

Luminescence of excitons & Highly excited semiconductors





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Chapter 7 & Chapter 8





Classification of luminescence by intensity

Low-fluence (Weak) excitation (0.01- 10 W/cm²): gas-discharge lamp, incandescent lamp, continuous-wave gas laser

-  - Recombination of **free excitons** (FE or X; FE-LO phonon)
-  - Recombination of a **bound exciton** (BE): (D⁰-X; A⁰-X; D⁺-X; A⁻X, isoelectronic impurity)
-  - Recombination of **donor-acceptor pairs** (D⁰-A⁰)
-  - Recombination of **free hole/electron** with **neutral donor/acceptor** (h-D⁰; e-A⁰; e-h)

$$1 \text{ sun} = 0.1 \text{ mW/cm}^2 = 0.1 \text{ W/cm}^2 = 1 \text{ kW/m}^2$$

High-fluence (Strong) excitation (1 kW-10 MW/cm²):

-  - Radiative decay of **excitonic molecule** (EM or XX)
-  - Luminescence from inelastic collisions of excitons (X-X collisions)
-  - Luminescence of electron-hole liquid (EHL) or electron-hole plasma (EHP)
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These excitations can occur simultaneously, in these two chapters we focus on low-fluence excitation and neglected luminescence from excitons

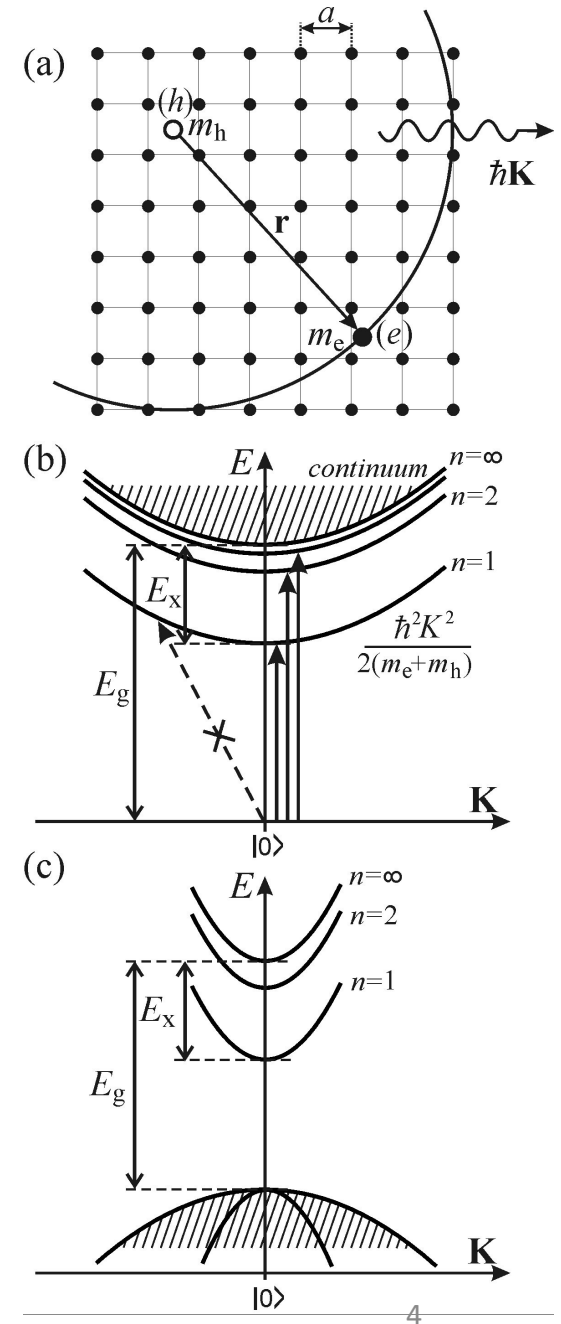
Excitons

- Electron and hole not independent, treated as one quasi particle
 - Energy $< E_g$ because of Coulomb interaction
- Three types
 1. Frenkel exciton
 - Distance about one unit cell
 - Molecular crystals
 2. Charge transfer exciton
 - A bit larger radius
 - Ionic crystals
 3. **Wannier exciton**
 - Separation over many lattice constants
 - Moves freely, delocalized wavefunction (if not bound, we will discuss later)
 - Free excitons
- **Bound excitons**

Wannier exciton

- Radius estimated to be $a_x = 100 \cdot a_b = 5 \text{ nm}$
- Binding energy estimated to be 13 meV
 - Low T needed
- Proper treatment if wavefunction \rightarrow radius and binding energy
- Excitons with kinetic energy
 - Bound excitons have less kinetic energy, localized
- Dispersion relation $E(\mathbf{k})$
 - Here we do not have a free electron approximation

$$E_{(n)}(\mathbf{K}) = E_g - E_{X(n)} + E_{\text{kin}} = E_g - \frac{(m_r/m_0) Ry(H)}{\epsilon^2 n^2} + \frac{\hbar^2 K^2}{2(m_e + m_h)} \quad (7.5)$$



Wannier exciton – Absorption spectrum

Direct bandgap

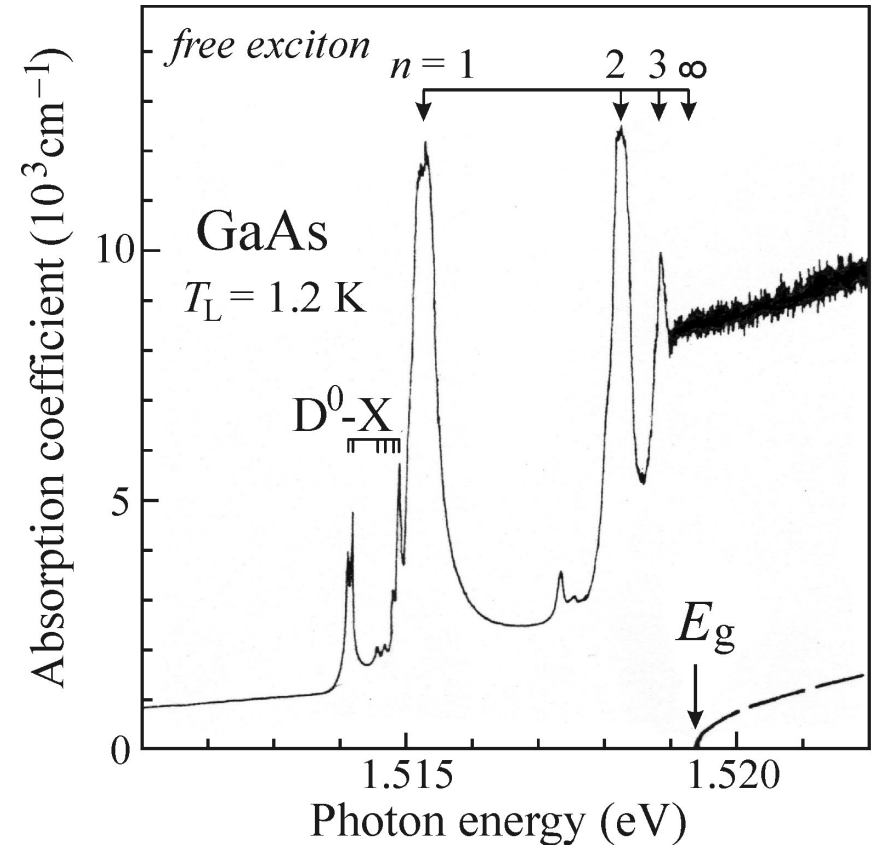
- Dipole approximation

$$\alpha(h\nu) \approx (h\nu - E_g)^{1/2}, \quad h\nu \geq E_g.$$

- $T = 0\text{K}$, no kinetic energy \rightarrow only vertical transitions (\mathbf{k})
- As in hydrogen atom, absorption line series

