Skriftlig Tentamen
IE1204 Digital Design
2024-10-24 Med Lösningar

Examiner/Examinator: Carl-Mikael Zetterling
Responsible teacher/Ansvarig lärare: Carl-Mikael Zetterling, 08-790 4344

Swedish/Svenska:
Tentamenstexten ska lämnas in när lösningarna lämnas in.
Inga tillåtna hjälpmedel utom linjal.
Examen består av två delar:
Del 1 har 16 uppgifter med max 1 poäng per uppgift som ska besvaras på ”Answer Form”.
Del 2 har 4 uppgifter med max 4 poäng per uppgift som ska besvaras på separat papper.
Lämna in båda delar samtidigt. Disponera tiden själv mellan delarna.
Uppgifterna är inte ordnade efter svårighetsgrad.
Om slutsumman av tentan har halvpoäng avrundas det uppåt.

Totalt max 32 poäng på tentamen.
För godkänt krävs
(minst 8 poäng på del 1) OCH (minst 16 poäng totalt)
Fx om något villkor ej är uppfyllt med en poängs marginal.

Det betyder att
För E krävs minst 16 poäng totalt och minst 8 på del 1.
För Fx krävs 16 poäng totalt och minst 7 på del 1
eller 15 poäng totalt och minst 8 på del 1.

Betygskalan för tentamen förutsatt att studenten har minst 8 poäng från del 1.

0-15	16-18	19-21	22-24	25-27	28-31	32
F	E	D	C	B	A	A+

Resultat meddelas inom tre veckor.

Written Exam
IE1204 Digital Design
2024-10-24 With Solutions

Examiner/Examinator: Carl-Mikael Zetterling
Responsible teacher/Ansvarig lärare: Carl-Mikael Zetterling, 08-790 4344

English:
The exam text should be handed in after the exam.
No aids allowed except ruler.
The exam consists of two parts:
Part 1 has 16 exercises for max 1 point per exercise to be answered on the "Answer Form".
Part 2 has 4 exercises for max 4 points per exercise, to be answered on a separate paper.
Hand in both parts at the same time. Plan the time yourself between the parts.
The exercises are not in order of difficulty.
If the total sum of the exam has half points this will be rounded up.

Total max of 32 points on the exam.
To pass the exam requires
(at least 8 points from part 1) AND (at least 16 points in total)
Fx if any condition is not fulfilled by one point's margin.

This means
For E, a minimum of 16 points in total and at least 8 in part 1 is required.
For Fx, 16 points are required in total and at least 7 on part 1
or 15 points in total and at least 8 in part 1.

Grades are given as follows provided the student has at least 8 points from part 1.

0-15	16-18	19-21	22-24	25-27	28-31	32
F	E	D	C	B	A	A+

The result will be announced within three weeks.

Del 1/Part 1, 1 point per exercise, fill in on “Answer Form”

1 Number Conversion

Swedish: Konvertera de positiva talen och utför beräkningen.
Svara med ett decimalt tal.

English: Convert the unsigned numbers and perform the calculation.
Answer with a decimal number.

$Y = 101101_2 + 24_8 - 25_{10} + 26_{16}$
$= 45 + 20 - 25 + 38 = 78$

2 Addition

Swedish: A och B är hexadecimala 8 bitars två-komplement kodade tal.
Beräkna A + B och svara med ett 8 bitars två-komplement hexadecimalt kodat tal.
Tänk på att du kan kontrollera dina beräkningar med decimala tal.

English: A and B are hexadecimal 8-bit binary (two’s complement) numbers.
Calculate A + B and answer with an 8-bit binary (two’s complement) hexadecimal number.
You can check your calculations using decimal numbers.

$A = 0xB3 = B3_{16}$ 10110011
$B = 0x25 = 25_{16}$ + 00100101
A + B = 11011000 = 0xD8

or calculate directly from 0xB3 + 0x25

3 Subtraction

Swedish: A och B är hexadecimala 8 bitars två-komplement kodade tal.
Beräkna A - B och svara med ett 8 bitars två-komplement hexadecimalt kodat tal.
Tänk på att du kan kontrollera dina beräkningar med decimala tal.

English: A and B are hexadecimal 8-bit binary (two’s complement) numbers.
Calculate A - B and answer with an 8-bit binary (two’s complement) hexadecimal number.
You can check your calculations using decimal numbers.

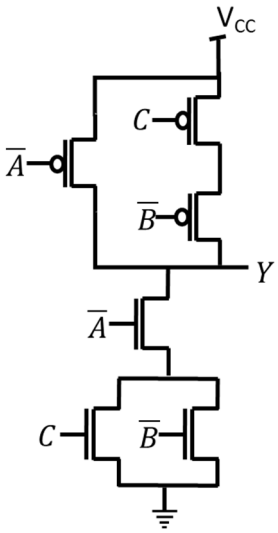
$A = 0xC9 = C9_{16}$ 11001001
$B = 0x44 = 44_{16}$ = 01000100
- B = + 10111100
A – B = 10000101 = 0x85

or calculate directly from 0xC9 – 0x44

4 Analysis of CMOS circuits

Swedish: Ta fram enklast möjliga booleska uttryck för CMOS-kretsen.

English: Derive the simplest possible Boolean expression for the CMOS circuit.



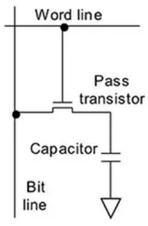
$$Y = A + B \cdot \overline{C}$$

5 Memory circuits

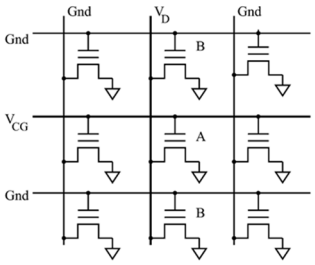
Swedish: Vilken av kretsarna är en SRAM?

English: Which circuit is an SRAM?

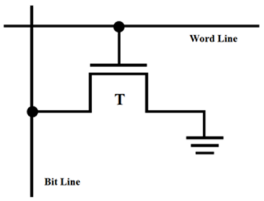
#1



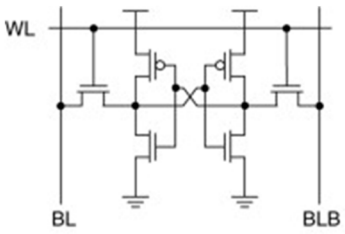
#2



#3



#4



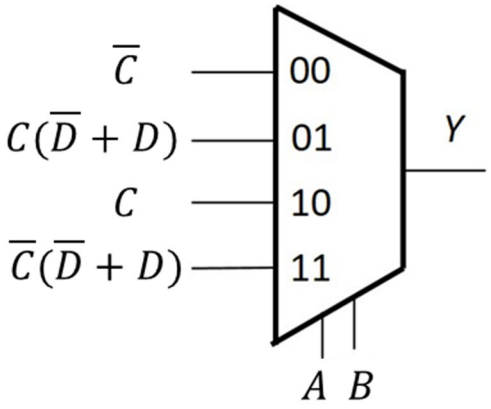
#4

6 Analysis: MUX to K-map

Swedish: Fyll i K-Map från MUX-kopplingen.
Ta fram enklast möjliga booleska uttryck för Y från K-map. Välj PoS eller SoP.

