Exercise session 4

Thursday Sept 30, 2021

Problems with odd numbers will be solved in class

1- Given that 200 W/m^2 of randomly polarized light is incident normally on a stack of ideal linear polarizers that are positioned one behind the other with the transmission axis of the first vertical, the second at 30°, the third at 60°, the fourth at 90°. How much light emerges? (Solution: 42 W/m^2)

2- An ideal polarizer is rotated at a rate ω between a similar pair of stationary crossed polarizers. Show that the emergent flux density will be modulated at four times the rotational frequency. In other words, show that

$$I = \frac{I_1}{8}(1 - cos4\omega t)$$

where I_1 is the flux density emerging from the first polarizer and I is the final flux density.

3- The specific rotatory power for sucrose dissolved in water at 20°C (λ = 589.3nm) is +66.45° per 10 cm of path traversed through a solution containing 1 g of active substance (sugar) per cm^3 of solution. A vertically polarized state (sodium light) enters at one end of a 1m tube containing 1000 cm^3 of solution, of which 10g is sucrose. At what orientation will the light emerge? (Solution: 6.645° from vertical, clock-wise)

4- Suppose you were given a linear polarizer and a quarter-wave plate. How could you determine which was which, assuming you also had a source of natural light? (Hint: consider light transmission through quarter-wave plate followed by polarizer and through polarizer followed by quarter-wave plate).

5- Confirm that the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

will serve as a Mueller matrix for a quarter-wave plate with its fast axis at +45°. Shine linear light polarized at 45°through it. What happens? (Solution: light unchanged) What emerges when a horizontally polarized state enters the device? (Solution: light with circular polarization R)

6- We are building a setup for measurement of polarization for a quantum entanglement experiment. We have a detector with a fixed polarizer in front. The polarizer is oriented horizontally. We have a half-wave plate and a quarter-wave plate that can be placed in front of the polarizer and that can be rotated to any angle.

a) How do you measure horizontal (H = 0º), vertical (V = 90º), diagonal (D = 45º), and anti-diagonal (A = -45º) polarization? Which wave plate should be used and under what angle?
(Solution: H: no wave plate needed or half-wave plate at 0º. V: Half-wave plate at 45º. D: half-wave plate at 22.5º. A: Half-wave plate at -22.5º.)

b) How do you measure left-handed circular (L) and right-handed circular (R) polarization? Which wave plate should be used and under what angle?
(Solution: L: Quarter-wave plate at 45º. R: Quarter wave plate at -45º.)

7- Two incoherent llght beams represented by the Stokes vectors (1, 1, 0, 0) and (3, 0, 0, 3) are superimposed.

(a) Describe the polarization states of each of these.
(Solution: see Hecht, Table 8.5)
(b) Determine the resulting Stokes parameters of the combined beam and describe its polarization state.
(Solution: (4, 1, 0, 3) with both H and R components)
(c) What is its degree of polarization?
(Solution: V=0.79)
(d) What is the resulting light produced by overlapping the incoherent beams (1, 1, 0, 0) and (1, -1, 0, 0)?
(Solution: unpolarized light)

8- Show by direct calculation, using Mueller matrices, that a unit-irradiance beam of natural light passing through a vertical linear polarizer is converted into a vertically polarized state. Determine its relative irradiance and degree of polarization. (Solution: relative irradiance 0.5; V=1)

9- Show by direct calculation, using Mueller matrices, that a unit-irradiance beam of natural light passing through a linear polarizer with its transmission axis at +45° is converted into a polarized state at +45°. Determine its relative irradiance and degree of polarization. (Solution: relative irradiance 0.5; V=1)

10- An optical filter can be described by a Jones matrix

$$\begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$$

Obtain the form of the emerging beam when the incident light is plane polarized at angle Θ to the horizontal. Deduce from this result the nature of the filter and confirm your deduction by testing it mathematically. (Solution: see Hecht, Exercise 8.93)

11- a) The Jones vector for an arbitrary linearly polarized state at an angle Θ with respect to the horizontal is (cosΘ , sin Θ). Prove that this is in agreement with the Jones vector for linear polarized light at 45° [1/√2, 1/√2].
b) Find a Jones vector E_2, representing a polarization stale orthogonal to E_1 = (1, -2i).

12- Red plane waves from a ruby laser (λ = 694.3 nm) in air impinge on two parallel slits in an opaque screen. A fringe pattern forms on a distant wall, and we see the fourth bright band 1.0° above the central axis. Calculate the separation between the slits. (Solution: $1.6 \cdot 10^{-4}$m)

13- A thin film of ethyl alcohol (n = 1.36) spread on a flat glass plate and illuminated with white light shows a color pattern in reflection. If a region of the film reflects only green light (500 nm) strongly, how thick is it? (Solution: $9.2 \cdot 10^{-6}$m)

14- Suppose we place a chamber 10.0 cm long with flat parallel windows in one arm of a Michelson Interferometer that is being illuminated by 600 nm light. If the refractive index of air is 1.00029 and all the air is pumped out of the cell, how many fringe-pairs will shift by in the process? (Solution: N=97)

15- Sunlight incident on a screen containing two long narrow slits 0.20 mm apart casts a pattern on a white sheet of paper 2.0 m beyond. What is the distance separating the violet (λ = 400 nm) in the first-order band from the red (λ = 600 nm) in the second-order band? (Solution: Δy=8mm)