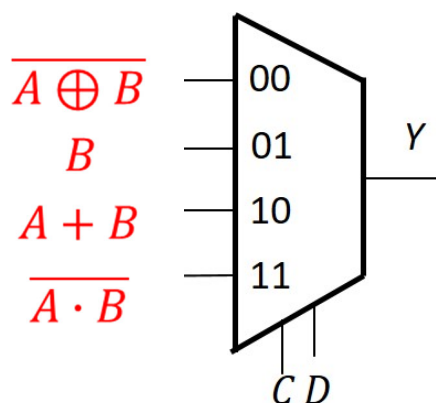
$$\overline{B} \cdot \overline{D} + A \cdot D = (\overline{B} + D)(A + \overline{D})$$

## 7 K-Map to MUX

**Swedish:** Använd en 4:1 MUX och valfria grindar eller 0 och 1 och gör en krets för K-map med CD som select-signaler.

**English:** Use a 4:1 MUX and any logic gates or 0 or 1 to draw a circuit for the K-map with CD as select signals.

Y	CD 00	CD 01	CD 11	CD 10
AB 00	1	0	1	0
AB 01	0	1	1	1
AB 11	1	1	0	1
AB 10	0	0	1	1

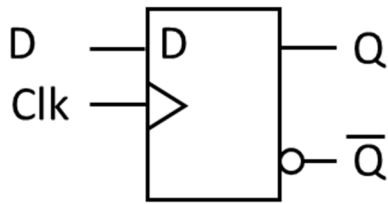




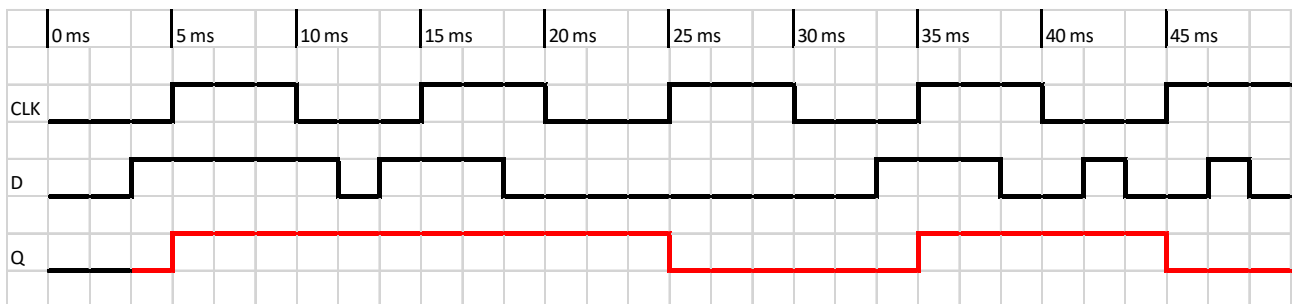
## 8 Timing diagram

**Swedish:** Rita tidsdiagram för D-vippan (D flip-flop) i "Answer Form".

**English:** Draw the timing diagram for the D flip-flop in the "Answer Form".



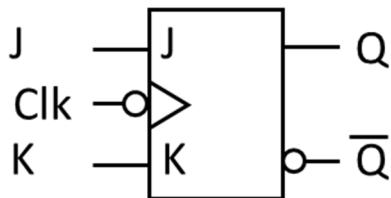
**NOTE: positive edge**



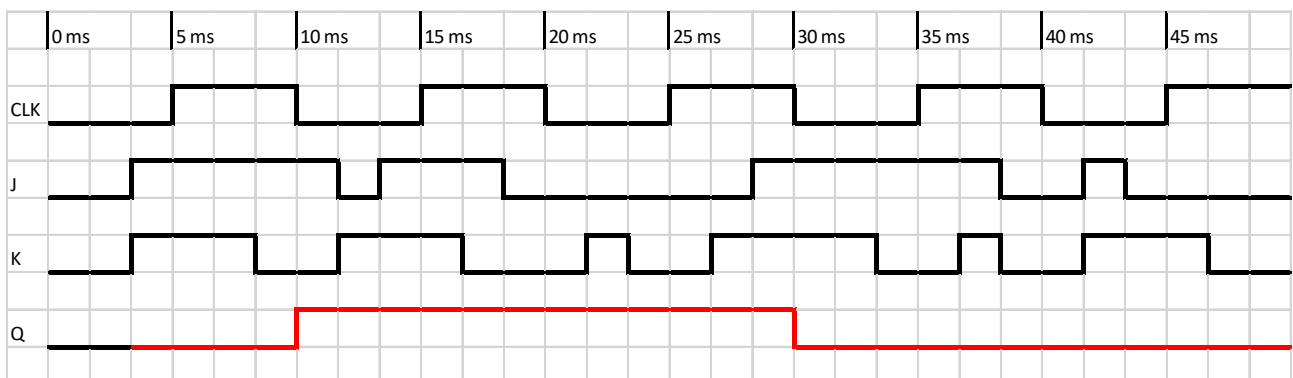
## 9 Timing diagram

**Swedish:** Rita tidsdiagram för JK-vippan (JK flip-flop) i "Answer Form".

**English:** Draw the timing diagram for the JK flip-flop in the "Answer Form".



**NOTE: negative edge**



## 10 Timing calculation

**Swedish:** Beräkna maximal klockfrekvens för kretsen nedan.

**English:** Calculate the maximum clock frequency for this circuit.

Delay Per Register

$$t_{pcq} = 45 \text{ ps}$$

$$t_{ccq} = 30 \text{ ps}$$

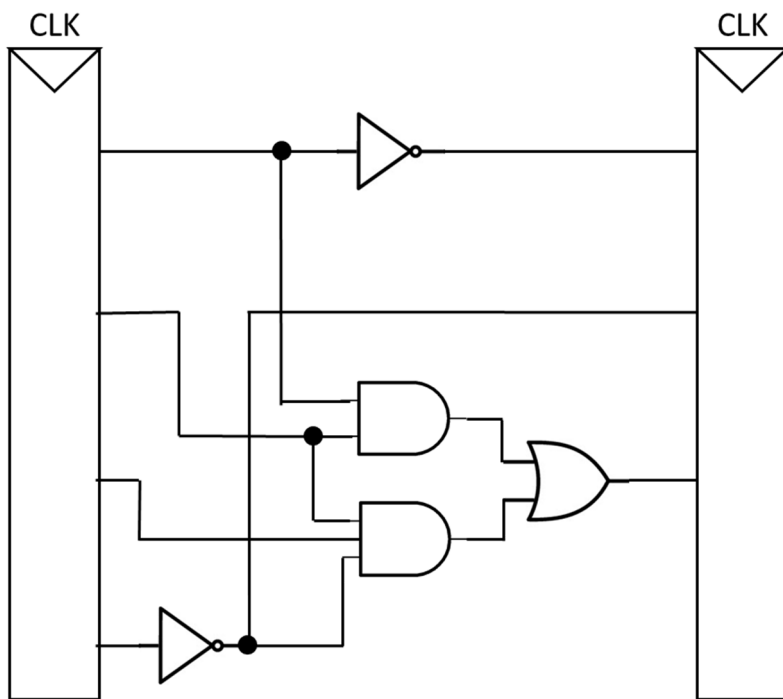
$$t_{\text{setup}} = 55 \text{ ps}$$

$$t_{\text{hold}} = 60 \text{ ps}$$

Delay Per gate:

$$t_{pd} = 50 \text{ ps}$$

$$t_{cd} = 20 \text{ ps}$$



The longest path is three gates

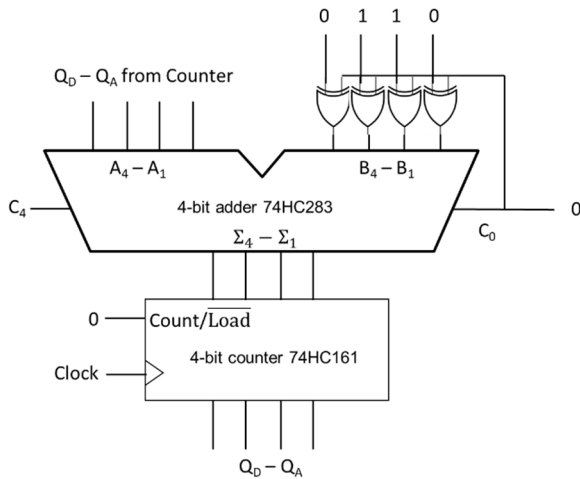
$$T_c \geq t_{pcq} + t_{pd} + t_{\text{setup}} = 45 + 3 \times 50 + 55 = 250 \text{ ps}$$