$$h\nu = E_g - \frac{E_X}{n^2}, \quad n = 1, 2, 3, \dots \infty;$$

- Intensity is proportional to $1/n^3$
- Also, enhanced absorption above E_g



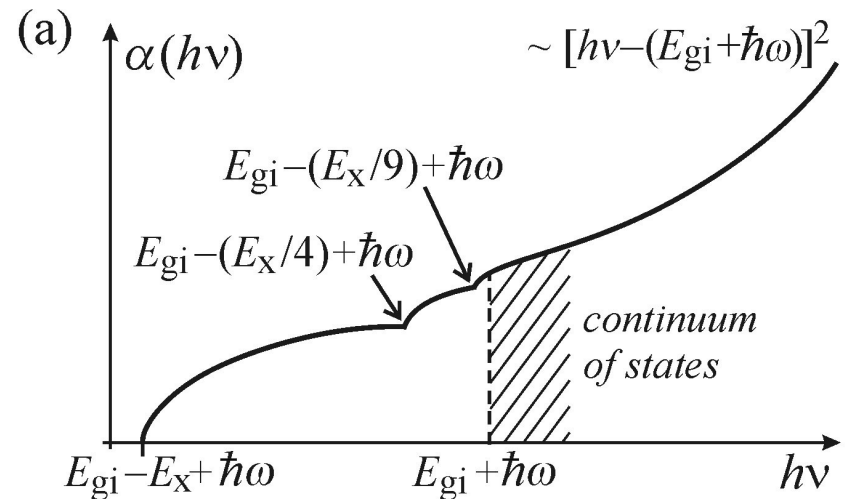
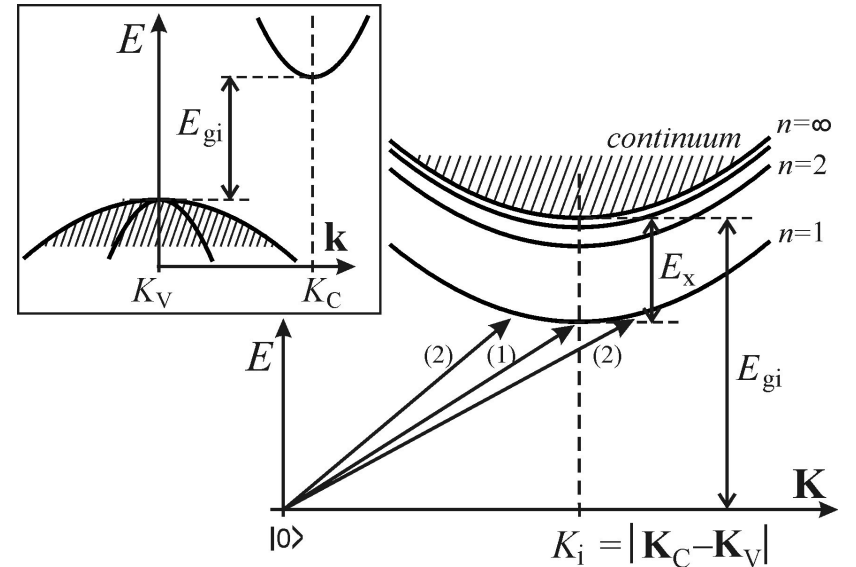
Wannier exciton – Absorption Spectrum

Indirect bandgap

- Group velocity of e and h equal

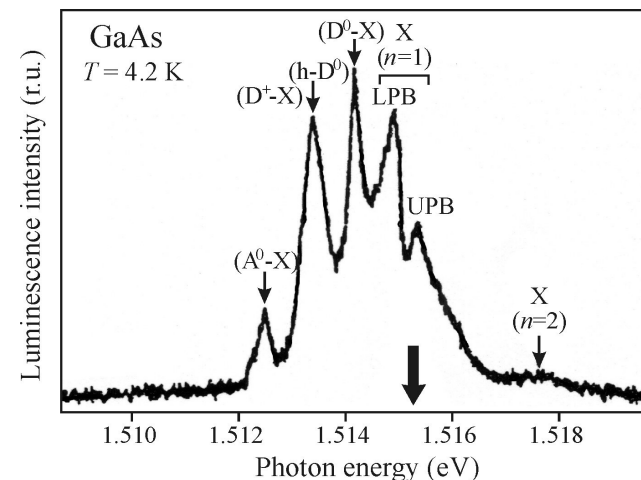
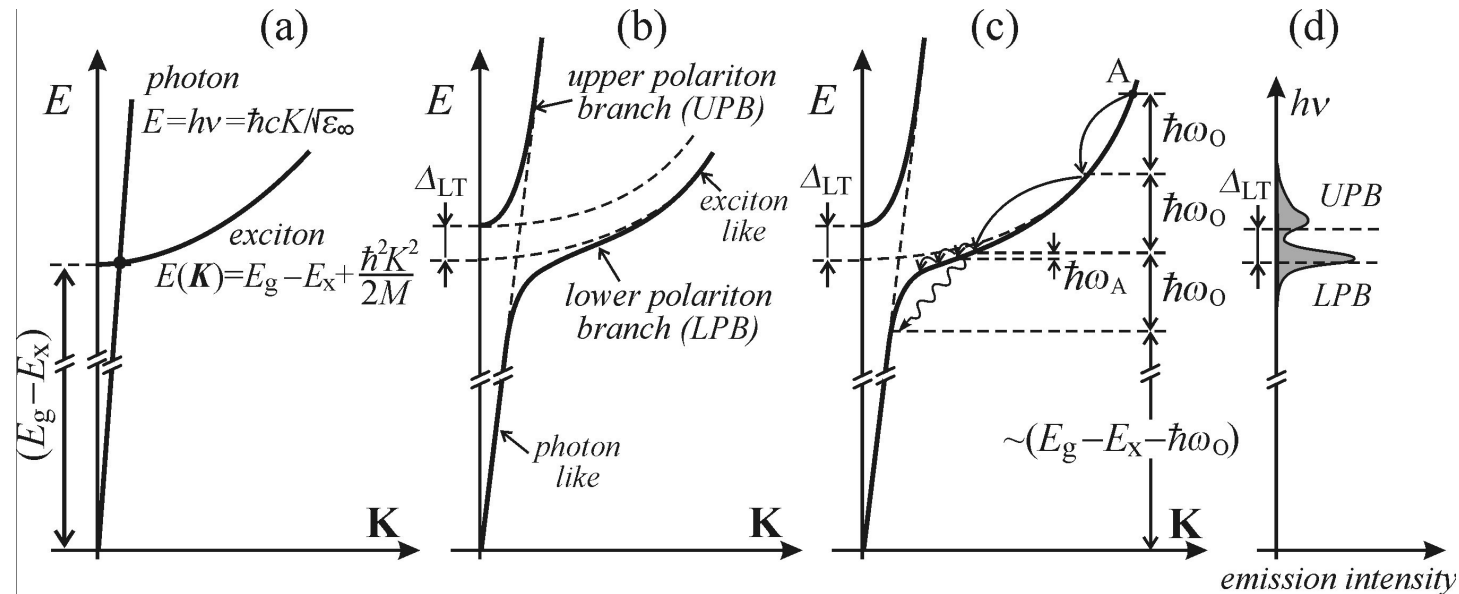
$$\mathbf{V}_g = \frac{1}{\hbar} \frac{\partial E}{\partial \mathbf{k}}$$

- Minimum energy if $K_i = |\mathbf{K}_c - \mathbf{K}_v|^2$
- Onset of absorption $h\nu = E_{gi} - E_X + \hbar\omega$
- Transition (1) and (2) equally probable \rightarrow density of states important
- Note: excitons with non-zero kinetic energy possible!



Resonant luminescence : direct E_g : free exciton-polaritons

- Absorption != emission
- Strong reabsorption
- Photon-Exciton dispersion curves combined
- Degeneracy lifted : band splitting
- No simple equation

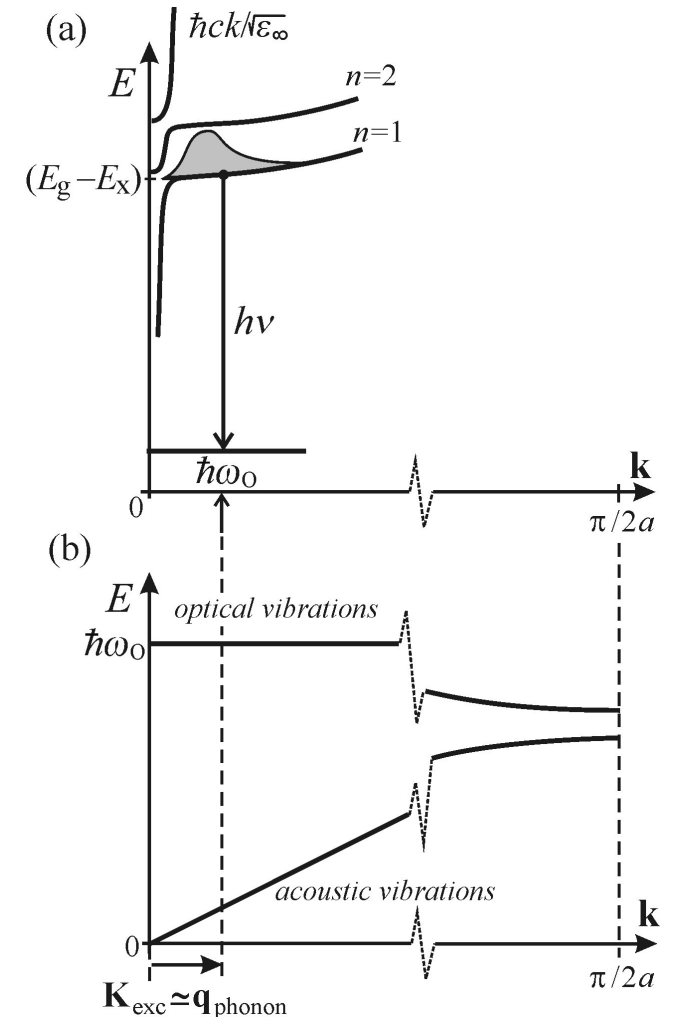


Luminescence : direct E_g : free excitons and OP

$$h\nu_{X-mLO} \approx (E_g - E_X) - m\hbar\omega_0, \quad m = 1, 2, 3, \dots$$

- Very strong lines, why?
 - Energy of photon $< E_g$
 - All excitons regardless of \mathbf{k} can participate
- Lineshape Maxwell-Boltzmann + variation
 - Probability of phonon creation

$$I_{sp}^{(m)}(h\nu) \approx (h\nu - [(E_g - E_X) - m\hbar\omega_0])^{1/2} \\ \times \exp\left[-\frac{h\nu - [(E_g - E_X) - m\hbar\omega_0]}{k_B T}\right] \\ W^{(m)}(m\mathbf{q}_{\text{phonon}} \approx \mathbf{K}_{\text{exc}}),$$