English: Fill in the K-Map from the MUX circuit.
Derive simplest possible Boolean expression from the K-map. Select PoS or SoP.

Y	CD 00	CD 01	CD 11	CD 10
AB 00	1	1	0	0
AB 01	0	0	1	1
AB 11	1	1	0	0
AB 10	0	0	1	1



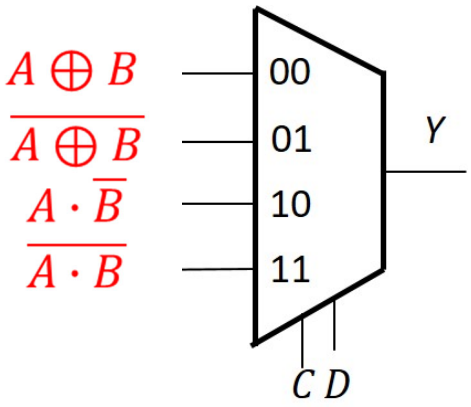
$\overline{A}\cdot\overline{B}\cdot\overline{C}+\overline{A}\cdot B\cdot C+A\cdot B\cdot\overline{C}+A\cdot\overline{B}\cdot C$

7 Design: K-Map to MUX

Swedish: Använd en 4:1 MUX och valfria grindar eller 0 och 1 och gör en krets för K-map med CD som select-signaler.

English: Use a 4:1 MUX and any logic gates or 0 or 1 to draw a circuit for the K-map with CD as select signals.

Y	CD 00	CD 01	CD 11	CD 10
AB 00	0	1	1	0
AB 01	1	0	1	0
AB 11	0	1	0	0
AB 10	1	0	1	1

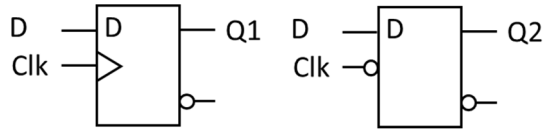


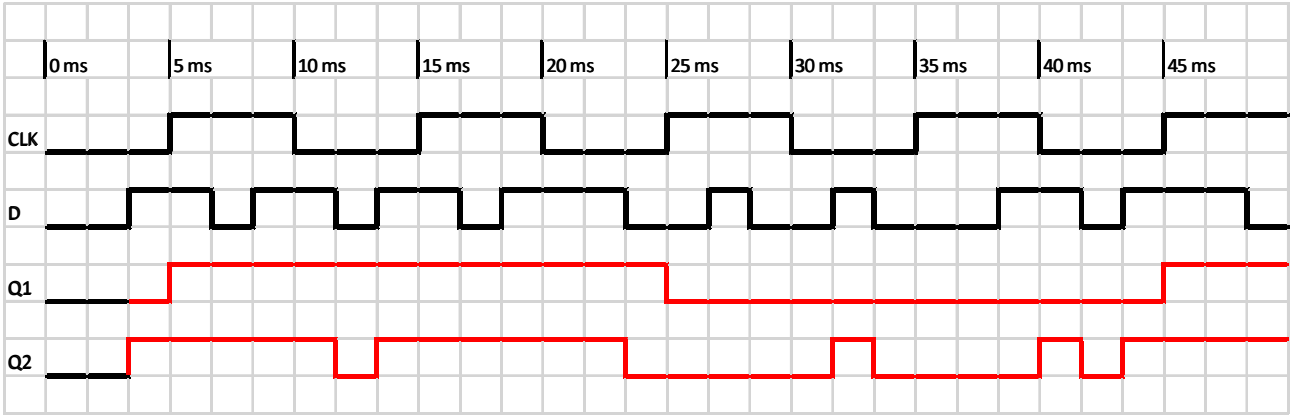
0.5 point deducted if third and fourth expressions are interchanged.

8 Timing diagram (Analysis)

Swedish: Rita tidsdiagram för kretsen i ”Answer Form”.

English: Draw the timing diagrams for the circuit in the “Answer Form”.

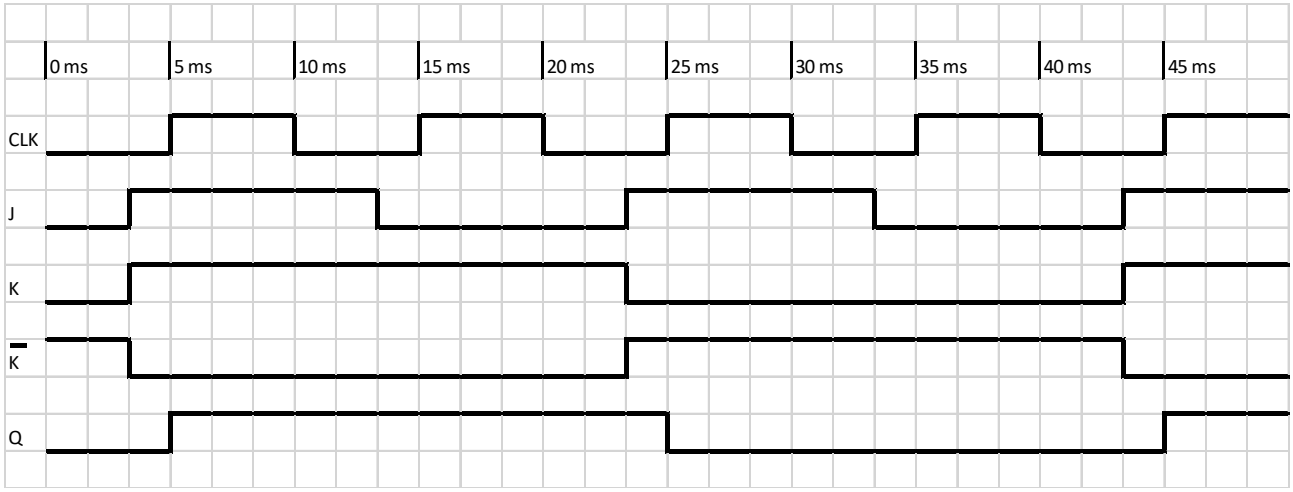


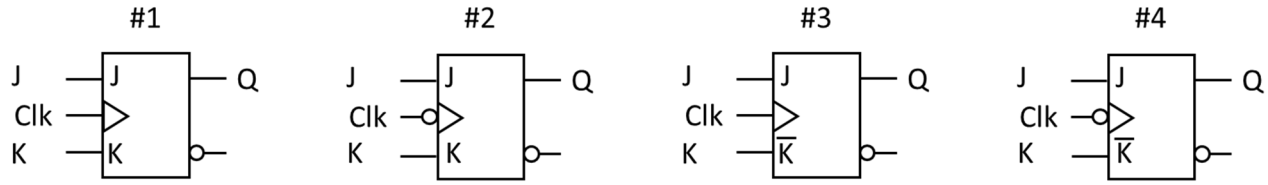


9 Timing diagram (Design)

Swedish: Vilken vippa ger tidsdiagrammet nedan? OBS att alla fyra är olika.