$$f_c = 1/T_c = 4 \text{ GHz}$$

## 11 Counter

**Swedish:** Räkaren nedan har kommit till tillstånd  $Q_D Q_C Q_B Q_A = 1001$   
Vad blir nästa tillstånd?

**English:** The counter below has counted to state  $Q_D Q_C Q_B Q_A = 1001$   
What is the next state?



Add 6 for each clock, after 1001 (=9) comes 1111 (=15)

## 12 Multiplication

**Swedish:** A och B är 8 bitars två-komplement kodade tal.  
Beräkna  $A \times B$  (binärt) och svara med ett 16 bitars två-komplement kodat tal.  
Tänk på att du kan kontrollera dina beräkningar med decimala tal.

**English:** A and B are 8-bit binary (two's complement) numbers.  
Calculate  $A \times B$  (binary) and answer with a 16-bit binary (two's complement) number.  
You can check your calculations using decimal numbers.

$$A = 11110101_2$$

$$B = 00100111_2 \quad = 39_{10}$$

A is negative, convert to positive:  $-A = 00001011_2 = 11_{10}$

Multiply  $-A \times B = B \times (-A)$ :

$$\begin{array}{r}
 00100111 \\
 \times 00001011 \\
 \hline
 00100111 \quad 1^{\text{st}} \text{ multiplication} \\
 + 00100111 \quad 2^{\text{nd}} \text{ multiplication} \\
 \hline
 001110101 \quad \text{add intermediate result first two rows} \\
 + 00100111 \quad \text{add 3}^{\text{rd}} \text{ multiplication} \\
 \hline
 0000000110101101 \quad = 429_{10} \quad (511 - 82) = 39 \times 11
 \end{array}$$

$$1111111001010011 \quad \text{Two's complement} = A \times B$$

### 13 Division

**Swedish:** A och B är 8 bitars två-komplement kodade tal.  
Beräkna A / B (binärt) och svara med kvot och rest (8 bitars två-komplement kodade tal).  
Tänk på att du kan kontrollera dina beräkningar med decimala tal.