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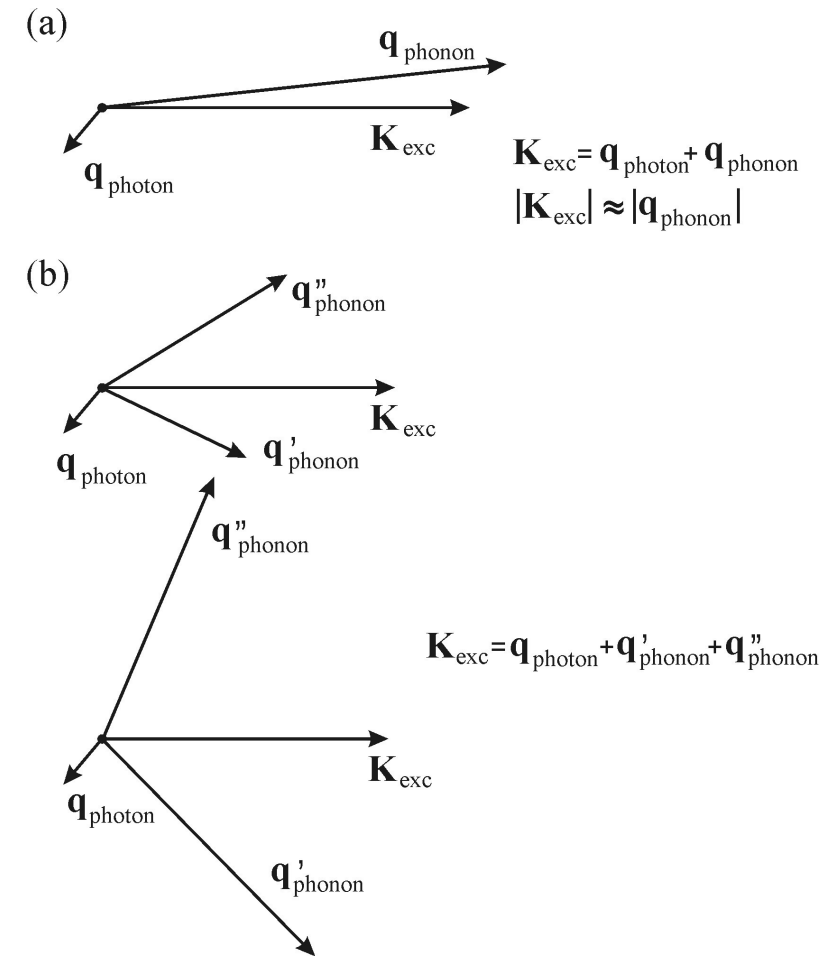
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$$W^{(m)}(m\mathbf{q}_{\text{phonon}} \approx \mathbf{K}_{\text{exc}}),$$

- $m = 1 \rightarrow$ phonons with λ similar to linear size of exciton are more probable $W^{(1)}(\mathbf{q}_{\text{phonon}} \approx \mathbf{K}_{\text{exc}}) \sim K_{\text{exc}}^2$

$$I_{sp}^{(1)}(h\nu) \approx (h\nu - [(E_g - E_X) - \hbar\omega_0])^{3/2} \exp\left[-\frac{h\nu - [(E_g - E_X) - \hbar\omega_0]}{k_B T}\right]$$



Luminescence : direct E_g : free excitons and OP

$$h\nu_{X-mLO} \approx (E_g - E_X) - m\hbar\omega_0, \quad m = 1, 2, 3, \dots$$

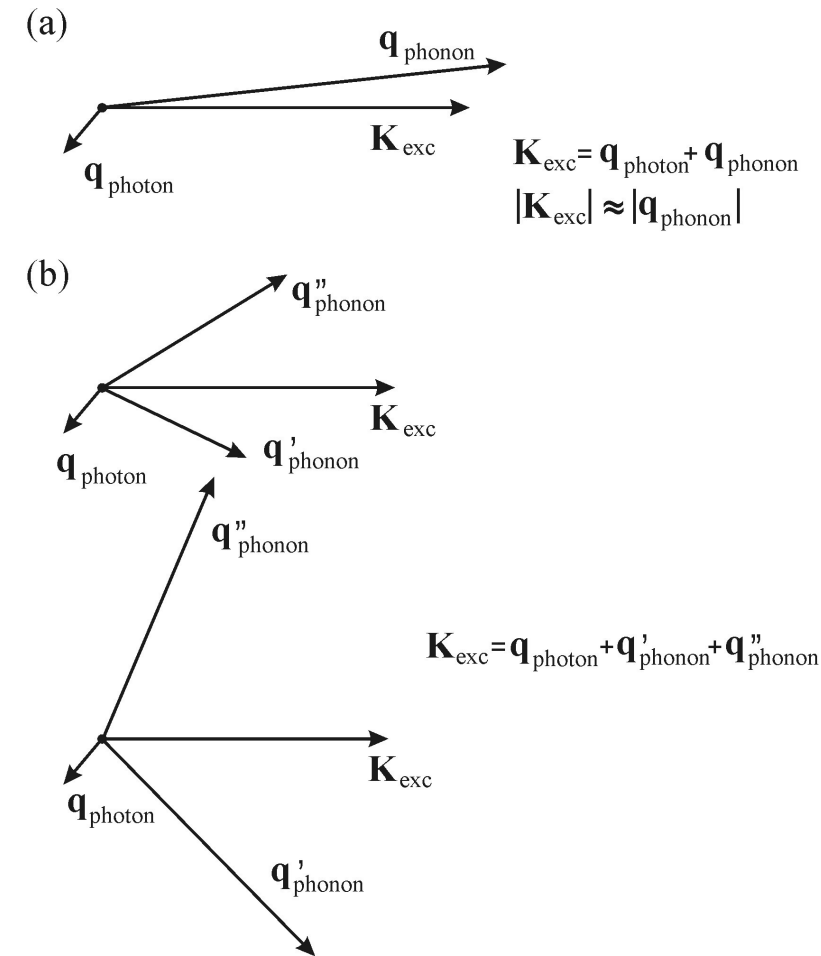
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$$W^{(m)}(m\mathbf{q}_{\text{phonon}} \approx \mathbf{K}_{\text{exc}}),$$

- $m = 2 \rightarrow$ many possibilities, Maxwell like

$$I_{sp}^{(2)}(h\nu) \approx (h\nu - [(E_g - E_X) - 2\hbar\omega_0])^{1/2} \exp\left[-\frac{h\nu - [(E_g - E_X) - 2\hbar\omega_0]}{k_B T}\right]$$



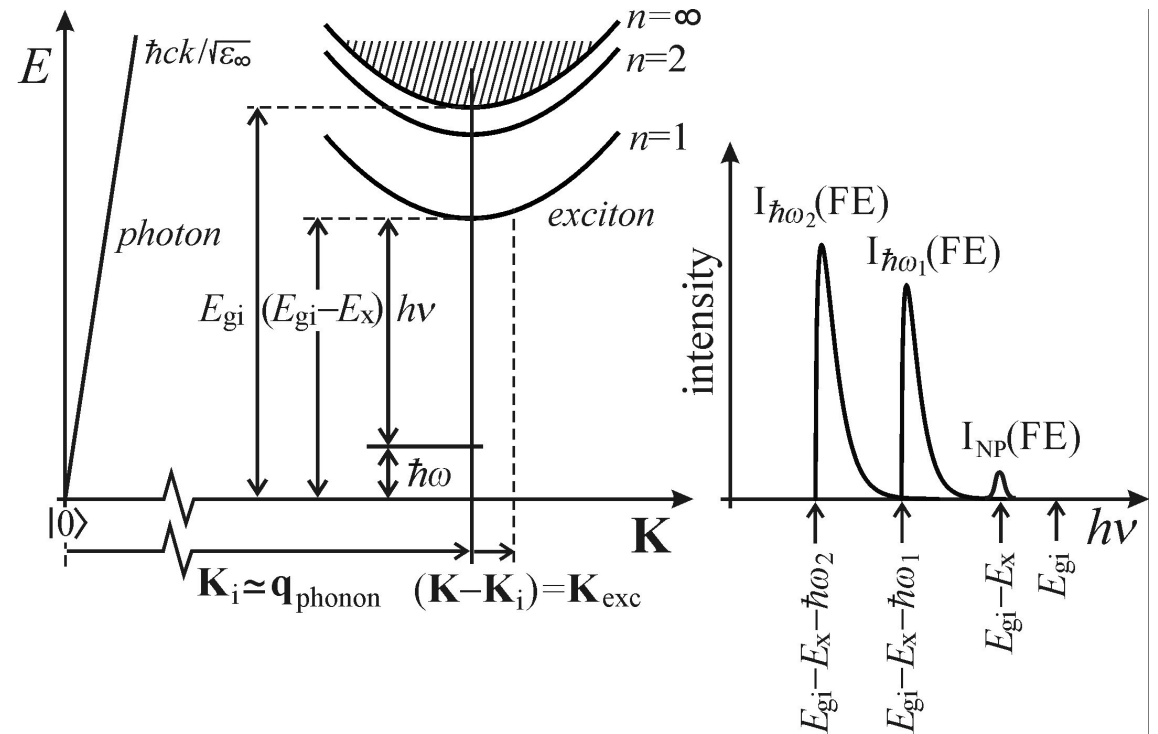
Luminescence : direct E_g : free excitons and OP

$$h\nu_{X-m\text{LO}} \approx (E_g - E_X) - m \hbar\omega_0, \quad m = 1, 2, 3, \dots$$