English: Which flip-flop has the timing diagram below? NOTE that all four are different.





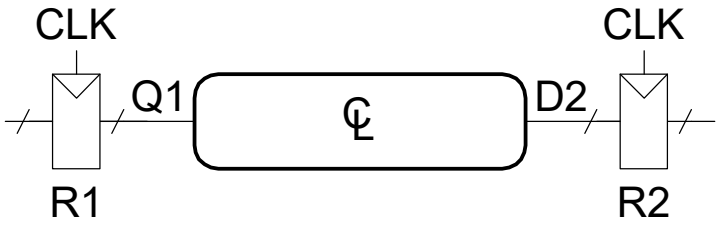
Answer: #3 positive edge triggered $J\overline{K}$ Flip-flop

10 Timing calculation

Swedish: Beräkna maximala fördröjningstiden t_{pd} för kretsen. Är Hold-villkoret uppfyllt?

English: Calculate the maximum delay time t_{pd} for the circuit. Is the Hold time constraint ok?

t_{pcq} = 40 ps
t_{ccq} = 20 ps
t_{setup} = 50 ps
t_{hold} = 45 ps
t_{cd} = 20 ps
T_c = 250 ps

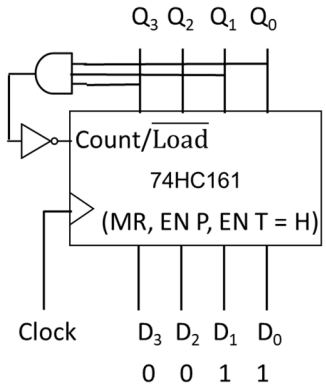


$t_{pd} \leq 250 - 40 - 50 = 160$ ps 20 + 20 < 45 No

11 Counter analysis

Swedish: Räknaren nedan har laddat in tillståndet $Q_3Q_2Q_1Q_0 = D_3D_2D_1D_0$.
Hur många klockpulser innan den upprepar sig? Vad är sista tillståndet?

English: The counter below has loaded state $Q_3Q_2Q_1Q_0 = D_3D_2D_1D_0$.
How many clock pulses until it repeats? What is the end state?

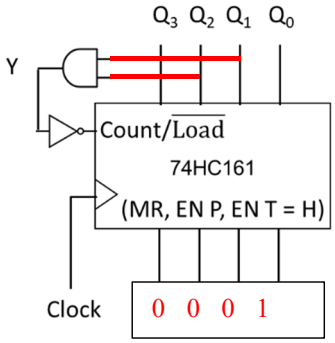


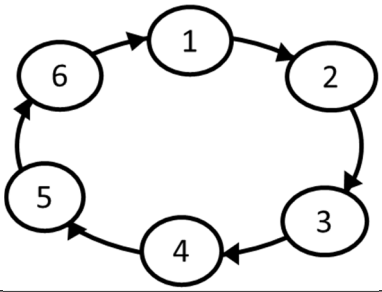
N = 9 Sequence: 0011 – 1011, Last state is 1011
The last state will cause a load operation on the next clk.
Count the number of states including the loaded state:
Last – Loaded (first) + 1

12 Counter design

Swedish: Hur ska AND-grinden och D_3 – D_0 kopplas för att få en räknare med tillståndsdiagrammet nedan?

English: How should the AND-gate and D_3 – D_0 be connected to get a counter with the state diagram below?





The AND gate should detect the last state, Y= $Q_2 \cdot Q_1$
$D_3D_2D_1D_0$ should be connected to load the first state 0001

13 Multiplication

Swedish: A och B är 8 bitars två-komplement kodade tal.
Beräkna A x B (binärt) och svara med ett 16 bitars två-komplement **hexadecimalt kodat tal**.
Tänk på att du kan kontrollera dina beräkningar med decimala tal.

English: A and B are 8-bit binary (two's complement) numbers.
Calculate A x B (binary) and answer with a 16-bit binary (two's complement) **hexadecimal number**.
You can check your calculations using decimal numbers.

$A = 01100110_2$
$B = 01111101_2$

A x B = B x A (both are positive) put B above since it has more '1's

```
        01111101
      x 01100110
        01111101   1st multiplication
    + 01111101     2nd multiplication
      1011101110
 + 01111101        3rd multiplication
   1001010001110
+ 01111101         4th multiplication
0011000111001110 = 31CE16
```

14 Division

Swedish: A och B är 8 bitars två-komplement kodade tal.
Beräkna A / B (binärt) och svara med kvot och rest
(8 bitars två-komplement **hexadecimalt kodade tal**).
Tänk på att du kan kontrollera dina beräkningar med decimala tal.

English: A and B are 8-bit binary (two's complement) numbers.
Calculate A / B (binary) and answer with quotient and remainder
(8-bit binary two's complement **hexadecimal numbers**).
You can check your calculations using decimal numbers.

$A = 01111101_2$
$B = 00110000_2$

A and B are both positive

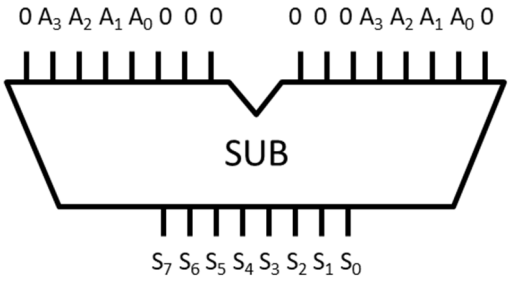
```
          10  Quotient = 2
110000/01111101
      - 110000
        11101 Remainder = 1D
```

15 Full Subtractor

Swedish: Vad blir resultatet från kretsen nedan?
Svara med 8 bitar (S_7 S_6 S_5 S_4 S_3 S_2 S_1 S_0). Ignorera carry out (C_8).

English: What is the result for the circuit below?
Answer with 8 bits (S_7 S_6 S_5 S_4 S_3 S_2 S_1 S_0). Ignore carry out (C_8).

