Fractal Geometry
Assignment 2
Due on Tuesday, March 1st

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Question 1. Prove that if $f : F \to \mathbb{R}^n$ satisfies the Hölder condition

$$|f(x) - f(y)| \leq c|x - y|^\alpha$$

where $c > 0$ and $0 < \alpha \leq 1$, then the upper and lower box-counting dimensions satisfy the following:

$$\underline{\dim}_B f(F) \leq \left(\frac{1}{\alpha}\right) \underline{\dim}_B F \text{ and } \overline{\dim}_B f(F) \leq \left(\frac{1}{\alpha}\right) \overline{\dim}_B F.$$

Question 2. Verify from the definition that the s-dimensional Hausdorff measure, H^s, satisfies the following properties:

1. $H^s(\emptyset) = 0$,
2. $H^s(E) \subset H^s(F)$ if $E \subset F$,
3. $H^s(\cup_{i=1}^\infty F_i) \leq \sum_{i=1}^\infty H^s(F_i)$.

Question 3. What is the Hausdorff dimension of $F \times F \subset \mathbb{R}^2$, where F is the middle third Cantor set?

Question 4. What is the Hausdorff dimension of the "Cantor tartan" which is given by $\{(x, y) \in \mathbb{R}^2 : \text{ either } x \in F \text{ or } y \in F\}$, where F is the middle third Cantor set?