- Free exciton recombination is a **linear** function of excitation intensity
- Acoustic phonons have too small momentum
- Has to be optical ones – are they allowed?
 - Symmetry argument : YES

Luminescence : indirect E_{gi} : free excitons

- Exciton and phonon dispersion curves don't cross – no polaritons
- Phonons assist: whole population can recombine
- Probability doesn't depend on phonon momentum \rightarrow Maxwell-Boltzmann







$$I_{\text{sp}(i)}^{(1)}(h\nu) \cong (h\nu - [(E_{gi} - E_x) - \hbar\omega])^{1/2} \exp - \frac{h\nu - [(E_{gi} - E_x) - \hbar\omega]}{k_B T}$$

Luminescence : indirect E_g : free excitons

- Which phonon will participate?
 - Selection rules based on symmetry
 - Si: LO, TO, LA, TA but LA and LO degenerate at X
- Impurities in crystals make transitions without phonon possible
- Linear dependence on pump intensity





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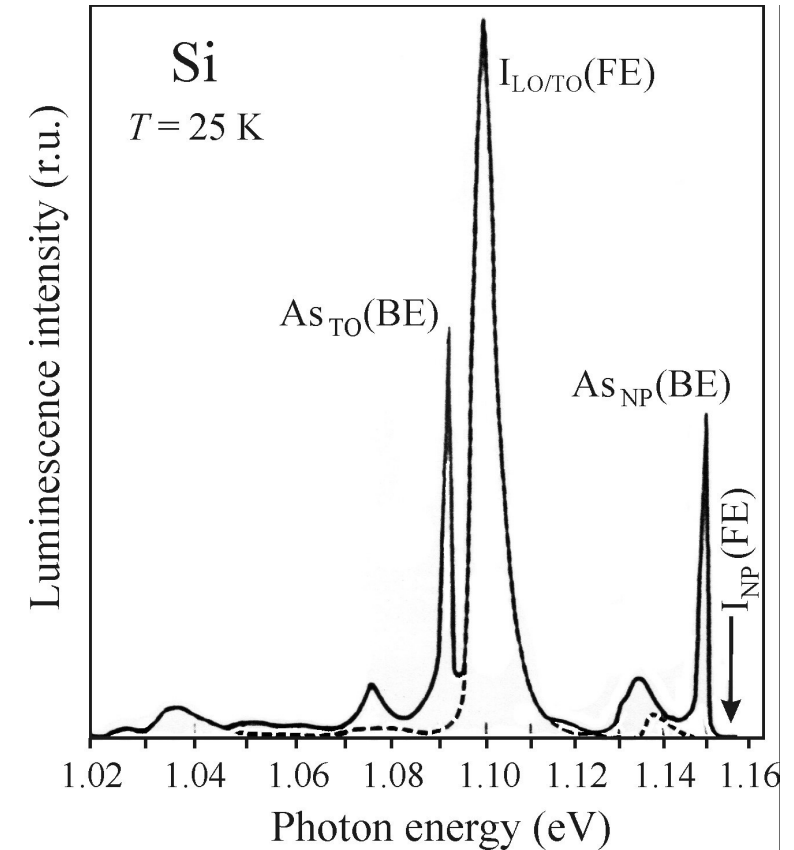
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Bound Excitons

- Localized, loses kinetic energy
- Extrinsic
- Dominates free excitation
 - Large radius: likely to encounter a trap
 - Giant strength from all unit cells within the volume
- PL can give us defects!
- No kinetic energy --> no broadening with T
- Linear dependency on intensity until saturation
- Lower $h\nu$ compared to free excitation, no kinetic energy



Bound Excitations : shallow impurities

- eelectron,
- hhole,
- $\oplus e$neutral donor; also D^0 ,
- $\ominus h$neutral acceptor; also A^0 ,
- FE.....free exciton,
- \oplusionized donor; also D^+ ,
- \ominusionized acceptor; also A^- ,
- $\oplus eh$exciton bound to ionized donor; also (D^+-X) or (D^+, X) ,
- $\ominus eh$exciton bound to ionized acceptor; also (A^--X) or (A^-, X) ,
- $\oplus eeh$exciton bound to neutral donor; also (D^0-X) or (D^0, X) ,
- $\oplus heh$exciton bound to neutral acceptor; also (A^0-X) or (A^0, X) .

$$\left. \begin{array}{l} \oplus + FE \rightarrow \oplus eh + D_1 \\ \oplus eh \rightarrow \oplus + h\nu_{BE} \end{array} \right\} h\nu_{BE} = FE - D_1 \cong (E_g - E_X) - D_1$$

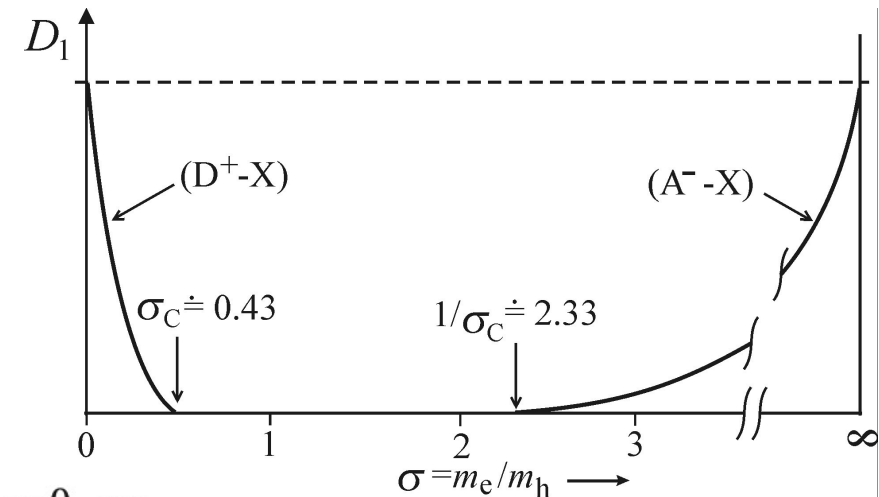
Bound Excitations : shallow impurities

D1 • Ionized

- $\oplus eh$exciton bound to ionized donor; also (D^+-X) or (D^+, X) ,
- $\ominus eh$exciton bound to ionized acceptor; also (A^--X) or (A^-, X) ,

- D1 hard to compute theoretically

- Effective mass
 - Light hole in +eh will break away from the neutral +e
 - Heavy = small kinetic energy
- A—X unlikely holes are usually heavier