A = 1110

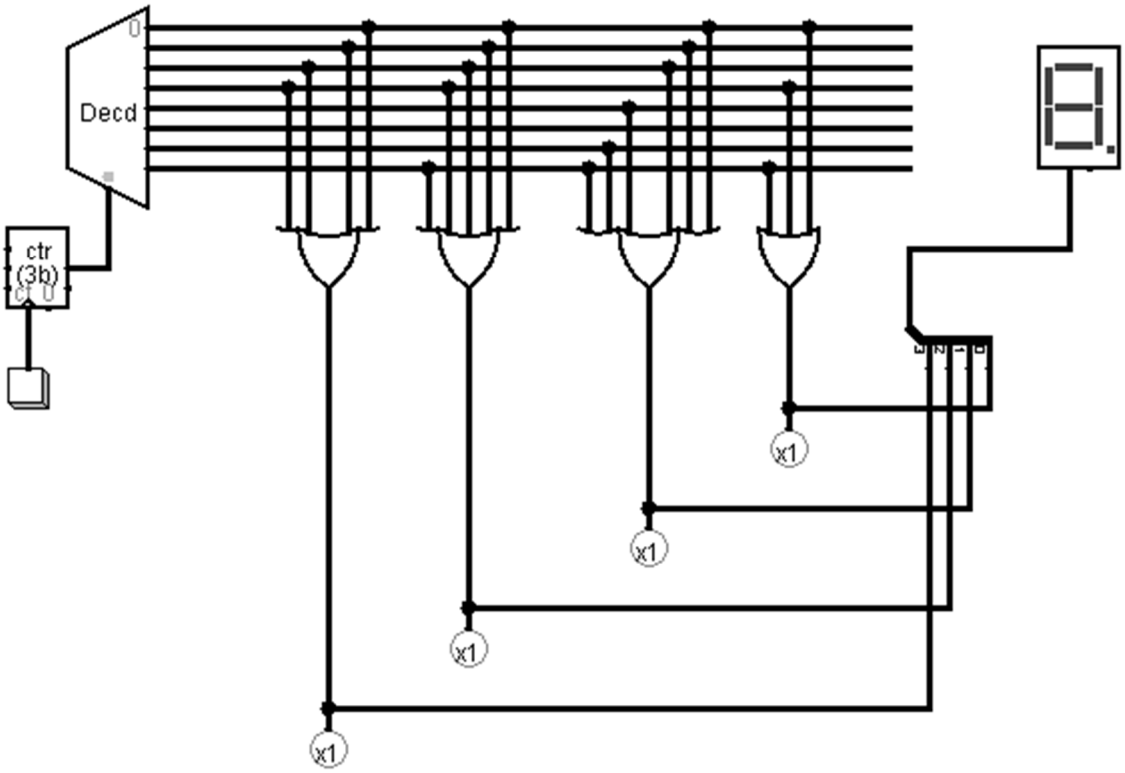



```
   11100100 (-14)
  + 1110    (14*8)
(1)01010100 (14*6=84)
Ignore leading 1 for
subtraction
```

16 Memory

Swedish: Detta ROM har åtta 4 bitars tal, vilka är talen?
Svara med 8 **hexadecimala siffror**, från adress 000 till 111.

English: This ROM has eight 4-bit numbers stored, what are the numbers?
Answer with 8 **hexadecimal digits**, from address 000 to 111.



The hexadecimal number is F E E D 2 0 2 7

Del 2/Part 2, 4 points per exercise, answer on separate paper

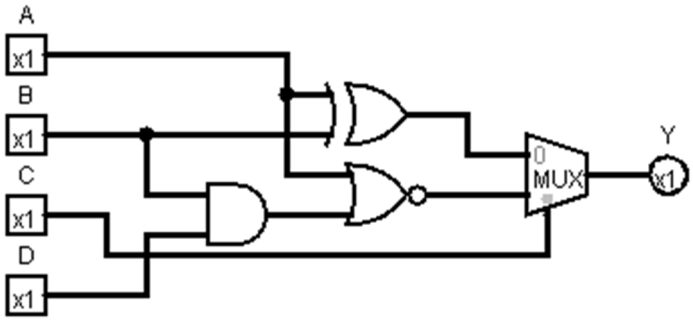
17 Analysis of Combinational Circuit

Swedish:

1. Ta fram booleskt uttryck för kretsen nedan.
2. Rita K-map för kretsen med variabelordning som i figuren.
3. Förenkla uttrycket med hjälp av K-map.
4. Rita ny krets med enbart NOR-grindar.

English:

1. Derive the Boolean expression for the circuit below.
2. Draw a K-map for the circuit with variables as in the figure.
3. Simplify the expression using the K-map.
4. Draw a new circuit using only NOR gates.



Y	CD 00	CD 01	CD 11	CD 10
AB 00	0	0	1	1
AB 01	1	1	0	1
AB 11	0	0	0	0
AB 10	1	1	0	0

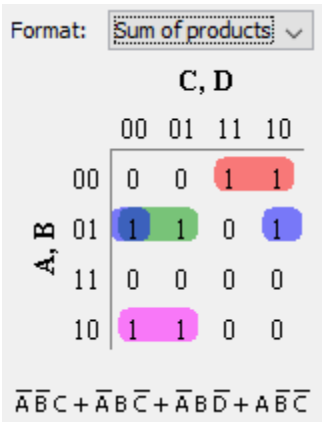
Rita av K-map i dina inlämnade svar.

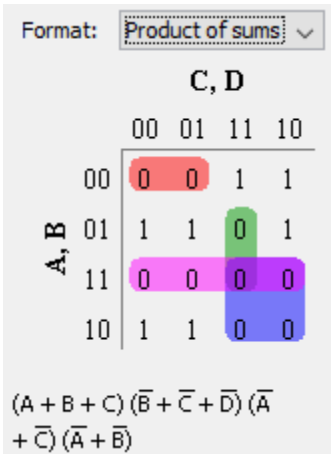
Redraw the K-map in your answer sheets.

Boolean expression and truth table

$$Y = \overline{C} \cdot (A \oplus B) + C \cdot (\overline{A + B \cdot D}) = \overline{A} \cdot B \cdot \overline{C} + A \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot \overline{B} \cdot C + \overline{A} \cdot C \cdot \overline{D}$$

K-Map and simplified expressions





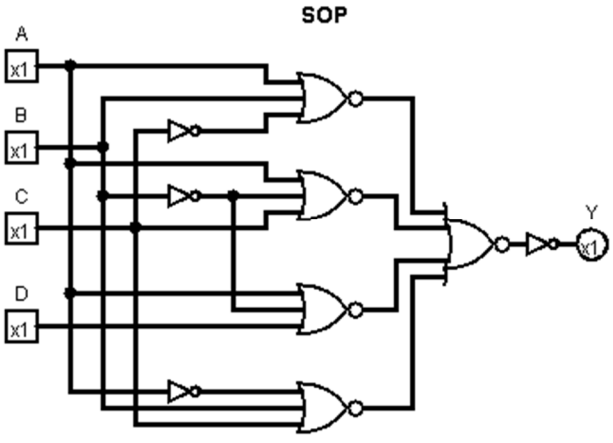
A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

