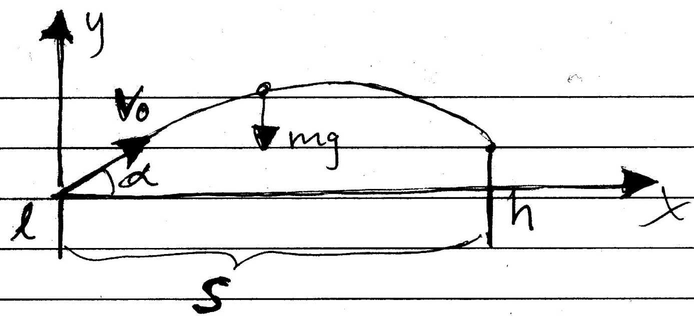
1



NII $y: \quad m\ddot{y} = -mg$
$x \quad m\ddot{x} = 0$

B.V. Då $t=0$ har vi $x = y = 0$
$\dot{x} = V_0 \cos\alpha$
$\dot{y} = V_0 \sin\alpha$

Integrera två gånger och utnyttja begynnelsevillkoren. Detta ger

$y = -\frac{1}{2}gt^2 + V_0 \sin\alpha\, t$

$x = V_0 \cos\alpha\, t$

Bollen går i korgen vid $x = s$ och $y = h-l$

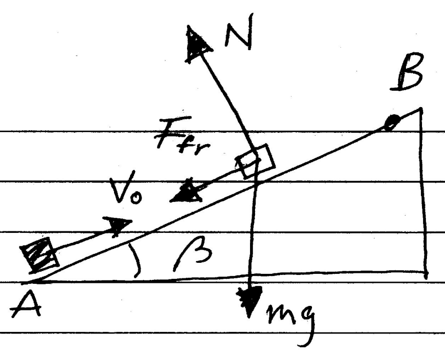
$h-l = -\frac{1}{2}gt^2 + V_0 \sin\alpha\, t$

$s = V_0 \cos\alpha\, t$

Eliminera V_0 och lös för t:

$t = \sqrt{\dfrac{2(s\tan\alpha - (h-l))}{g}}$

2.



NII $\quad 0 = N - mg\cos\alpha \rightarrow N = mg\cos\alpha$

$$F_{fr} = \mu N = \mu mg\cos\alpha$$

Lagen om kinetiska energin från A till B:

$$0 - \frac{1}{2}mV_0^2 = (-F_{fr} - mg\sin\alpha)l$$

$$\frac{1}{2}\cancel{m}V_0^2 = (\mu \cancel{m}g\cos\alpha + \cancel{m}g\sin\alpha)l$$

$$l = \frac{V_0^2}{2g(\mu\cos\alpha + \sin\alpha)}$$

Lagen om kinetiska energin från B till A

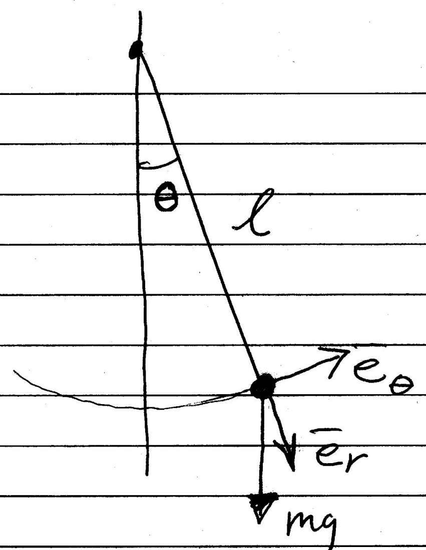
$$\frac{1}{2}mV_s^2 = (-F_{fr} + mg\sin\alpha)l \Rightarrow$$

$$V_s^2 = 2gl(\sin\alpha - \mu\cos\alpha) =$$

$$= V_0^2\frac{\sin\alpha - \mu\cos\alpha}{\sin\alpha + \mu\cos\alpha} \Rightarrow$$

$$V_s = V_0\sqrt{\frac{\sin\alpha - \mu\cos\alpha}{\sin\alpha + \mu\cos\alpha}}$$

3



NII $\bar{e}_\theta$: $\cancel{m}(r\ddot{\theta}-2\dot{r}\dot{\theta}) = -\cancel{m}g\sin\theta$