E_{BX} • Neutral : stable for any effective mass ratio

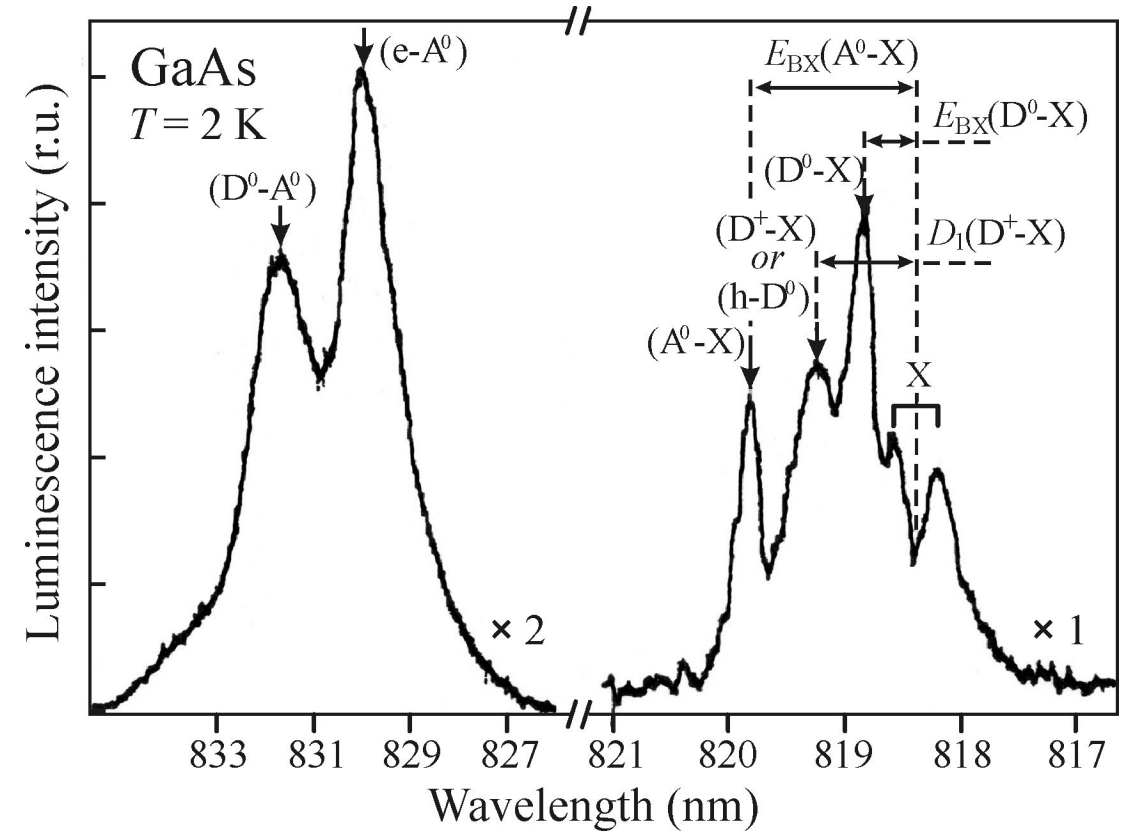
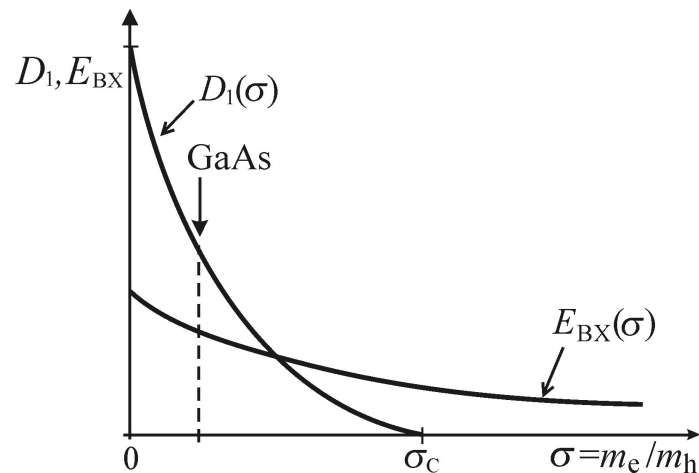
- $\oplus eeh$exciton bound to neutral donor; also (D^0-X) or (D^0, X) ,
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$$h\nu_{BE} = FE - E_{BX} = (E_g - E_X) - E_{BX} \quad \text{Direct}$$

$$h\nu_{BE} = FE - E_{BX} = (E_{gi} - E_X - \hbar\omega) - E_{BX} \quad \text{Indirect}$$

Bound Excitations : shallow impurities

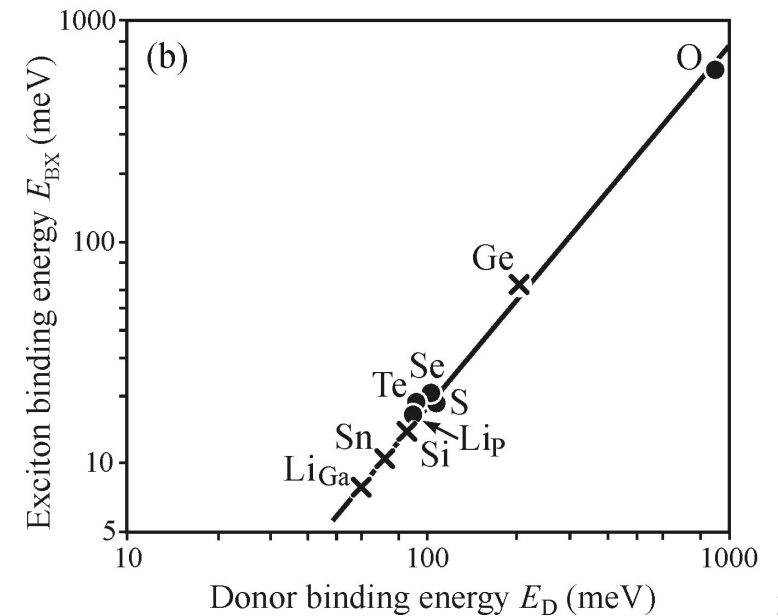
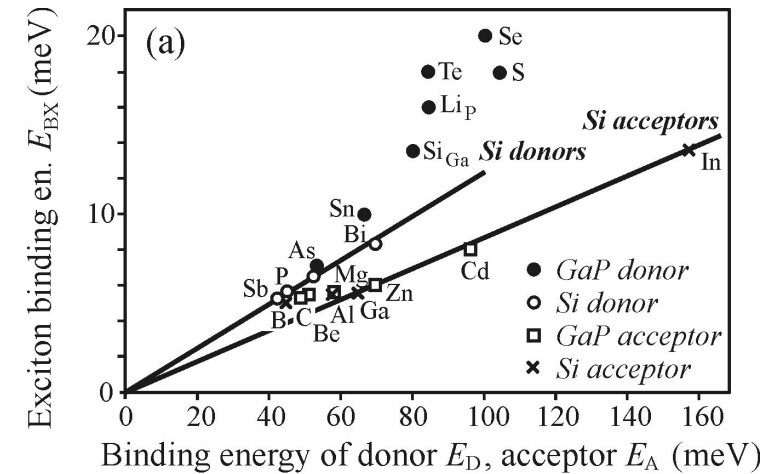
- Here: $D_1 > E_{BX}$
 - $D_1(D^+-X)$ emission more redshifted than $E_{BX}(D^0-X)$
- This is material dependent



Bound Excitons : shallow impurities

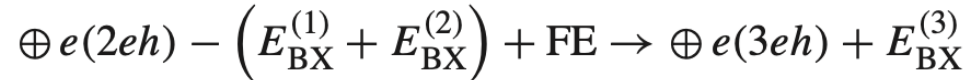
- Hayne's rule
 - Not only effective masses, also atom itself!
 - Si: $a = 0$, $b = 0.1$

$$E_{\text{BX}} = a + b E_{\text{D}}$$

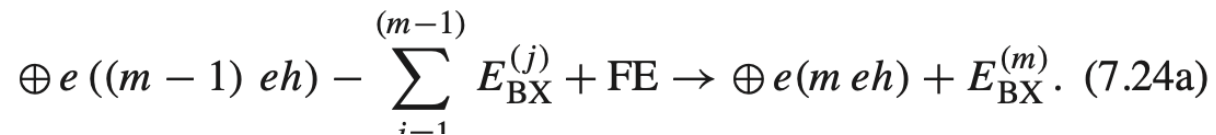


Bound multiexciton complexes

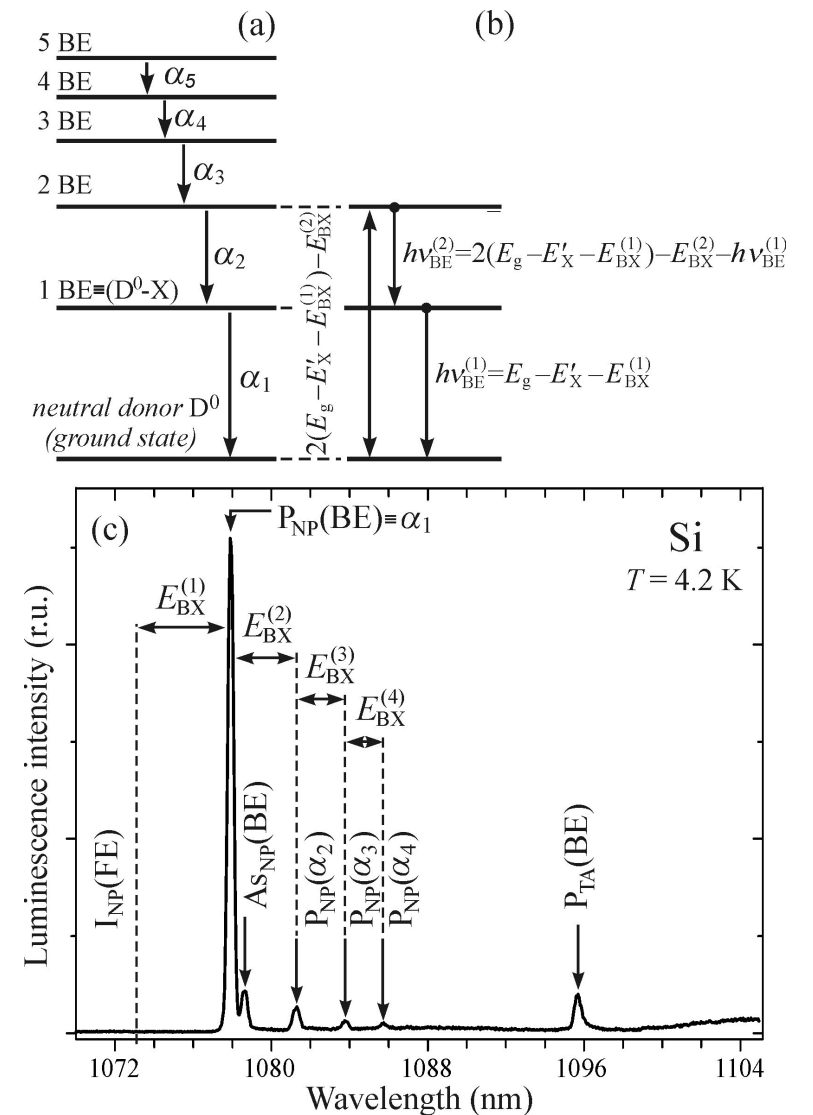
- Shift of emission line by sum of all binding energies