**English:** A and B are 8-bit binary (two's complement) numbers.  
Calculate A / B (binary) and answer with quotient and remainder (8-bit binary two's complement numbers).  
You can check your calculations using decimal numbers.

$$\begin{aligned} A &= 00111100_2 & = 60_{10} \\ B &= 00001011_2 & = 11_{10} \end{aligned}$$

(A and B are both positive, nothing to convert)

$$\begin{array}{r} \phantom{1011/} \underline{101} \quad \text{Quotient} = 5, \text{ check: } 5 \times 11 = 55 \\ 1011 \overline{)00111100} \\ \phantom{1011/} - \phantom{00} \underline{1011} \\ \phantom{1011/} \phantom{00} \phantom{00} 10000 \\ \phantom{1011/} \phantom{00} \phantom{00} - \phantom{00} \underline{1011} \\ \phantom{1011/} \phantom{00} \phantom{00} \phantom{00} 101 \quad \text{Remainder of 5} \end{array}$$

### 14 Fixed point

**Swedish:** Ett format för fixed point tal med tecken är att använda två-komplementtal med 4 bitar för talet och 4 bitar för bråkdelen. Omvandla A till ett decimalt tal.

**English:** One format for fixed point signed numbers is to use two's complement numbers with 4 bits for the integer and 4 bits for the fraction. Convert A to a decimal number.

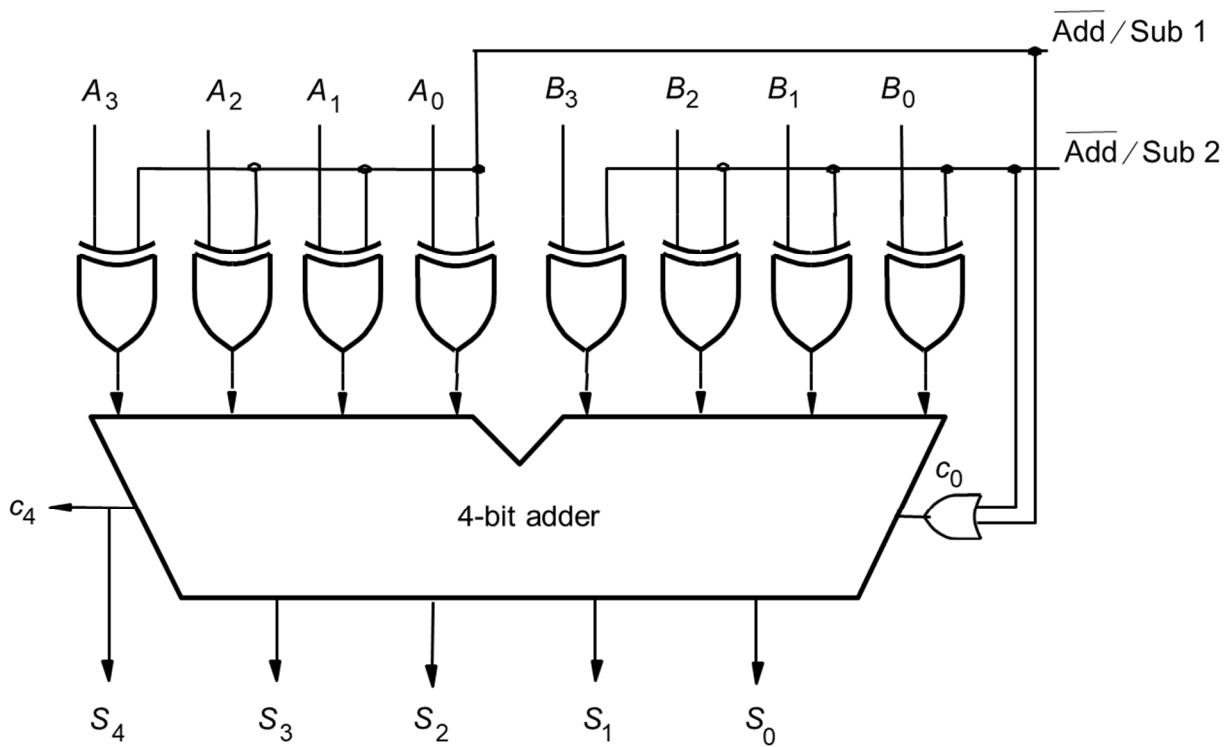
$$\begin{aligned} A &= 1100.1100_2 \\ A &= -8 + 4 + \frac{1}{2} + \frac{1}{4} = -4 + 0.75 = -3.25 \end{aligned}$$

## 15 Full Adder

**Swedish:** Vad blir resultatet från heladderarkretsen nedan?  
Svara med 5 bitar (S4 S3 S2 S1 S0)

**English:** What is the result for the full adder circuit below?  
Answer with 5 bits (S4 S3 S2 S1 S0)

A = 1100    B = 0011     $\overline{\text{Add/Sub}}\ 1 = 1$      $\overline{\text{Add/Sub}}\ 2 = 0$



A will be subtracted since  $\overline{\text{Add/Sub}}\ 1 = 1$

B is not affected since  $\overline{\text{Add/Sub}}\ 2 = 0$

$S = B - A = 0011 - 1100 = 0011 + 0011 + 1 = 00111$  (no carry out)