$r = l$ $\quad \dot{r} = 0$ $\quad$ Detta ger

$l\ddot{\theta} = -g\sin\theta$

Små svängningar $\Rightarrow$ $\sin\theta \approx \theta$

$\ddot{\theta} + \frac{g}{l}\theta = 0$

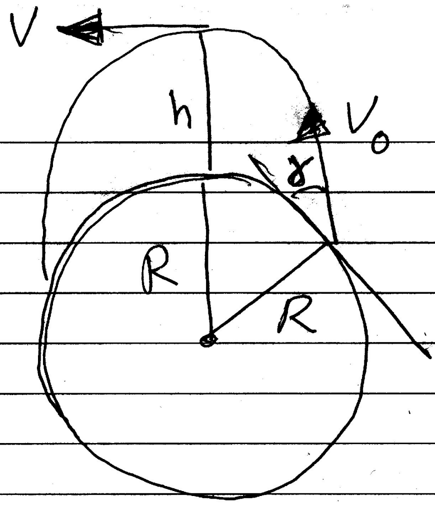
Svängningsekvation med $\omega_n = \sqrt{\frac{g}{l}}$

och perioden $T = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{l}{g}}$

$100T = T_0 \Rightarrow 100\sqrt{\frac{l}{g}}\,2\pi = T_0$

$g = \frac{(100\cdot 2\pi)^2}{T_0^2}l = 4\pi^2\cdot 10^4\frac{l}{T_0^2}$

(4)



Rörelsemängdsmomentets och energins bevarande

$$V_0 \cos\gamma \, R = V(R+h) \quad (1)$$

$$\frac{1}{2} m V_0^2 - \frac{m g R^2}{R} = \frac{1}{2} m V^2 - \frac{m g R^2}{R+h} \quad (2)$$

(1) ger: $V = \dfrac{V_0 \cos\gamma \, R}{R+h}$

Detta insatt i (2) ger:

$$V_0^2 - 2gR = \frac{V_0^2 \cos^2\gamma \, R^2}{(R+h)^2} - \frac{2gR^2}{R+h} \Rightarrow$$

$$(V_0^2 - 2gR)(R+h)^2 = V_0^2 \cos^2\gamma \, R^2$$

$$-2gR^2(R+h) \Rightarrow$$

$$(V_0^2 - 2gR)(R^2 + h^2 + 2Rh) = V_0^2\cos^2\gamma\, R^2$$

$$-2gR^3 - 2gR^2h \Rightarrow$$

$$(V_0^2 - 2gR)h^2 + V_0^2\, 2Rh - 4gR^2h$$

$$+ V_0^2R^2 - \cancel{2gR^3} = V_0^2\cos^2\gamma\, R^2 - \cancel{2gR^3} - \cancel{2gR}$$

$$(V_0^2 - 2gR)h^2 + (V_0^2 - gR)\,2Rh$$

$$+ V_0^2R^2(1 - \cos^2\gamma) = 0$$

$$h^2 + \frac{gR - V_0^2}{2gR - V_0^2}\,2Rh - \frac{V_0^2R^2\sin^2\gamma}{2gR - V_0^2}$$

$$= 0 \Rightarrow$$

$$h = -\frac{gR - V_0^2}{2gR - V_0^2}\,R$$

$$\overset{+}{(-)}\, R\sqrt{\left(\frac{gR - V_0^2}{2gR - V_0^2}\right)^2 + \frac{V_0^2\sin^2\gamma}{2gR - V_0^2}}$$

$$= R\Bigg(-\frac{gR - V_0^2}{2gR - V_0^2}$$

$$\overset{+}{(-)}\,\frac{\sqrt{(gR - V_0^2)^2 + (2gR - V_0^2)V_0^2\sin^2\gamma}}{2gR - V_0^2}$$

$\Rightarrow$ ---

$h \geqslant 0$ för alla värden på V_0

$\Rightarrow$ Välj den positiva roten

$$h = R\left(-\frac{gR - V_0^2}{2gR - V_0^2} + \frac{\sqrt{g^2R^2 + \cos^2\gamma\,(V_0^4 - 2gRV_0^2)}}{2gR - V_0^2} \right)$$

$$= R\,\frac{\sqrt{g^2R^2 + V_0^2\cos^2\gamma\,(V_0^2 - 2gR)} - (gR - V_0^2)}{2gR - V_0^2}$$