⋮



- Intensity decreases for higher m
- More probable in indirect because longer lifetime



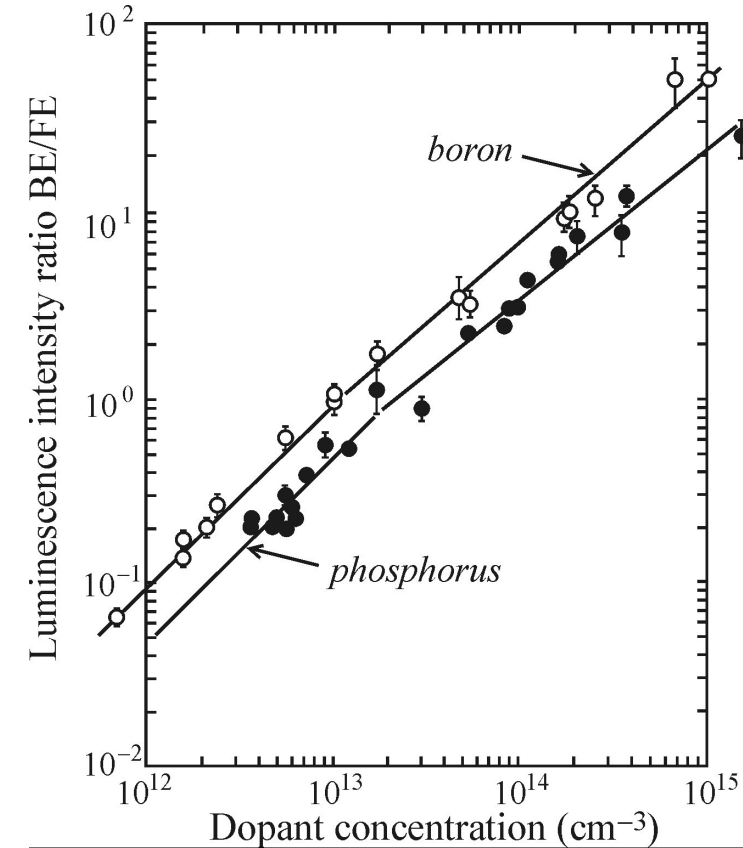
Quantitative analysis of shallow impurities in Si

- BE/FE
- Challenges:
 - Detector range
 - High S/N
 - High resolution
 - Need low excitation

Table 7.2 Spectral positions of luminescence lines (TO-replicas) due to excitons bound at various impurities in Si; $T = 4.2 \text{ K}^*$)

Impurity	Type	Wavelength (nm)
P	SD	1135.13
As	SD	1135.97
Sb	SD	1135.04
Bi	SD	1138.32
Li	ID	1133.74
B	SA	1134.39
Al	SA	1135.60
Ga	SA	1136.15
In	SA	1144.83
Tl	SA	1177.59
C	I	1164.36
free exciton	-	1129.76

*)SD stands for a substitutional donor, ID interstitial donor, SA substitutional acceptor, I isoelectronic impurity.



Bound Excitons : isoelectric impurities

- Impurities from the same group
 - Same number of valence electrons
 - N for P in GaP & I for Br in AgBr
- Electronegativity
- Lineshape
 - Short range forces : no broadening
 - Exciton-phonon interaction
 - Weak --> narrow line (small reaction from surrounding lattice)
 - Strong --> broader line (larger interaction with surrounding lattice)
- Effective enhancement!

Iodide impurity emission

$$z_{\text{ef}} = \frac{I_{\text{LO}}(\text{BE})}{I_{\text{TO}}(\text{FE})} \frac{\tau_{\text{Iod}}}{\tau_{\text{FE}}} \approx 10^4 \times \left(10^3/3\right) \approx 3 \times 10^6(!)$$

Free exciton emission

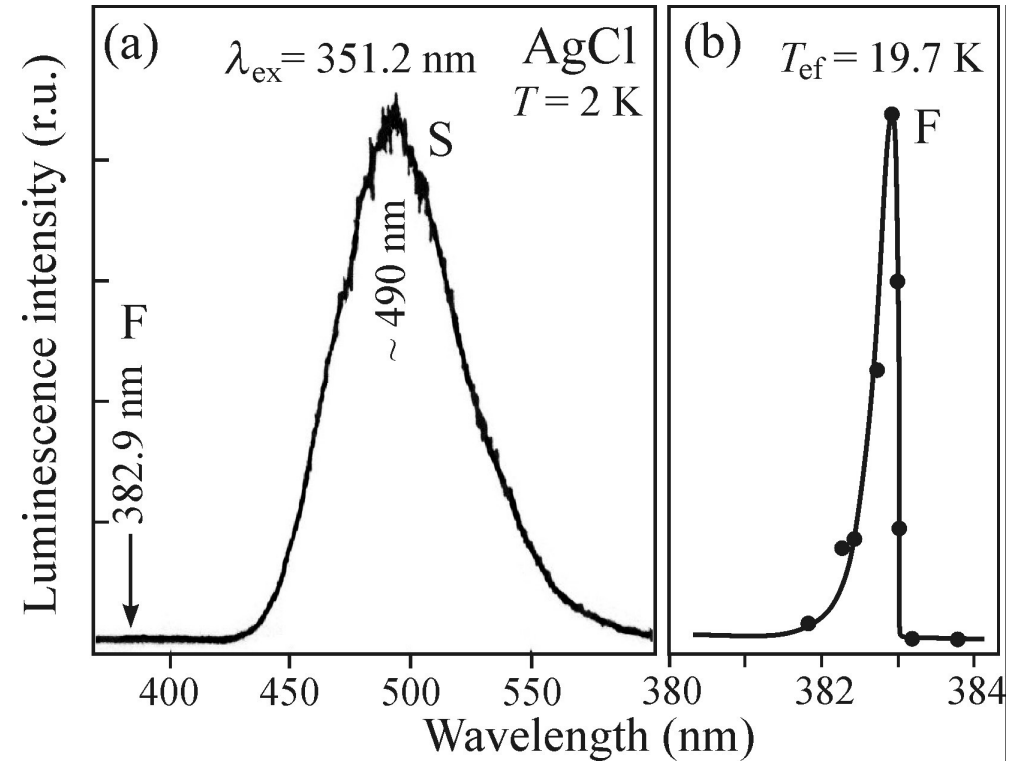
Bound Excitons : isoelectric impurities

- Why such effective enhancement?
 - Trapping
 - No Auger recombination
 - Strong localization in x \rightarrow strong delocalization in k
 - $\Delta x \Delta k \leq 1/2$
 - Can stretch all the way to the direct E_g !

$$z_{\text{ef}} = \frac{I_{\text{odLO}}(\text{BE})}{I_{\text{TO}}(\text{FE})} \frac{\tau_{\text{Iod}}}{\tau_{\text{FE}}} \approx 10^4 \times \left(10^3/3\right) \approx 3 \times 10^6 (!)$$





Self trapped excitons

- Strong exciton phonon interaction
- A moving exciton polarize its surroundings (hole is the heavier, drives)
 - \rightarrow moves slower \rightarrow more polarization \rightarrow slows down even more \rightarrow more polarization \rightarrow stands still = localized
- Local energy minima







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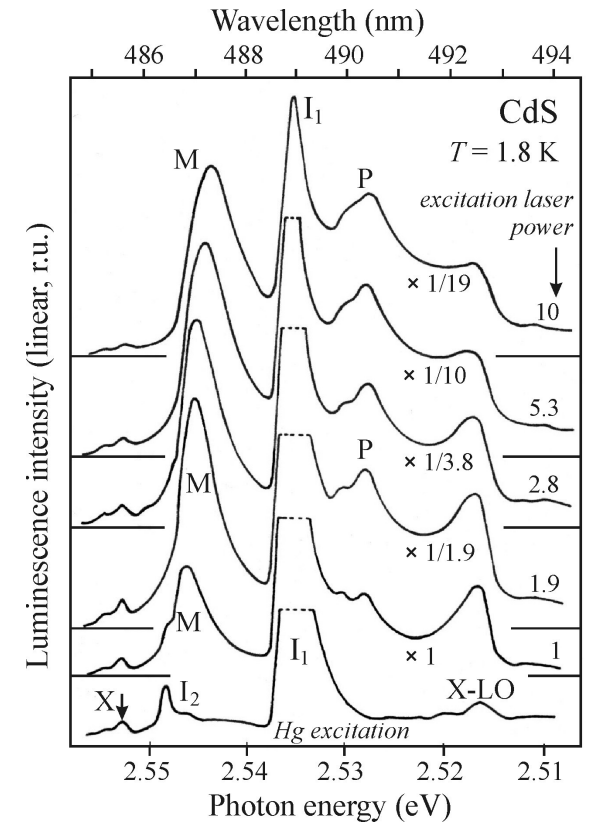
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Highly Excited Semiconductors

