Chapter 5

Bell basis and Photon Detection

For two particle entanglement, there rare four possible Bell states that define a basis:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}[|H_1H_2\rangle + |V_1V_2\rangle] \tag{5.1}$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}[|H_1H_2\rangle - |V_1V_2\rangle] \tag{5.2}$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}[|H_1V_2\rangle + |V_1H_2\rangle] \tag{5.3}$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}[|H_1V_2\rangle - |V_1H_2\rangle] \tag{5.4}$$

We can generalize these into:

$$|\Phi\rangle = \frac{1}{\sqrt{2}}[|H_1H_2\rangle + e^{i\theta}|V_1V_2\rangle] \tag{5.5}$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|H_1V_2\rangle + e^{i\theta}|V_1H_2\rangle] \tag{5.6}$$

The state $|\Phi\rangle$ has positive correlations: both measurements yield the same results. The state $|\Psi\rangle$ has negative correlations: each measurement yields opposite results.

But what is the physical meaning of the phase factor? Is there a measurable difference between $|\Phi^+\rangle$ and $|\Phi^-\rangle$?

To look for an answer, we can use another basis than H and V, we can use D and A: diagonal and anti-diagonal.

We express $|V\rangle$ and $|V\rangle$ in the $|D\rangle$ and $|A\rangle$ basis:

$$|H\rangle = \frac{1}{\sqrt{2}}[|D\rangle + |A\rangle] \tag{5.7}$$

$$|V\rangle = \frac{1}{\sqrt{2}}[|D\rangle - |A\rangle] \tag{5.8}$$

We can then express $|HH\rangle$ and $|VV\rangle$ in terms if $|A\rangle$ and $|D\rangle$:

$$|HH\rangle = |H\rangle |H\rangle \tag{5.9}$$

$$= \frac{1}{\sqrt{2}}[|D\rangle + |A\rangle][|D\rangle + |A\rangle] \tag{5.10}$$

$$= \frac{1}{\sqrt{2}}[|DD\rangle + |AA\rangle + |AD\rangle + |DA\rangle] \tag{5.11}$$

$$|VV\rangle = |V\rangle |V\rangle \tag{5.12}$$

$$= \frac{1}{\sqrt{2}}[|D\rangle - |A\rangle][|D\rangle - |A\rangle] \tag{5.13}$$

$$= \frac{1}{\sqrt{2}}[|DD\rangle - |AS\rangle - |DA\rangle + |AA\rangle] \tag{5.14}$$

We can rewrite the general state $|\Phi\rangle = \frac{1}{\sqrt{2}}[|HH\rangle + e^{i\theta}|VV\rangle]$ in the D,A basis:

$$|\Phi\rangle = \frac{1}{2\sqrt{2}}[|DD\rangle + |AA\rangle + |AD\rangle + |DA\rangle + e^{i\theta}(|DD\rangle - |DA\rangle - |AD\rangle + |AA\rangle)] \tag{5.15}$$

$$= \frac{1}{2\sqrt{2}}[(1+e^{i\theta})(|DD\rangle + |AA\rangle) + (1-e^{i\theta})(|DA\rangle - |AD\rangle)] \tag{5.16}$$

We see that θ has a physical meaning! For example, for $\theta = 0$ we measure high countsfor positive correlations (DD and AA) but for $\theta = \pi$ that is $\phi^+ = HH + VV$ we measure high count rates for positive correlations (DD + AA).

5.1 Single Photon detection

A near infrared photon carries 10^{-19} Joules, a typical electrical pulse used for computer communication carries 10^{10} more energy. This means that to detect a single photon and process the detection event we need a very large amplification factor of the order of 10^{10}. It is under debate whether the human eye is able to detect single photons, it might be able to detect small numbers of photons but with a time resolution that makes it irrelevant for most experiments in physics. The photomultiplier tube has been used since the 1930s to detect single photons, it is based on the photoelectric effect where an incoming photon knock off an electron from a metal that is then accelerated in vacuum towards another metal plate to release more electrons, this process is repeated several times to produce a large current.

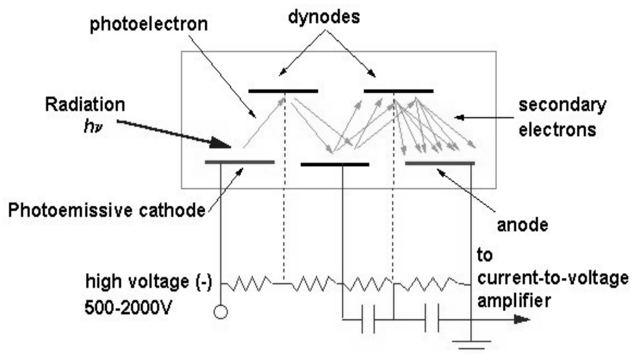

Figure 5.1: The photomultiplier tube.