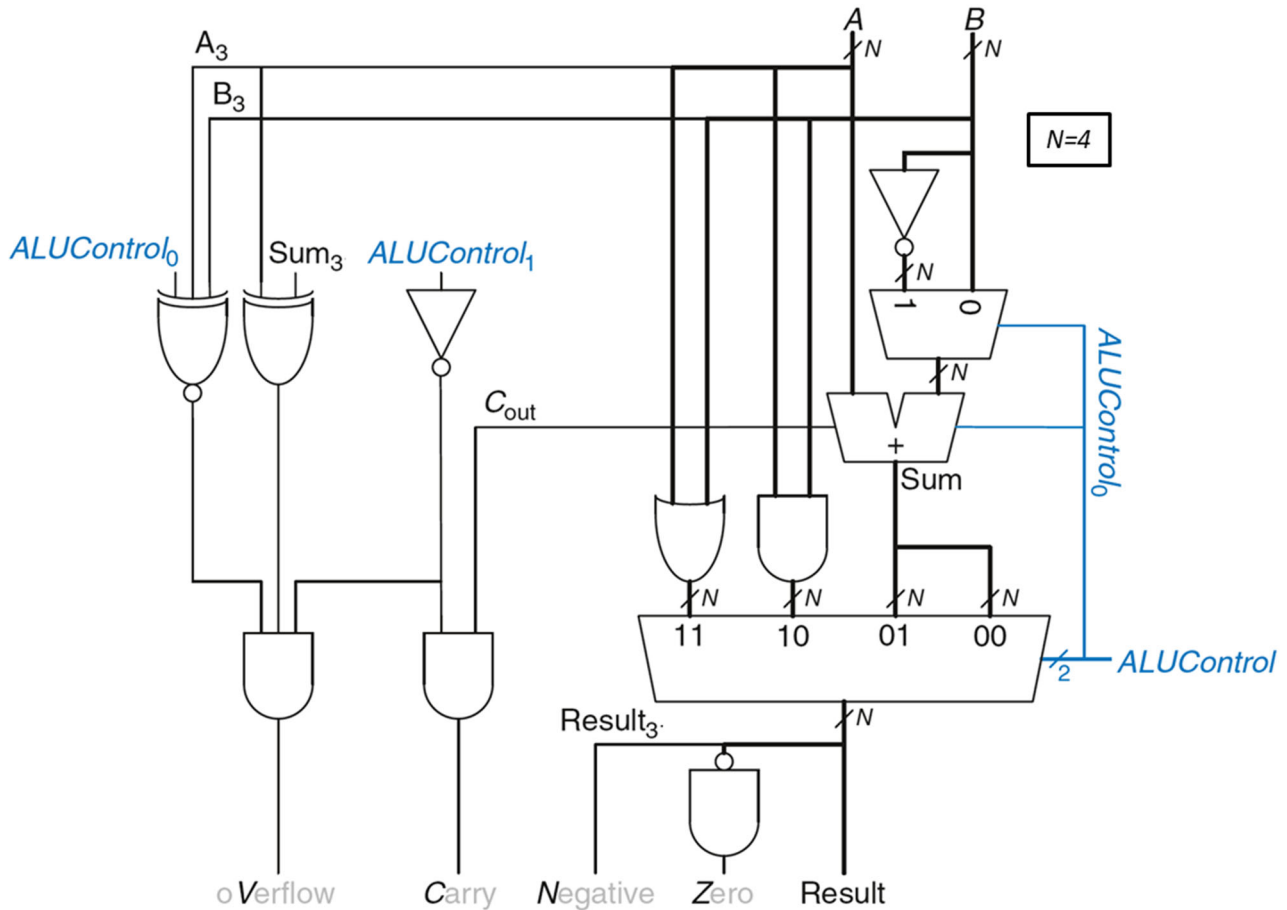
Check: A = -4, B = 3, S = 3 - (-4) = 7

## 16 ALU

**Swedish:** Med A och B enligt nedan blev flaggorna i ALU som visas. Vilken instruktion genomfördes? Svara med 2 bitar för ALUControl.

**English:** Using A and B below, the flags gave the result shown. Which instruction was executed? Answer with 2 bits for ALUControl.

A = 1000    B = 0110    →    V C N Z = 1100



$ALUControl_{1,0}$	Function
0 0	Add
0 1	Subtract
1 0	AND
1 1	OR

Since  $V = 1$  it was an arithmetic operation (Add or Subtract)

$A + B = 1000 + 0110 = 1110$  does not generate a carry ( $C = 0$ ) but  $N = 1$

$A - B = 1000 - 0110 = 1000 + 1010 = 10010$  generates a carry ( $C = 1, N = 0$ ), **Answer: 01**

## Del 2/Part 2, 4 points per exercise, answer on separate paper

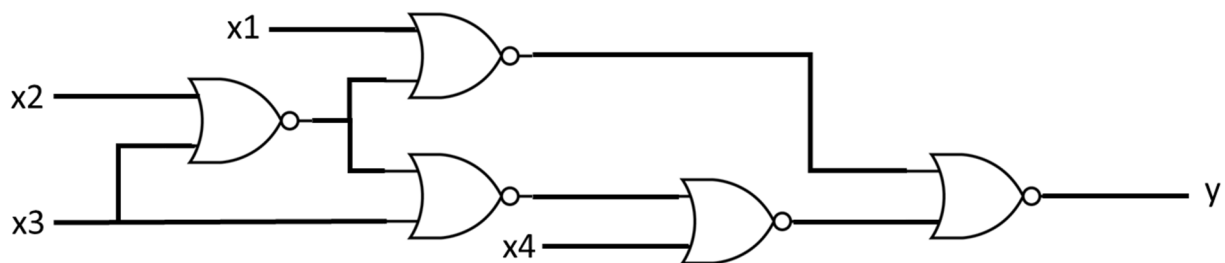
### 17 Analysis of Combinational Circuit

#### Swedish:

1. Ta fram booleskt uttryck för kretsen nedan.
2. Rita K-map för kretsen med variabelordning som i figuren.
3. Förenkla uttrycket med hjälp av K-map.
4. Rita ny krets med enbart NOR-grindar.