- Until now only $0.01 - 10 \text{ W/cm}^2$
- Now: $1\text{k}-1\text{MW/cm}^2$
 - Pulsed laser
- New emission lines:
 - Radiative decay of excitonic molecules (=biexcitons) **M**
 - *Collisions between excitons* **P**
 - *Luminescence of electron hole liquid or electron hole plasma*
 - *Bose Einstein condensation of excitons or biexcitons*



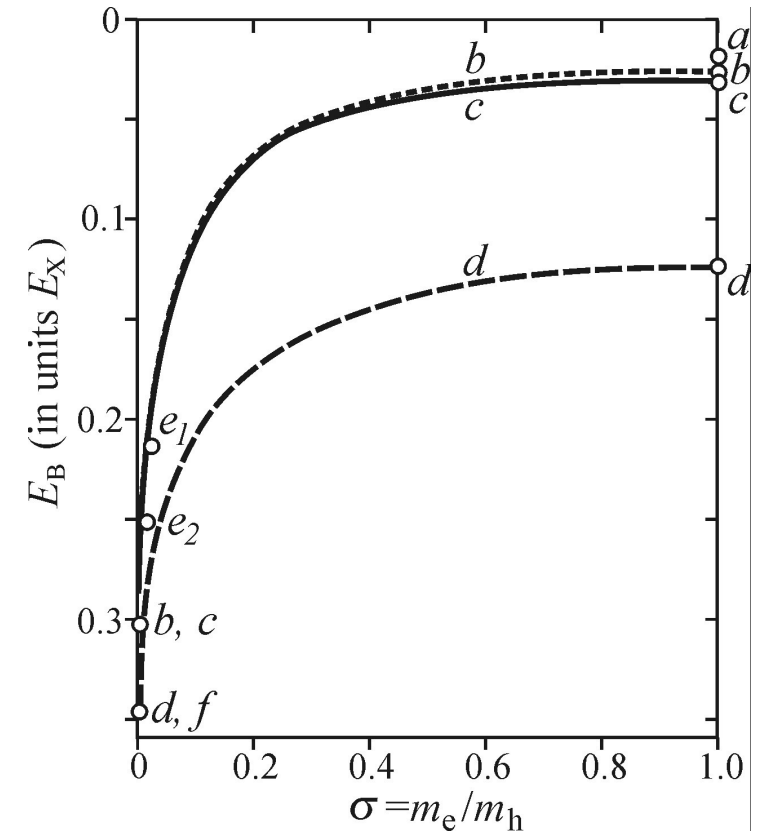
} Not today ☹️

Highly Excited Semiconductors

- When power is increased the concentration of free excitons increase
- When the mean separation is about two times the separation between electron and hole \rightarrow interaction
- Reality: density not homogenous but rapid decrease from surface
 - Two photon excitation
- Indirect $E_g \rightarrow$ lower threshold
- Mechanical threshold!

Excitonic molecules / Biexcitons

- Fusion of two excitons into one quasi-particle
- Stability depends on ratio between effective masses
- Typical values 0.5-0.1 $\rightarrow E_b = 0.1E_x$
 - Less resistant to thermal dissociation
 - 1.3-50K needed
- Radiative recombination:
 - Biexcitation \rightarrow free exciton ($n=1$ state) and one photon
 - M line

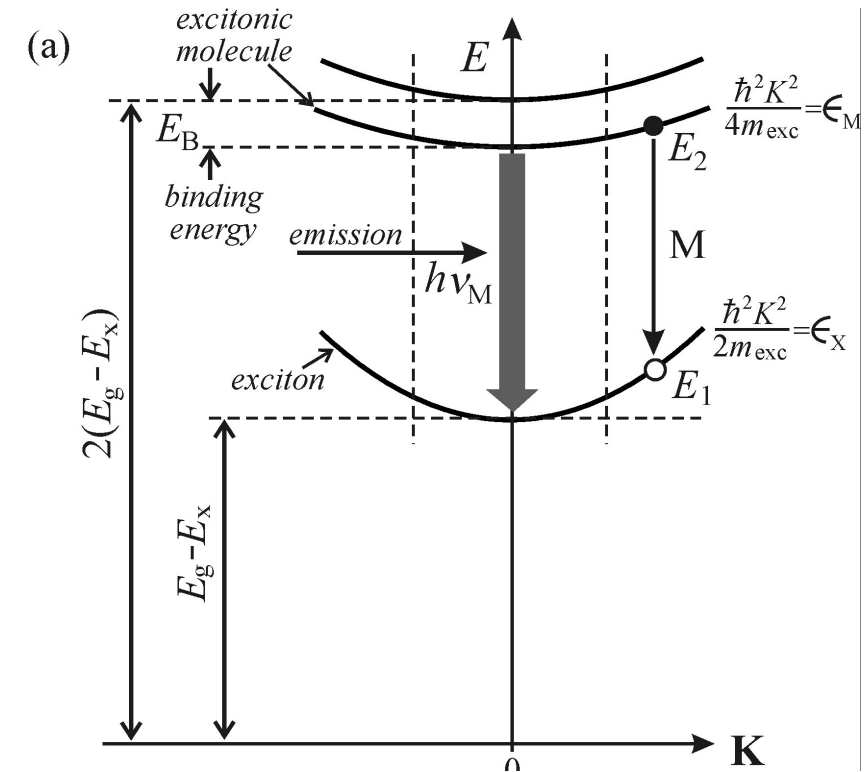


Biexcitons EM

Direct semiconductor

- Fermi's golden rule
- M_M independent of $h\nu$
- Effective temperature T_M
- Population factor f_M : Even number of fermions: total spin integer: bosons: don't have to take population factor into account
- Inverse Maxwell-Boltzmann distribution
- Recombining exciton takes a recoil \mathbf{k}

$$I_{sp}^M(h\nu) \sim \rho_M(h\nu) f_M(h\nu) |M_M|^2.$$



$$\begin{cases} 2(E_g - E_X) - E_B = h\nu_M + (E_g - E_X) \\ h\nu_M = (E_g - E_X) - E_B \end{cases}$$

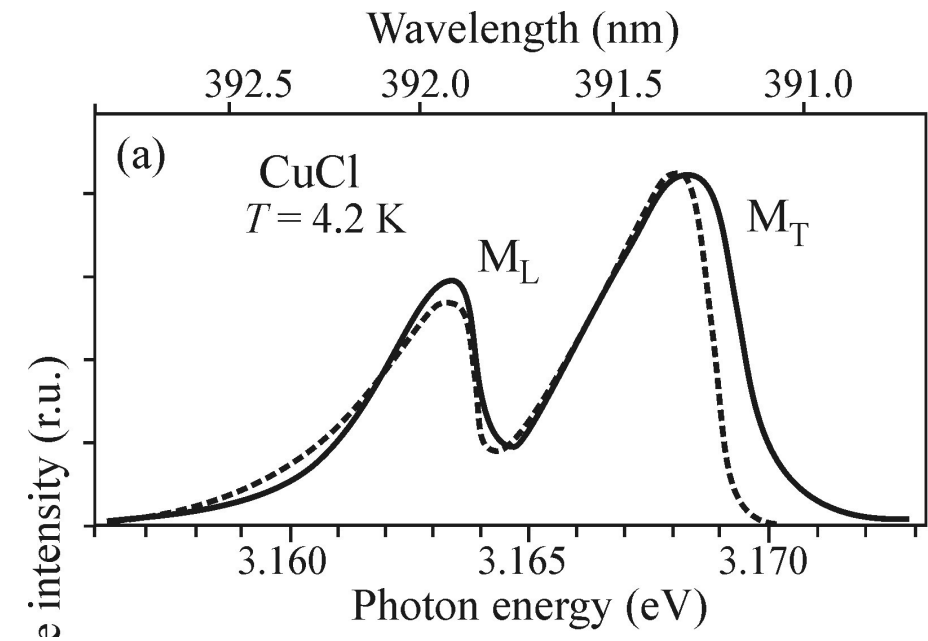
$$I_{sp}^M(h\nu) \cong [(E_g - E_X - E_B) - h\nu]^{1/2} \exp\{-[(E_g - E_X - E_B) - h\nu] / k_B T_M\}$$