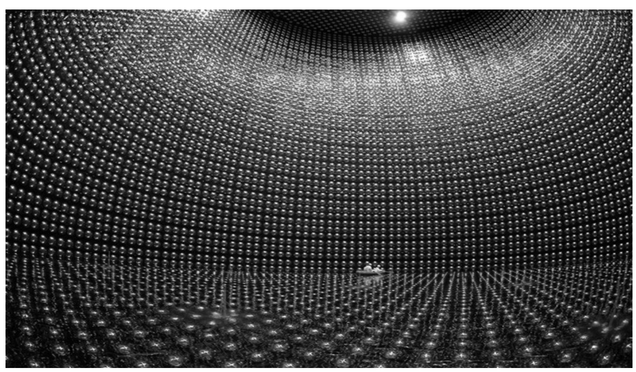

Figure 5.2: The Super Kamiokande: Photo Multiplier tubes are used to detect single photons linked to neutrinos.

Limitations with the detection efficiency of the photomultiplier tube were addressed with avalanche photodiodes that are monolithic semiconductor devices. Detection efficiencies in excess of 60% can be achieved in the near-visible but time resolution and noise levels are not as good as one might wish for. The avalanche photodiode is made of a pin semiconductor diode, silicon can be used but then is limited by its bandgap of 1.1 eV: photons with energy lower than 1.1 eV are not absorbed by silicon and are not detected. Other semiconductors such as InGaAs can be used, with a lower bandgap to be able to detect light at telecom frequencies. These devices can be made very small and consume a small amount of power, they are however still not ideal as their detection efficiency is still far below 100%, they have important dark counts (detection events are generated even in the absence of incoming light), they have limited time resolution (the timing between a photon reaching the detector and a detection pulse being generated fluctuates widely).

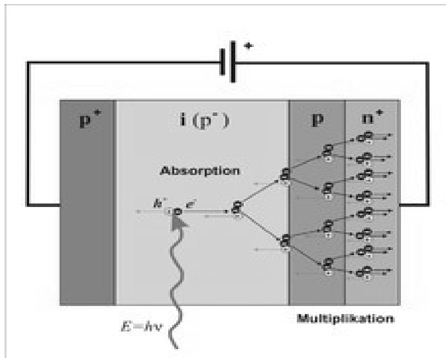

Figure 5.3: The avalanche photodiode.

The most recent type of single photon detector to be developed is the superconducting single photon detector where superconducting nanostructures can switch from the superconducting state to the resistive state with the absorption of a single photon, this results in very high detection efficiencies in excess of 95%, in very good time resolution better than 10 ps and in very law noise levels in the mHz range. It is interesting to note that 10 ps is the time it take light to travel 3 mm, this is particularly useful for lidar applications where a 3 dimensional image is created by measuring the return time of a laser pulse.

Time resolution: uncertainty in the timing of the output electrical pulse for a fixed photon absorption time. Dark noise: detection pulses generated by the detector in the absence of incident photons. On the order of hundreds of photons per second for a usual avalanche photodiode. Can be far less than one event per second for state of the art superconducting single photons. Dead time: time interval following a detection event where the detector is not able to detect an incoming photon. Quantum efficiency: probability for a photon to generate a detection event.

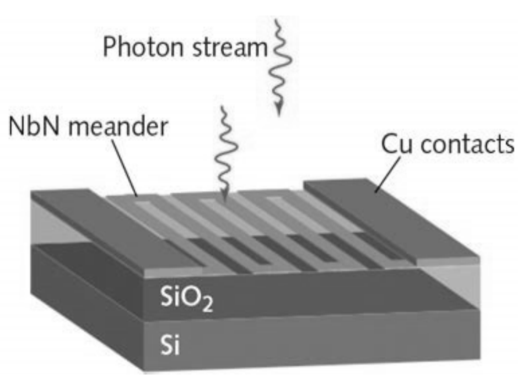

Figure 5.4: The Superconducting Nanowire Single Photon Detection.

A practical photon number detector remains to be invented, all we have at our disposal today are ‘click’ detectors: if one photon or more impinge the detector, it generates a detection pulse, irrespective of the number of impinging photons. Note that resolving the number of photons is not impossible, a bolometer does just that, it measures the amount of heat deposited by a pulse of light on a detector. When operated very precisely, it is possible to distinguish the heat associated with one photon from the heat associated with two photons at optical frequencies. This can be done with a Transition Edge Sensor where the heat deposited on a nanoscale piece of Tungsten is measured with an accuracy that allows to count the number of photons, this device is however very slow and operates a mK.

Trick question: if the detection efficiency of a photon number resolving detector is less than unity, what happens to the fidelity of the measurement?