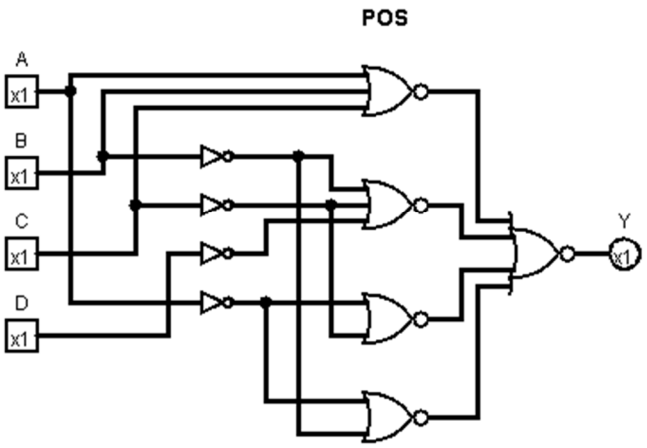
Can be built from SOP:

$$\begin{aligned} Y &= \overline{A} \cdot \overline{B} \cdot C + \overline{A} \cdot B \cdot \overline{C} + \overline{A} \cdot B \cdot \overline{D} + A \cdot \overline{B} \cdot \overline{C} \\ &= \overline{\overline{A} \cdot \overline{B} \cdot C} + \overline{\overline{A} \cdot B \cdot \overline{C}} + \overline{\overline{A} \cdot B \cdot \overline{D}} + \overline{A \cdot \overline{B} \cdot \overline{C}} \\ &= \overline{A + B + \overline{C}} + \overline{A + \overline{B} + C} + \overline{A + \overline{B} + D} + \overline{\overline{A} + B + C} \\ &= A + B + \overline{C} + A + \overline{B} + C + A + \overline{B} + D + \overline{A} + B + C \end{aligned}$$

Or better from starting from POS since it is NOR (inverters can be made with NOR gates):

$$\begin{aligned} Y &= (A + B + C)(\overline{B} + \overline{C} + \overline{D})(\overline{A} + \overline{C})(\overline{A} + \overline{B}) = \overline{(A + B + C)(\overline{B} + \overline{C} + \overline{D})(\overline{A} + \overline{C})(\overline{A} + \overline{B})} \\ &= \overline{(A + B + C)} + \overline{(\overline{B} + \overline{C} + \overline{D})} + \overline{(\overline{A} + \overline{C})} + \overline{(\overline{A} + \overline{B})} \end{aligned}$$





18 Design of Combinational Circuit

Swedish:

Decimaltal kan konverteras till binära tal med fixnotation. Tex så blir $\sqrt{2}$ ≈1,414213562 på detta sätt 1.011010100000100. Vi kan använda det som funktion för en sanningstabell, se nedan.

1. Rita en K-map för sanningstabellen med variabelordning som i figuren.
2. Ta fram enklast möjliga booleska uttryck.
3. Rita en krets för uttrycket med enbart NAND-grindar.
4. Rita en krets för K-map som använder en 4:1 Mux och grindar.

English:

Decimal numbers can be converted to binary with fix notation. For instance $\sqrt{2}$ ≈1,414213562 can be written as 1.011010100000100. We can use it as the truth table function, see below.

1. Draw a K-map for the truth table with variables as in the figure.
2. Derive the simplest possible Boolean expression from the K-map.
3. Draw the circuit for the expression using only NAND-gates.
4. Draw a circuit for the K-Map using a 4:1 Mux and gates.

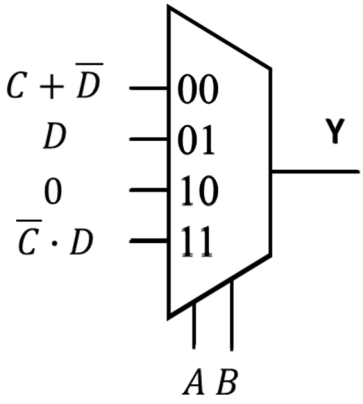
A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

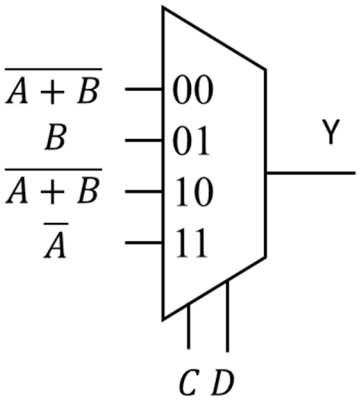
Y	CD 00	CD 01	CD 11	CD 10
AB 00	1	0	1	1
AB 01	0	1	1	0
AB 11	0	1	0	0
AB 10	0	0	0	0

Rita av K-map i dina inlämnade svar.

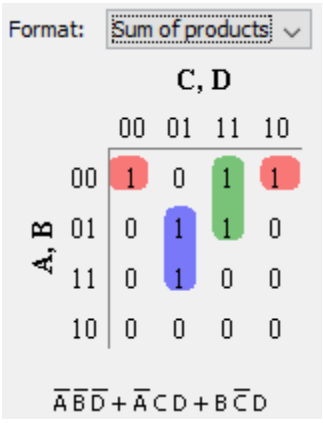
Redraw the K-map in your answer sheets.

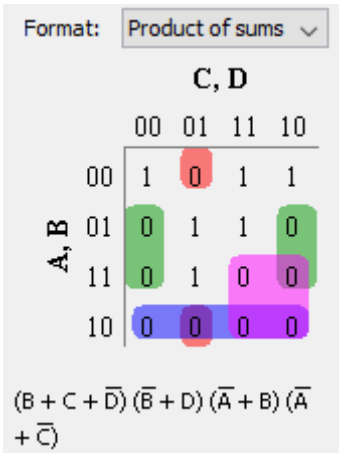
Two possible Mux solutions:





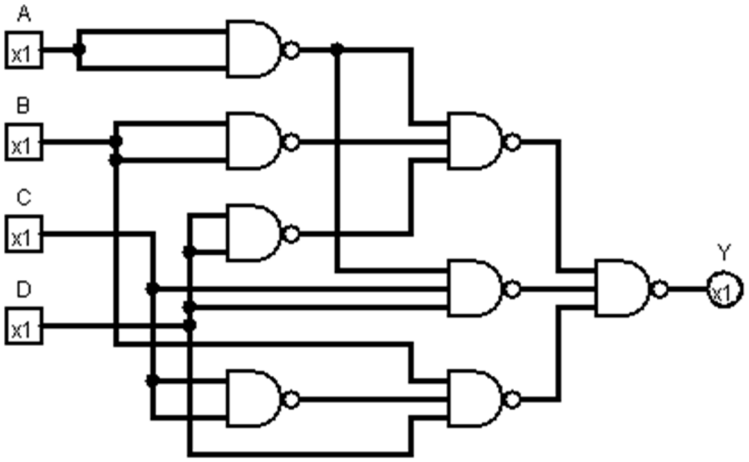
K-Maps and simplified expressions



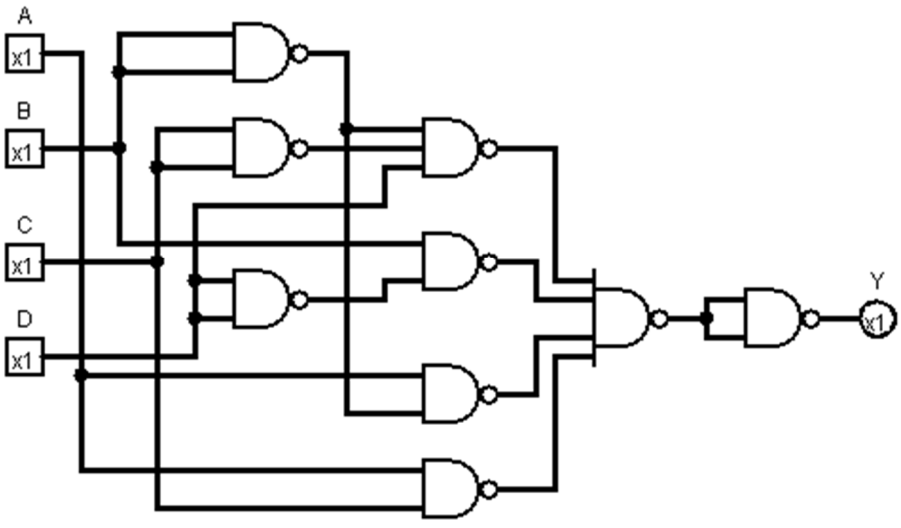


Normally use SOP for NAND only (inverters are ok if you note that they can be made with a NAND).

$$Y = \overline{A} \cdot \overline{B} \cdot \overline{D} + \overline{A} \cdot C \cdot D + B \cdot \overline{C} \cdot D = \overline{\overline{\overline{A} \cdot \overline{B} \cdot \overline{D}} \cdot \overline{\overline{A} \cdot C \cdot D} \cdot \overline{B \cdot \overline{C} \cdot D}}$$



For POS $Y = (B + C + \overline{D})(\overline{B} + D)(\overline{A} + B)(\overline{A} + \overline{C}) = \overline{(B + C + \overline{D})} \cdot \overline{(\overline{B} + D)} \cdot \overline{(\overline{A} + B)} \cdot \overline{(\overline{A} + \overline{C})} = \overline{B} \cdot \overline{C} \cdot D \cdot \overline{B} \cdot \overline{D} \cdot \overline{A} \cdot \overline{B} \cdot \overline{A} \cdot C = \overline{\overline{B} \cdot \overline{C} \cdot D \cdot B \cdot \overline{D} \cdot A \cdot \overline{B} \cdot \overline{A} \cdot C}$



19 Analysis of FSM

Swedish: Analysera vad tillståndsmaskinen (FSM) på nästa sida utför.

1. Ta fram Boolska uttryck för nästa tillstånd Q3+ Q2+ Q1+ Q0+ och utsignal Y.
2. Rita K-Maps för Q3+ Q2+ Q1+ Q0+ och utsignal Y.
3. Rita tillståndstabell.
4. Rita tillståndsdiagram.

Använd ordningen Q3 Q2 Q1 Q0

English: Analyze the state machine (FSM) on the next page.

1. Derive Boolean expressions for next state Q3+ Q2+ Q1+ Q0+ and output Y.
2. Draw K-Maps for Q3+ Q2+ Q1+ Q0+ and output Y.
3. Draw a state table.
4. Draw a state diagram.

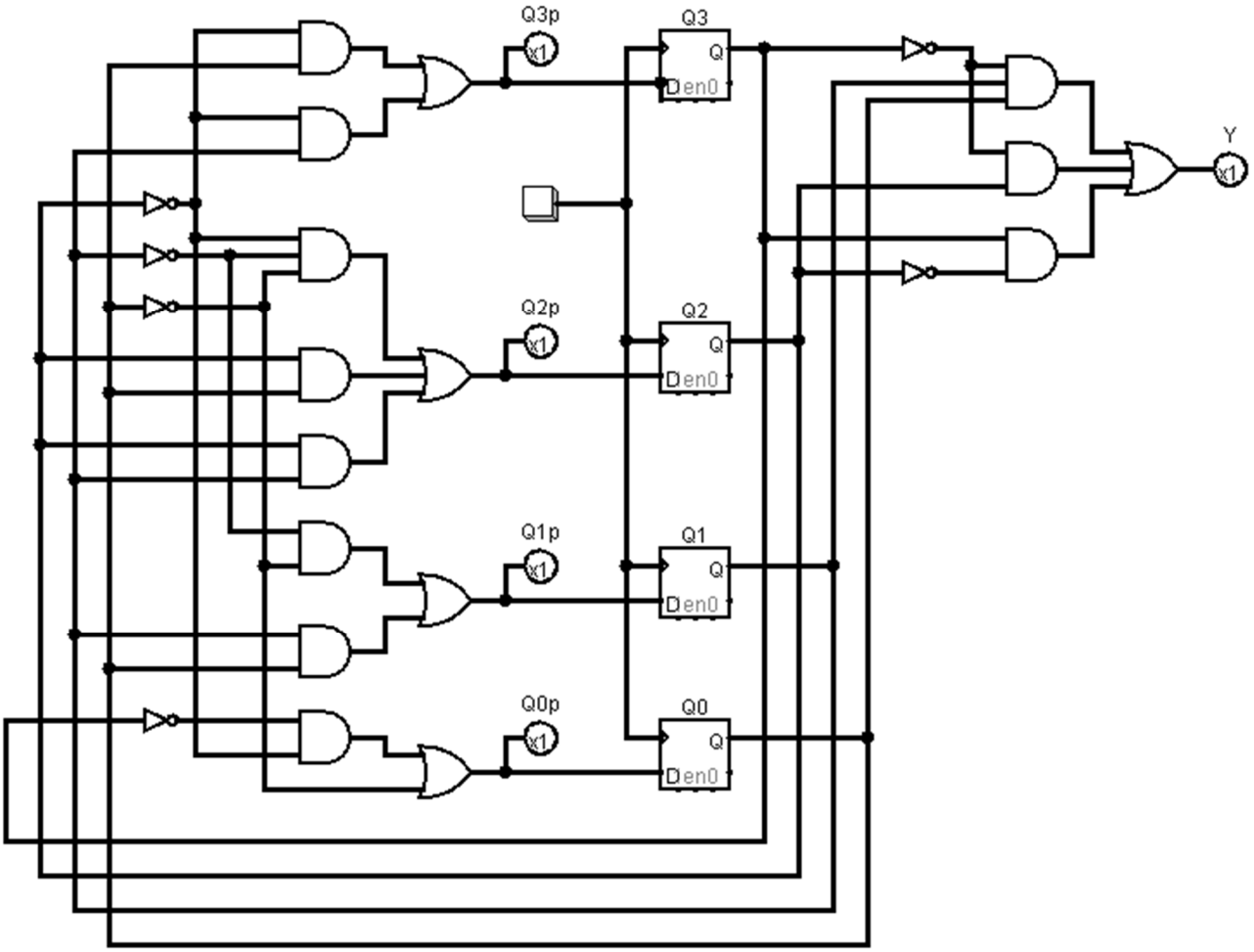
Use the order Q3 Q2 Q1 Q0

Q_3Q_2 \ Q_1Q_0 =	00	01	11	10
00				
01				
11				
10				

Rita av K-map i dina inlämnade svar.