Biexcitons EM

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Biexcitons EM

Indirect semiconductor

- Phonon assistance
- Can't use joint density of states

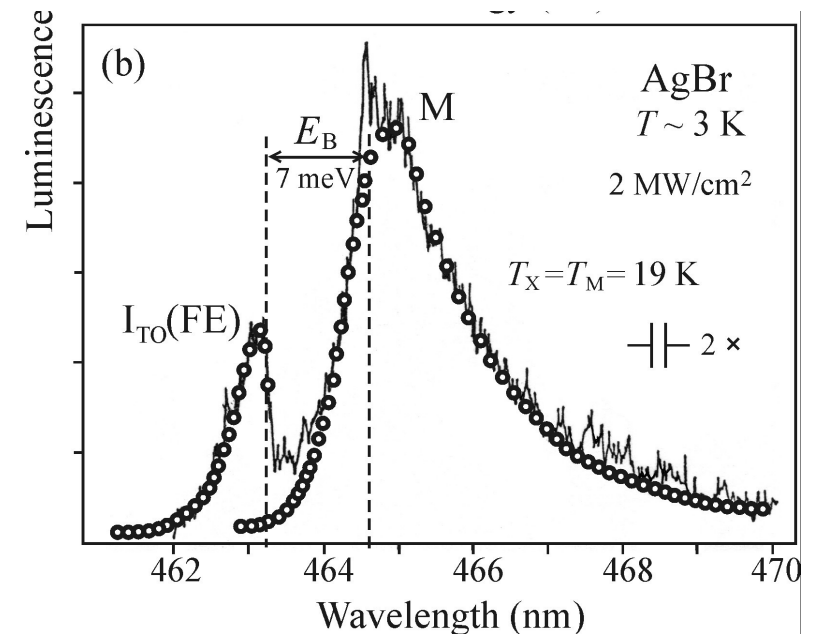
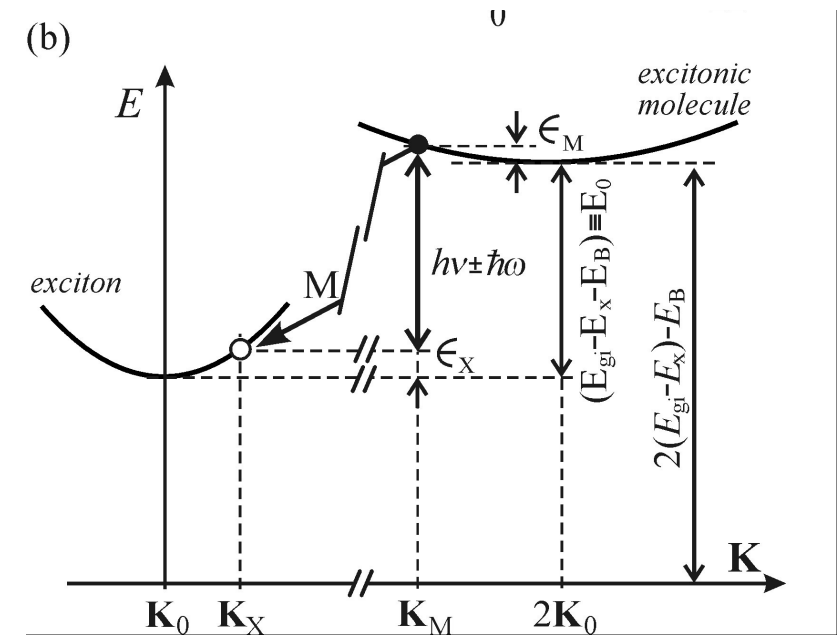
$$I_{\text{in}}^{\text{M}}(h\nu) \cong \int \rho_{\text{M}}(\epsilon_{\text{M}}) f_{\text{M}}(\epsilon_{\text{M}}) \rho_{\text{X}}(\epsilon_{\text{X}}) f_{\text{X}}(\epsilon_{\text{X}}) \left| M_{\text{in}}^{\text{M}} \right|^2 d\epsilon_{\text{M}}$$

- Parabolic approximation of densities of states
- Lower integration limit $\bar{h\nu} = h\nu + \hbar\omega - E_0$.
- Bosons in groundstate : ignore f_{x}

$$f_{\text{M}}(\epsilon_{\text{M}}) \approx \exp(-\epsilon_{\text{M}}/k_{\text{B}}T_{\text{M}})$$

- Matrix element not constant: changes with k

$$I_{\text{in}}^{\text{M}}(h\nu) \approx \int_{\bar{h\nu}}^{\infty} \sqrt{\epsilon_{\text{M}}} \sqrt{\epsilon_{\text{M}} - \bar{h\nu}} f_{\text{M}}(\epsilon_{\text{M}}) f_{\text{X}}(\epsilon_{\text{X}}) \left| M_{\text{in}}^{\text{M}} \right|^2 d\epsilon_{\text{M}}$$



Biexcitons EM

Intensity dependence

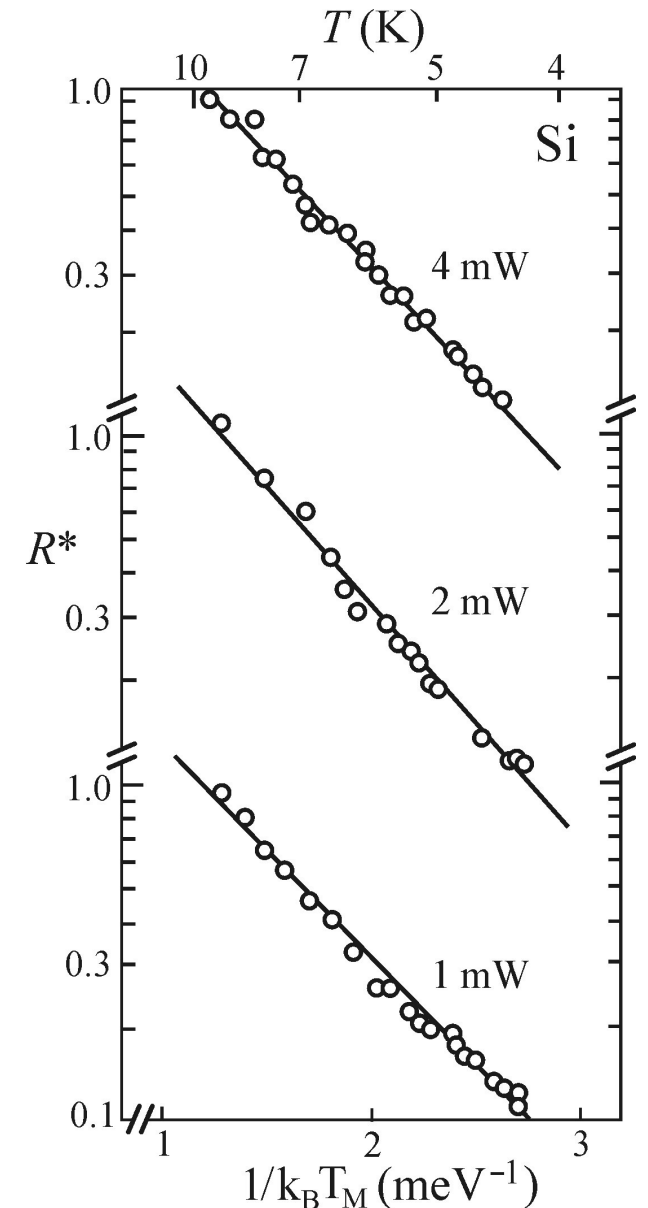
- Expect quadratic dependence: two excitons have to be created
- $n = 1.4 - 1.6$ in experiments $I^M \sim I_{\text{ex}}^n$,
- Model with characteristic material dependent excitation intensity

$$I^M \approx \text{const}(\sqrt{1 + I_{\text{ex}}/I_0} - 1)^2,$$

Biexcitons EM

Binding energy

- Direct: difference between FE and BE
- Indirect: Same but use fit of EM to get energy position at dispersion curve minimum
- Also: thermodynamic by FE and M position as function of temperature
 - M line drops quicker than FE line when T increases, lower binding energy







Summary

- **Free excitons**
 - **Direct bandgap:** FE or X or FE-LO phonon
 - FE polariton or (optical) phonon assisted
 - **Indirect bandgap:**
 - Phonon assisted
 - Linear function of excitation intensity
- **Bound excitons**
 - Shallow impurities (donors/acceptors)
 - Narrow line
 - Isoelectric impurities
 - Narrow line if low electron-phonon coupling
 - No temperature broadening
 - Linear function of excitation intensity until saturation
 - Lower $h\nu$ but stronger intensity
- **Biexcitons (Excitonic molecules)**
 - Superlinear function of intensity
 - Sort of inverse Maxwell-Boltzmann distribution





Classification of luminescence by intensity

Low-fluence (Weak) excitation (0.01- 10 W/cm²): gas-discharge lamp, incandescent lamp, continuous-wave gas laser

-  - Recombination of **free excitons** (FE or X; FE-LO phonon)
-  - Recombination of a **bound exciton** (BE): (D⁰-X; A⁰-X; D⁺-X; A⁻X, isoelectronic impurity)
-  - Recombination of **donor-acceptor pairs** (D⁰-A⁰)
-  - Recombination of **free hole/electron** with **neutral donor/acceptor** (h-D⁰; e-A⁰; e-h)

$$1 \text{ sun} = 0.1 \text{ mW/cm}^2 = 0.1 \text{ W/cm}^2 = 1 \text{ kW/m}^2$$

High-fluence (Strong) excitation (1 kW-10 MW/cm²):

-  - Radiative decay of **excitonic molecule** (EM or XX)
-  - Luminescence from inelastic collisions of excitons (X-X collisions)
-  - Luminescence of electron-hole liquid (EHL) or electron-hole plasma (EHP)
-  - Bose-Einstein condensation of excitons or excitonic molecules

These excitations can occur simultaneously, in these two chapters we focus on low-fluence excitation and neglected luminescence from excitons