#### English:

1. Derive the Boolean expression for the circuit below.
2. Draw a K-map for the circuit with variables as in the figure.
3. Simplify the expression using the K-map.
4. Draw a new circuit using only NOR gates.



#### 1. Boolean expression:

$$\begin{aligned}
 y &= \overline{\overline{x1 + \overline{x2 + x3}} + \overline{\overline{x2 + x3} + x3 + x4}} \\
 &= (x1 + \overline{x2 \cdot x3}) \cdot (\overline{x2 + x3} + x3 + x4) \\
 &= (x1 + \overline{x2} \cdot \overline{x3}) \cdot ((x2 + x3) \cdot \overline{x3} + x4) = (x1 + \overline{x2} \cdot \overline{x3}) \cdot (x2 \cdot \overline{x3} + x4) \\
 &= x1 \cdot x2 \cdot \overline{x3} + x1 \cdot x4 + \overline{x2} \cdot \overline{x3} \cdot x4
 \end{aligned}$$

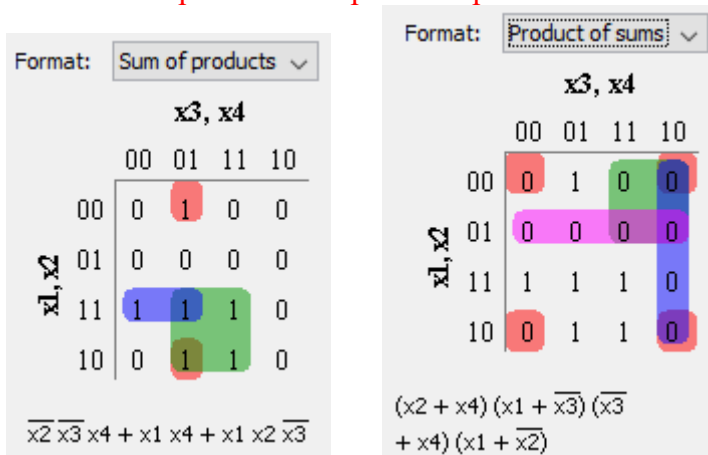
Output:

$$\overline{\overline{x1 + \overline{x2 + x3}} + \overline{\overline{x2 + x3} + x3 + x4}}$$


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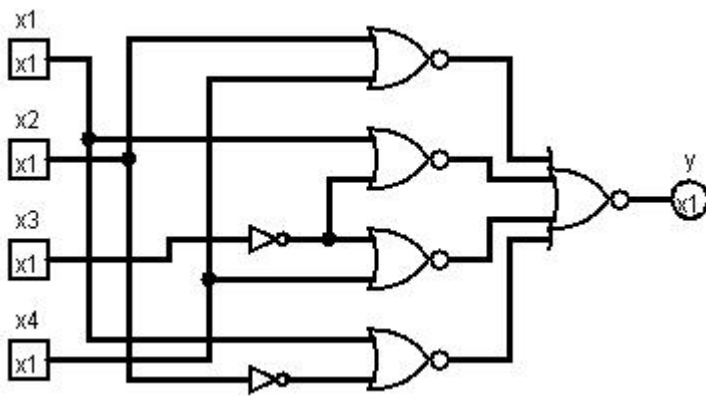

$$\sim(\sim(x1 + \sim(x2 + x3)) + \sim(\sim(\sim(x2 + x3) + x3) + x4))$$

## 2. K-Map with 3. Simplified expression



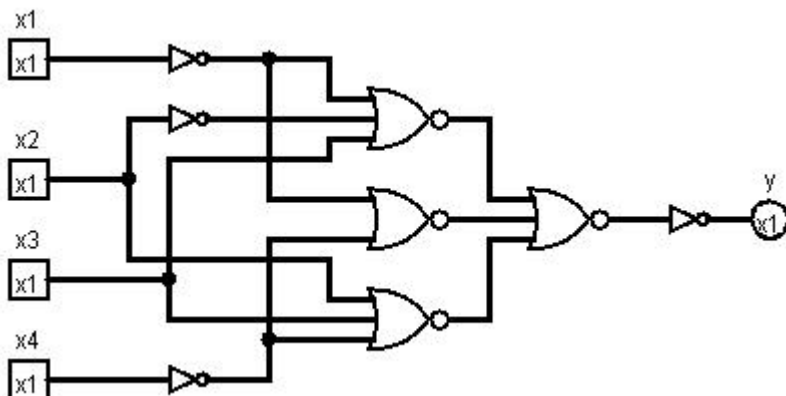
**Use POS for NOR only (inverters are ok if you note that they can be made with a NOR)  
No deductions if not simplest possible.**

$$y = \overline{\overline{(x2 + x4)(x1 + \overline{x3})(\overline{x3} + x4)(x1 + \overline{x2})}} = \overline{\overline{x2 + x4 + x1 + \overline{x3} + \overline{x3} + x4 + x1 + \overline{x2}}}$$



**For SoP you need double double inversion, and an extra NOR/Inverter at the output**

$$y = \overline{\overline{x2 \cdot x3 \cdot x4 + x1 \cdot x4 + x1 \cdot x2 \cdot x3}} = \overline{\overline{x2 + x3 + x4 + x1 + x4 + x1 + x2 + x3}} = \overline{\overline{x2 + x3 + \overline{x4} + \overline{x1} + \overline{x4} + \overline{x1} + \overline{x2} + x3}}$$





## 18 Design of Combinational Circuit

### Swedish:

En djuraffär vill ha en varningskrets som varnar om en kund köper en olämplig uppsättning djur.