Redraw the K-map in your answer sheets.

19 Analysis of FSM cont.



Boolean expressions

$$Q_3^+ = \overline{Q}_2 \cdot Q_0 + \overline{Q}_2 \cdot Q_1$$
$$Q_2^+ = \overline{Q}_2 \cdot \overline{Q}_1 \cdot \overline{Q}_0 + Q_2 \cdot Q_0 + Q_2 \cdot Q_1$$
$$Q_1^+ = \overline{Q}_1 \cdot \overline{Q_0} + Q_1 \cdot Q_0$$
$$Q_0^+ = \overline{Q_3} \cdot \overline{Q_2} + \overline{Q_0}$$
$$Y = \overline{Q_3} \cdot Q_1 \cdot Q_0 + \overline{Q_3} \cdot Q_2 + Q_3 \cdot \overline{Q_2}$$

Y	Q_1Q_0 00	01	11	10
Q_3Q_2 00	0	0	1	0
01	1	1	1	1
11	0	0	0	0
10	1	1	1	1

K-Maps and state table

Q3+	Q_1Q_0 00	01	11	10
Q_3Q_2 00	0	1	1	1
01	0	0	0	0
11	0	0	0	0
10	0	1	1	1

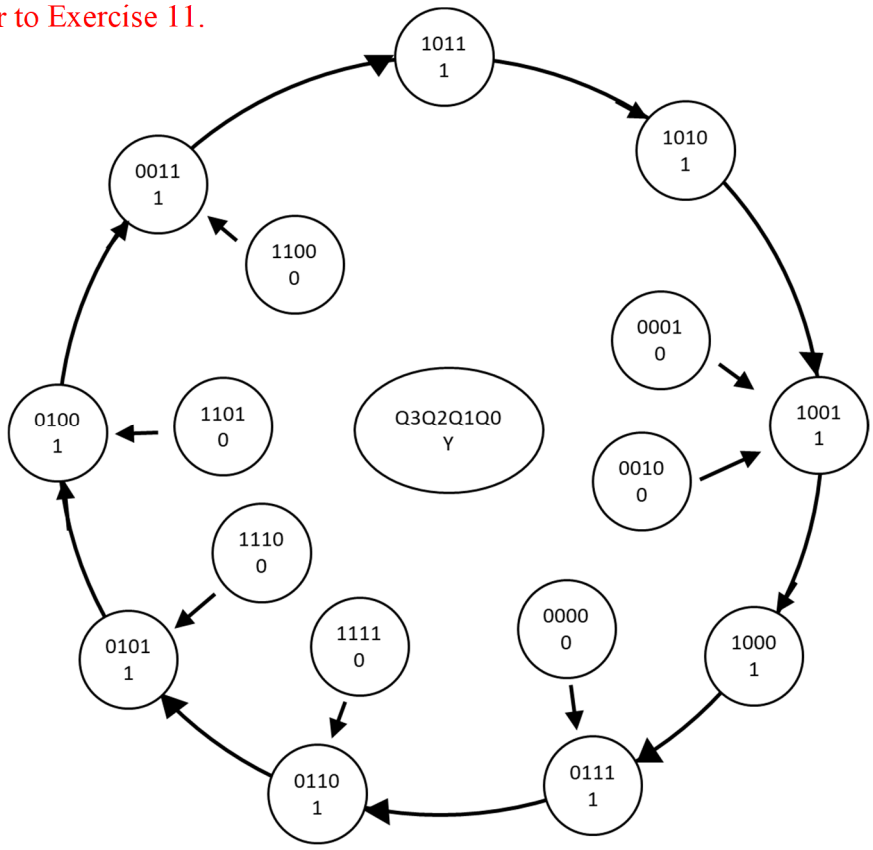
Q2+	Q_1Q_0 00	01	11	10
Q_3Q_2 00	1	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	0	0	0

Q1+	Q_1Q_0 00	01	11	10
Q_3Q_2 00	1	0	1	0
01	1	0	1	0
11	1	0	1	0
10	1	0	1	0

Q0+	Q_1Q_0 00	01	11	10
Q_3Q_2 00	1	1	1	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

Present state				Next state				Output
Q3	Q2	Q1	Q0	Q3+	Q2+	Q1+	Q0+	Y
0	0	0	0	0	1	1	1	0
0	0	0	1	1	0	0	1	0
0	0	1	0	1	0	0	1	0
0	0	1	1	1	0	1	1	1
0	1	0	0	0	0	1	1	1
0	1	0	1	0	1	0	0	1
0	1	1	0	0	1	0	1	1
0	1	1	1	0	1	1	0	1
1	0	0	0	0	1	1	1	1
1	0	0	1	1	0	0	0	1
1	0	1	0	1	0	0	1	1
1	0	1	1	1	0	1	0	1
1	1	0	0	0	0	1	1	0
1	1	0	1	0	1	0	0	0
1	1	1	0	0	1	0	1	0
1	1	1	1	0	1	1	0	0

State diagram; a counter for 0011 – 1011 reversed, Y = 0 if outside the sequence.
Similar to Exercise 11.



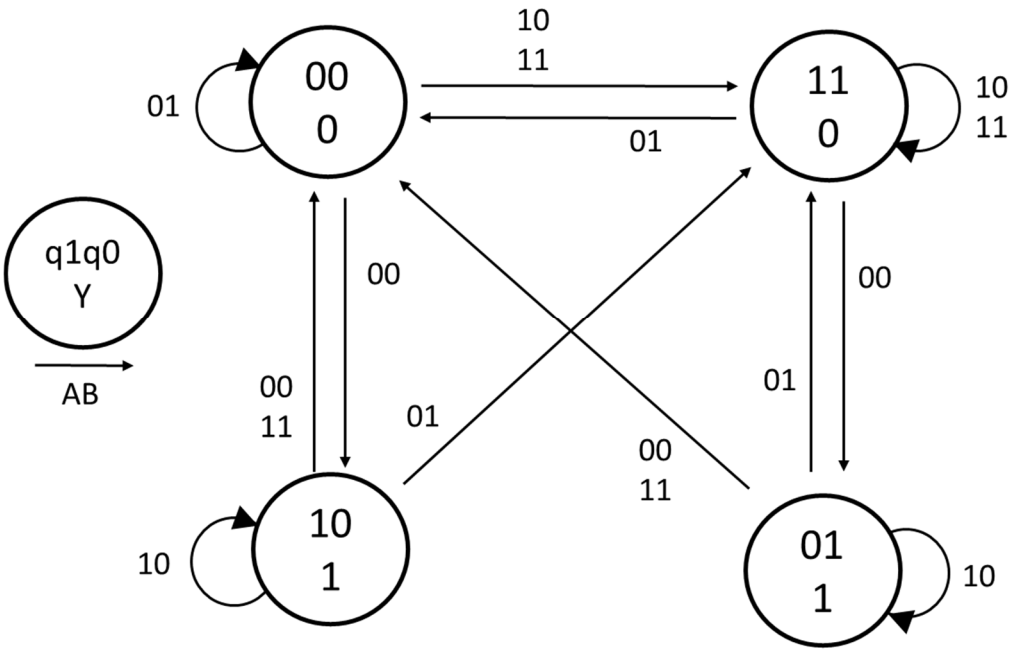
20 Design of FSM

Swedish: Konstruera en tillståndsmaskin (FSM) enligt tillståndsdiagrammet nedan.

