Fractal Geometry
Assignment 4
Due on Tuesday, May 3rd

Liviana Palmisano

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Question 1. For $\lambda > 1$ and $1 < s < 2$, let $f : [0,1] \to \mathbb{R}$ be a Weierstrass function modified to include given 'phases' θ_k :

$$f(t) = \sum_{k=1}^{\infty} \lambda^{(s-2)k} \sin(\lambda^k t + \theta_k).$$

Show that $\dim_B \mathrm{graph} f = s$, provided that λ is large enough.

Question 2. Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} 5x & x \leq 1 \\ 10 - 5x & 1 < x < 2 \\ 5x - 10 & x \geq 2. \end{cases}$$

Determine an IFS $S_1, S_2, S_3 : [0,5] \to [0,5]$ such that $f(S_i(x)) = x$ for each i. Show that the attractor F of this IFS is a repeller for f, and determine the Hausdorff and box dimension of F.

Question 3. Let $f_\lambda : [0,1] \to \mathbb{R}$ be given by $f_\lambda(x) = \lambda \sin \pi x$. Show that for λ sufficiently large, f_λ has a repeller F, in the sense that if $x \notin F$, then $f_\lambda^k(x) \notin [0,1]$ for some positive integer k. Find an IFS that has F as its attractor, and thus estimate $\dim_H F$ for large λ.

Question 4. Describe the Julia set of $f(z) = z^2 + 4z + 2$. (Hint: Write $z_1 = z + 2$.)