A = kunden köper ett akvarium

B = kunden köper en fågel

C = kunden köper en katt

D = kunden köper en hund

Varningskretsen  $W = 1$  om en katt köps tillsammans med något av de andra djuren.

$W = 0$  för alla andra kombinationer. Det finns inga ”don’t cares”, alla husdjur måste tas om hand!

1. Rita sanningstabellen.
2. Rita K-map för sanningstabellen med variabelordning som i figuren.
3. Ta fram enklast möjliga booleska uttryck från K-map.
4. Rita en krets för uttrycket med enbart 2- och 3-ingångars NAND-grindar.

### English:

A pet store wants a warning circuit that warns if a customer buys an incompatible set of animals.

A = customer buys an Aquarium

B = customer buys a Bird

C = customer buys a Cat

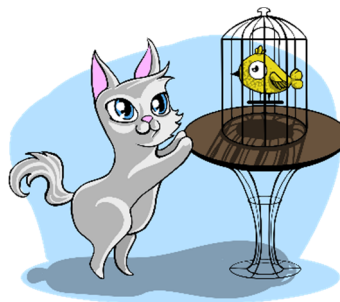
D = customer buys a Dog

The Warning circuit  $W = 1$  if a Cat is bought together with any of the other animals.

$W = 0$  for all other combinations. There are no don’t cares, all pets need to be taken care of!

1. Draw the truth table.
2. Draw a K-map for the truth table with variables as in the figure.
3. Derive simplest possible Boolean expression from the K-map.
4. Draw a circuit for the expression using only 2 and 3 input NAND-gates.

W	CD 00	CD 01	CD 11	CD 10
AB 00				
AB 01				
AB 11				
AB 10				



Rita om K-map i dina inlämnade svar.

Redraw the K-map in your answer sheets.



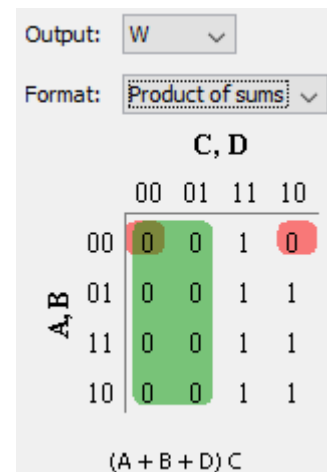
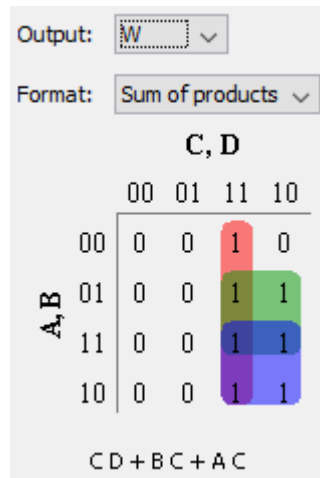
(Answer on next page)

# 18 Design of Combinational Circuit

## 1. Truth Table

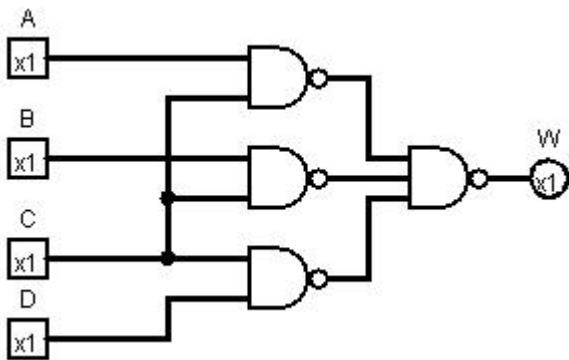
A	B	C	D	W
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

## 2. K-Map and 3. Boolean expressions

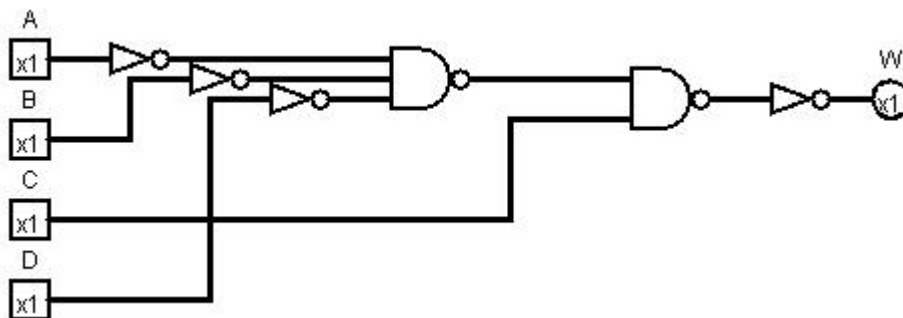


4. Use SOP for NAND only (inverters are ok if you note that they can be made with a NAND)  
No deductions if not simplest possible.

