Exercise session 2

Thursday Sept 9, 2021

Problems with odd numbers will be solved in class

1- a) What is the speed of light in diamond if the index of refraction is 2.42 ? (Solution: $v = 1.24 * 10^8$ m/s)
b) Given that the wavelength of a lightwave in vacuum is 540 nm, what will it be in water, where n = 1.33 ?
(Solution: $\lambda = 406$ nm)
c) Determine the index of refraction of a medium if it is to reduce the speed of light by 10% as compared to its speed in vacuum? (Solution: n = 1.11)

2- A student with a near point distance of 250 mm uses an objective with a focal length of 10 mm and an eye piece with a focal length of 25 mm to build a compound microscope. She uses a standard tube length of 160 mm and she builds the microscope such that a magnified virtual image is formed at infinity, such that a sharp image of an object is obtained with a relaxed eye.

a) Draw the microscope and relevant rays. (see Hecht, 5^{th} edition, Fig. 5.110)

b) Calculate the total magnification of the microscope. (Solution: MP = -160)

c) At what distance from the objective should an object be placed to obtain a sharp image with a relaxed eye? (Solution: 10.625 mm)

d) If the microscope contains a circular field stop at the plane of the intermediate image with a diameter of 10 mm, what is then the size of the largest object that can be observed with this microscope? (Solution: 0.625 mm)

3- a) Augustin Louis Cauchy (1789 – 1857) determined an empirical equation for n (λ) for substances that are transparent in the visible. His expression corresponded to the power series relation

$$n = C_1 + C_2 / \lambda^2 + C_3 / \lambda^4 + \ldots$$

where the C_s are all constants. What is the physical significance of C_1 ?
(Answer: $C_1 \sim n$ for >> λ)
b) Crystal quartz has refractive indexes of 1.557 and 1.547 at wavelengths of 410.0 nm and 550.0 nm, respectively. Using only the first two terms in Cauchy's equation, calculate C_1 and C_2 and determine the index of refraction of quartz at 610.0 nm. (Solution: $C_1 = 1.5345$, $C_2 = 3.78 * 10^3$ nm^2 , $n_{610nm} = 1.545$)

4- Yellow light from a sodium lamp (λ = 589 nm) traverses a tank of glycerin (of index 1.47), which is 20.0 m long, in a time t_1. If it takes a time t_2 for the light to pass through the same tank when filled with carbon disulfide (of index 1.63), determine the value of $t_2 - t_1$. (Solution: $t_2 - t_1 = 1.07 * 10^{-8}$ s)

5- A white floodlight beam crosses a large volume containing a molecular gas mixture of mostly oxygen and nitrogen. Compare the relative amount of scattering occurring for the yellow (580 nm) component with that of the violet (400 nm) component. (Solution: I_y / I_v = 22.6%)

6- A laserbeam impinges on an air-liquid interface at an angle of 55°. The refracted ray is observed to be transmitted at 40°. What is the refractive index of the liquid? (Solution: n = 1.27)

7- A ray of yellow light from a sodium discharge lamp falls on the surface of a diamond in air at 45°. If at that frequency n_d = 2.42, compute the angular deviation suffered upon transmission. (Solution: ΔΦ = 28°)

8- Prove that $t_\perp - r_\perp = 1$ for all Φ_p, first from the boundary conditions and then from the Fresnel equations.

9- Use the Fresnel equations to prove that light incident with polarization in the plane of incidence at $\Phi_p = \pi/2 - \Phi_t$ results in transmission =1

10- Making use of the Fresnel equations, show that $t_{||}$ (Φ_p) $t'_{||}$ (Φ'_p) = 1. Where $\Phi_p + \Phi'_p$ =pi/2

11- Semiconductor quantum dots are promising sources of single photons and entangled photons. They are buried in a high refractive index material. Because of this most of the light they emit do not make it out of the high index material due to total internal reflection at the surface of the semiconductor. We can illustrate this problem with simple ray tracing and geometrical considerations (you can ignore polarization and interference): assume that a quantum dot is a point source emitting isotropically in all directions and is located 10 μm below the surface of a semiconductor with refractive index n = 3.7, the medium outside of the semiconductor is vacuum (n = 1).

a) What is the angle of total internal reflection for light emitted by the quantum dot? Draw a schematic and label. What fraction of the total emitted light escapes the semiconductor? (Solution: emitted fraction without reflection 1.9%, interface reflection loss 33%, emitted fraction 1.25%)

b) To collect as much light as possible from a point source, we can choose objectives with high numerical aperture (NA). Approximately what fraction of the total emitted light can you collect using an objective with NA = 0.3 or one with NA = 0.82? (Solution NA = 0.3: emitted fraction 0.11%; NA = 0.82: emitted fraction 0.83%)

c) The easiest way to improve signal collection from such a buried quantum dot is to place a hemispherical solid immersion lens (SIL) with the same refractive index as the semiconductor on the surface directly above the dot, as this will avoid total internal reflection. However, since semiconductor SILs are not readily available and hard to work with, glass SILs are often used instead. How much light can escape the semiconductor substrate if a hemispherical SIL with n = 1.83 (LaSFN9 glass) is placed directly on top of the quantum dot? (Solution: emitted fraction without reflection 6.54%, interface reflection loss glass-semiconductor 11.4%, interface reflection loss glass-air 8.6%)

12- Compute the critical angle for the ordinary ray, that is, the angle for total internal reflection at the calcite-balsam layer of a Nicol prism. (Solution: $\Phi_c = 69°$)

13- What is Brewster's angle for reflection of light from the surface of a piece of glass ($n_g = 1.65$) immersed in water ($n_w = 1.33$) ? (Solution: $\Phi_p = 51.1°$)

14- Consider the common mirage associated with an inhomogeneous distribution of air situated above a warm roadway. Envision the bending of the rays as if it were instead a problem in total internal reflection. If an observer, at whose head $n_0 = 1.00029$, sees an apparent wet spot at $\Phi_c > 88.7°$ down the road, find the index of the air immediately above the road. (Solution: n = 1.000033)

15- We want to develop a new type of optical fibers based on materials with high refractive index.

a) What would be the advantage of a high refractive index fiber? What about disadvantages? (Answer: low critical angle for total internal reflection, high NA, low-loss waveguiding, but high reflection loss at end facet)

b) We make the fiber core with n = 2.0 and a cladding of silica (n = 1.5). What is the angle of total internal reflection at the core-cladding interface? Make a corresponding sketch. What would be the numerical aperture (or acceptance angle) of this fiber? (Solution: NA = 1.32)

c) This fiber will have spectral dispersion of 100 ps / (nm km). If we send short light pulses with square temporal and spectral profiles (10 ps pulse length) that have a spectral width of 0.5 nm through 20 km of fiber, what will be the pulse length at the end of the fiber? (Solution: $\Delta t = 1010$ ps)

d) The fiber core has a diameter of 100 µm. What time dispersion will this result in? Think of the shortest and longest possible paths. (Solution: $\Delta t = 45$ µs)