1. Rita tillståndstabell.
2. Ta fram K-map för nästa tillstånd.
3. Ta fram minimerade uttryck för nästa tillstånd.
4. Rita kretsschema för en FSM med DFFs och MUX eller vilka grindar som helst.

English: Design a state machine (FSM) according to the state diagram below.

1. Draw a state table.
2. Derive K-maps for next state.
3. Derive minimized expressions for next state.
4. Draw the FSM circuit diagram with DFFs and MUX or any gates.



	AB 00	AB 01	AB 11	AB 10
q1q0 00				
q1q0 01				
q1q0 11				
q1q0 10				

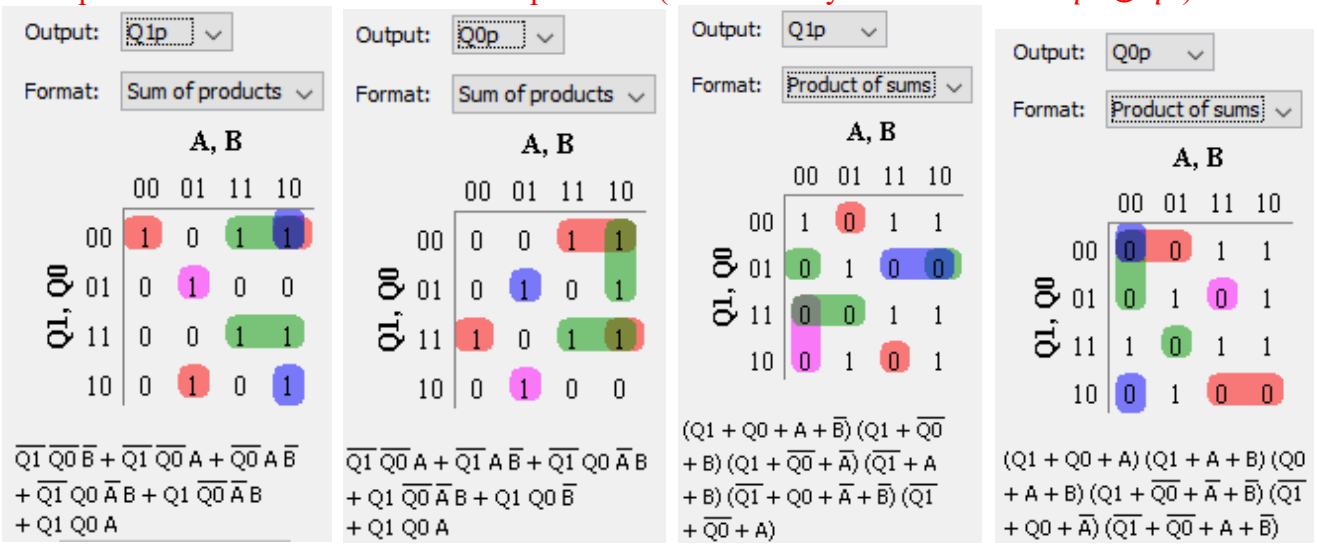
Rita av K-map i dina inlämnade svar.

Redraw the K-map in your answer sheets.

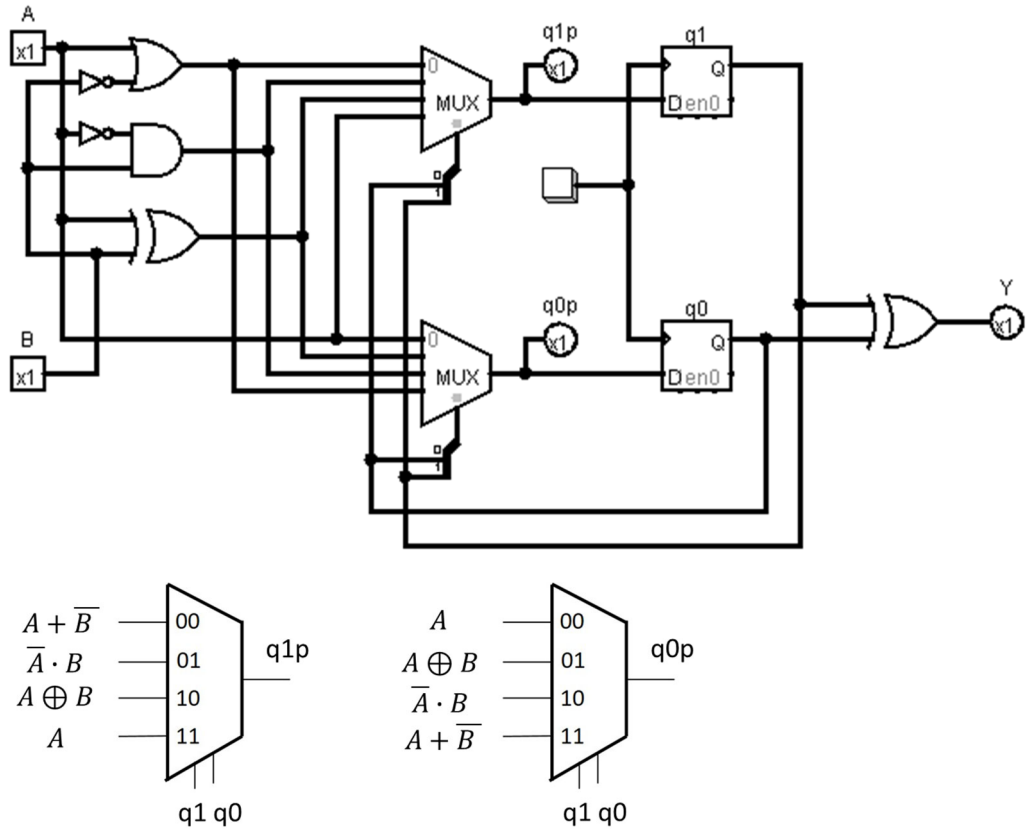
State table

		Next state								
Present state		AB = 00		AB = 01		AB = 11		AB = 10		Out Y
q1	q0	q1+	q0+	q1+	q0+	q1+	q0+	q1+	q0+	AB = 00
0	0	1	0	0	0	1	1	1	1	0
0	1	0	0	1	1	0	0	0	1	1
1	1	0	1	0	0	1	1	1	1	0
1	0	0	0	1	1	0	0	1	0	1

K-Maps for next state with minimized expressions (not necessary to draw for $Y = q1 \oplus q0$)



FSM circuit diagram (MUX version)



FSM Diagram SOP version