For SOP draw  $Y = \overline{\overline{C \cdot D + B \cdot C + A}} = \overline{\overline{C \cdot D} \cdot \overline{B \cdot C} \cdot \overline{A \cdot C}}$



For POS draw  $Y = \overline{\overline{(A + B + D) \cdot C}} = \overline{\overline{A} \cdot \overline{B} \cdot \overline{D} \cdot C}$   
(note the double inversion bars, extra inverters/NANDs are needed)



## 19 Analysis of FSM

**Swedish:** Analysera vad nedanstående tillståndsmaskin (FSM) utför.

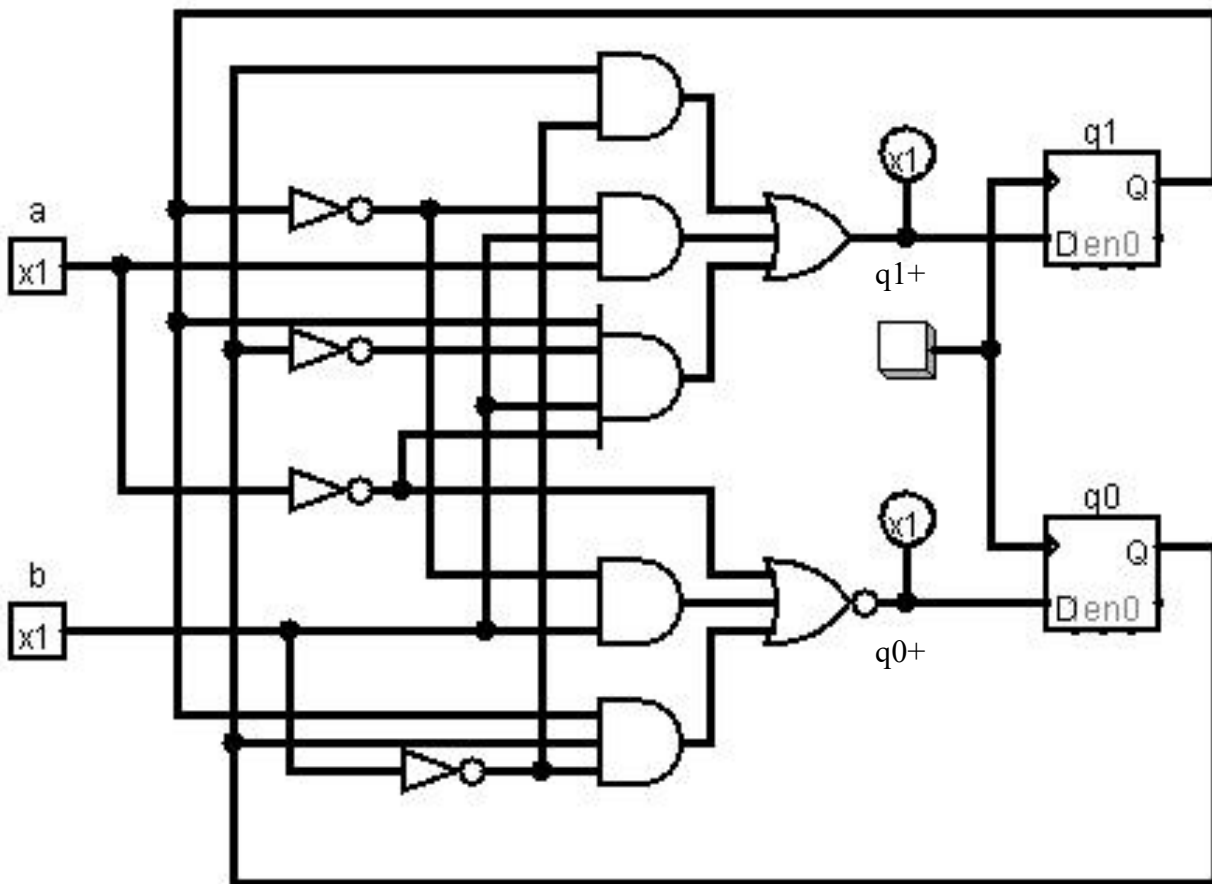
1. Ta fram Boolska uttryck för nästa tillstånd.
2. Rita K-Maps för  $q1+$  och  $q0+$ .
3. Rita tillståndstabell.
4. Rita tillståndsdigram.

Använd ordningen  $q1\ q0\ b\ a$  (det finns inga utsignaler förutom tillståndsvariablerna)

**English:** Analyze the state machine (FSM) below.

1. Derive Boolean expressions for next state.
2. Draw K-Maps for  $q1+$  and  $q0+$ .
3. Draw a state table.
4. Draw a state diagram.

Use the order  $q1\ q0\ b\ a$  (there are no outputs except for the state variables)



1. Boolean expressions for next state  $q1+$  and  $q0+$ , Note that  $q0+$  has a NOR gate.

$$q_1^+ = q_0 \cdot \bar{b} + \bar{q}_1 \cdot b \cdot a + q_1 \cdot \bar{q}_0 \cdot b \cdot \bar{a}$$

$$q_0^+ = \overline{\bar{a} + \bar{q}_1 \cdot b + q_1 \cdot q_0 \cdot \bar{b}}$$

2. K-Maps for  $q_1^+$  and  $q_0^+$

$q_1^+ = f(q_1, q_0, b, a)$ 

	ba			
	00	01	11	10
q <sub>1</sub> q <sub>0</sub>	00	01	11	10
00	0	0	1	0
01	1	1	1	0
11	1	1	0	0
10	0	0	0	1

