Fractal Geometry
Assignment 3
Due on Tuesday, April 5th

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Question 1. Let F be the set obtained by a Cantor-type construction in which each interval is replaced by two intervals, one of a quarter of the length at the left-hand end and one of half the length at the right-hand end. Thus, E_0 is the interval $[0, 1]$, E_1 consists of the intervals $[0, 1/4]$ and $[1/2, 1]$, and so on. Find the iterated function system (IFS) with attractor F, and thus find the Hausdorff and box dimensions of F.

Question 2.

[Part I] Describe the attractors of the following IFSs on $\mathbb{R}$.

(i) $S_1(x) = \frac{1}{4}x, S_2(x) = \frac{1}{4}x + \frac{3}{4}$,

(ii) $S_1(x) = \frac{1}{2}x, S_2(x) = \frac{1}{2}x + \frac{1}{2}$,

[Part II] Find a pair of similarity transformations on $\mathbb{R}$ for which the interval $[0, 1]$ is the attractor. How many such pairs of transformations you can find?

Question 3. Let $S_1, S_2 : [0, 1] \to [0, 1]$ be given by $S_1(x) = x/(2+x), S_2(x) = 2/(2+x)$. Show that the attractor F of this IFS satisfies $0.52 < \dim_H F < 0.81$.

Question 4. Let $f, g : [0, 1] \to \mathbb{R}$ be continuous functions such that the box dimension of their graphs exist. Show that $\dim_B \text{graph}(f + g)$ equals the greater of $\dim_B \text{graph } f$ and $\dim_B \text{graph } g$, provided that these dimensions are unequal. Give an example to show that this condition is necessary.