$q_0^+ = f(q_1, q_0, b, a)$ 

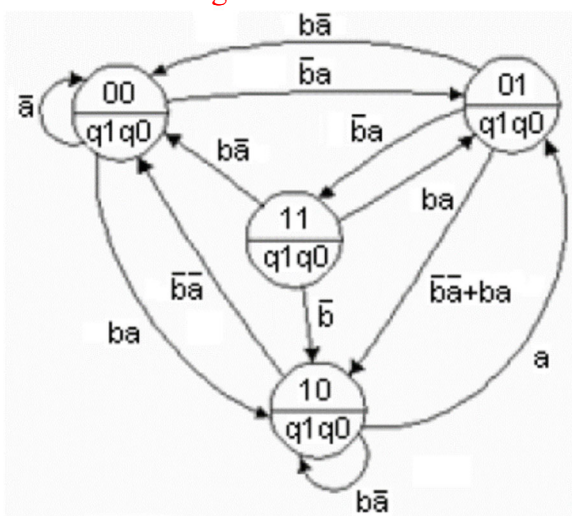
	ba			
	00	01	11	10
q <sub>1</sub> q <sub>0</sub>	00	01	11	10
00	0	1	0	0
01	0	1	0	0
11	0	0	1	0
10	0	1	1	0

3. State table

 $q_1^+ q_0^+ = f(q_1, q_0, b, a)$ 

	ba			
	00	01	11	10
q <sub>1</sub> q <sub>0</sub>	00	01	11	10
00	0 0	0 1	1 0	0 0
01	1 0	1 1	1 0	0 0
11	1 0	1 0	0 1	0 0
10	0 0	0 1	0 1	1 0

4. State diagram



Other notations on the arrows are also possible

If you find this state diagram familiar, you probably had FSM 20 in Lab 3

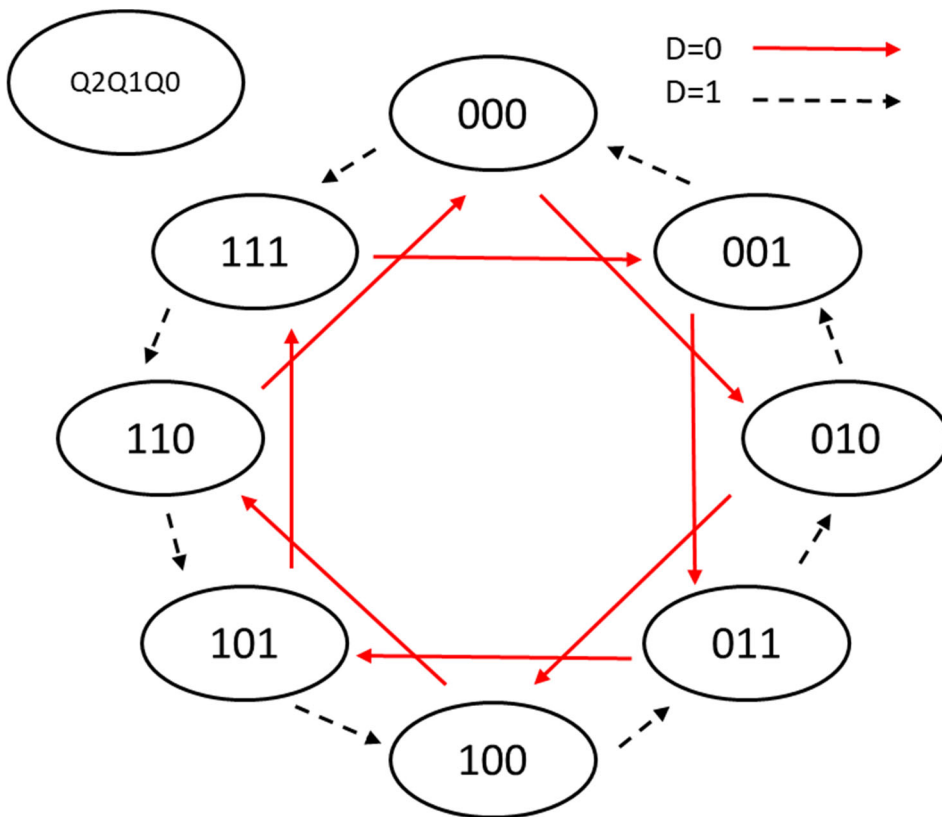
## 20 Design of FSM

**Swedish:** Konstruera en tillståndsmaskin (FSM) enligt tillståndsdigrammet nedan.

1. Rita tillståndstabell.
2. Ta fram K-map för nästa tillstånd.
3. Ta fram minimerade uttryck för nästa tillstånd.
4. Rita kretsschema för en FSM med DFFs och vilka grindar som helst.

**English:** Design a state machine (FSM) according to the state diagram below.

1. Draw a state table.
2. Derive K-maps for next state.
3. Derive minimized expressions for next state.
4. Draw the FSM circuit diagram with DFFs and any gates.



	$Q_1Q_0 =$			
	00	01	11	10
$D Q_2 =$ 00				
01				
11				
10				

Rita om K-map i dina inlämnade svar.

Redraw the K-map in your answer sheets.

(Answer on next page)

## 1. State Table

Present state			Next state D = 0			Next state D = 1		
Q2	Q1	Q0	Q2+	Q1+	Q0+	Q2+	Q1+	Q0+
0	0	0	0	1	0	1	1	1
0	0	1	0	1	1	0	0	0
0	1	0	1	0	0	0	0	1
0	1	1	1	0	1	0	1	0
1	0	0	1	1	0	0	1	1
1	0	1	1	1	1	1	0	0
1	1	0	0	0	0	1	0	1
1	1	1	0	0	1	1	1	0

## 2. K-Maps for next state Q2+, Q1+, and Q0+

## 3. Minimized Boolean expressions for next state (select SoP or PoS)

