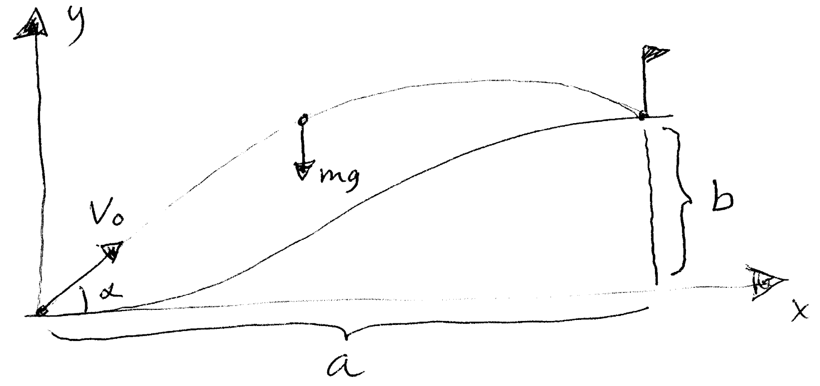
1



NII: $m\ddot{y} = -mg$

$m\ddot{x} = 0$

B.V. $x(0) = y(0) = 0$

$\dot{x}(0) = V_0 \cos\alpha$

$\dot{y}(0) = V_0 \sin\alpha$

Integration 2 gånger med utnyttjande av B.V. ger

$y = -\frac{1}{2} g t^2 + V_0 \sin\alpha\, t$

$x = V_0 \cos\alpha\, t$

Vid nedslaget ($t = t_1$) har vi $x = a$, $y = b$ $\Rightarrow$

$b = -\frac{1}{2} g t_1^2 + V_0 \sin\alpha\, t_1$

$a = V_0 \cos\alpha\, t_1$

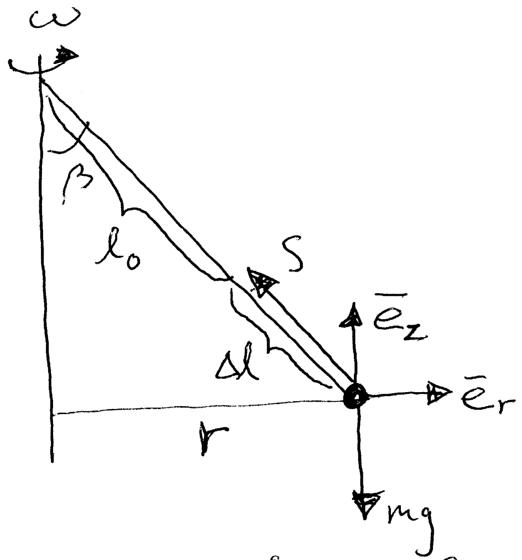
Eliminera t_1 och lös för V_0:

$$V_0 = a \sqrt{\frac{g}{2(a \sin\alpha \cos\alpha - b\cos^2\alpha)}}$$

b) I banans högsta punkt gäller att $v_y = \dot{y} = 0$ och farten ges av

$V_x = V_0 \cos\alpha$

2



$$N\text{II}: \quad \bar{e}_r: \quad m(\ddot{r} - r\dot{\theta}^2) = -S\sin\beta$$

$$\bar{e}_z: \quad 0 = S\cos\beta - mg$$

S är en fjäderkraft : $S = k\Delta l$

$r = (l_0 + \Delta l)\sin\beta$ är konstant $\Rightarrow \ddot{r} = 0$

och $\dot{\theta} = \omega$. Allt detta ger:

$$m(l_0 + \Delta l)\cancel{\sin\beta}\,\omega^2 = k\Delta l\cancel{\sin\beta} \qquad (1)$$

$$k\Delta l\cos\beta = mg \qquad (2)$$

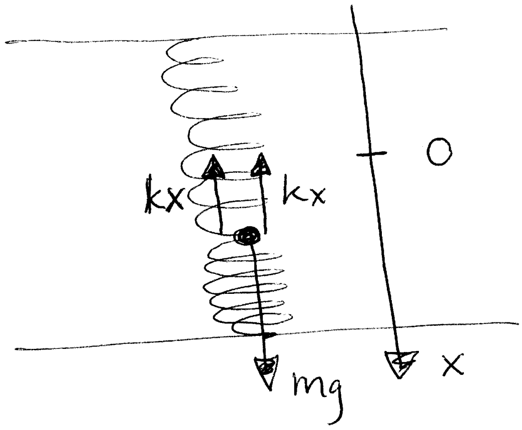
(1) ger $\Delta l(k - m\omega^2) = ml_0\omega^2$ (3)

(2) dividerat med (3) ger:

$$\frac{k\cos\beta}{k - m\omega^2} = \frac{g}{l_0\omega^2} \Rightarrow \cos\beta = \frac{g(k - m\omega^2)}{kl_0\omega^2}$$

$$\beta = \arccos\left(\frac{g(k - m\omega^2)}{kl_0\omega^2}\right)$$

3



NII $m\ddot{x} = mg - 2kx$

$$\ddot{x} + \frac{2k}{m}x = g$$

B.V. $x(0) = 0 \quad \dot{x}(0) = 0$

Inför $\omega_n = \sqrt{\frac{2k}{m}}$

$$x = x_h + x_p$$

$$x_p = \frac{mg}{2k} \qquad x_h = A\cos\omega_n t + B\sin\omega_n t$$

$$x = A\cos\omega_n t + B\sin\omega_n t + \frac{mg}{2k}$$

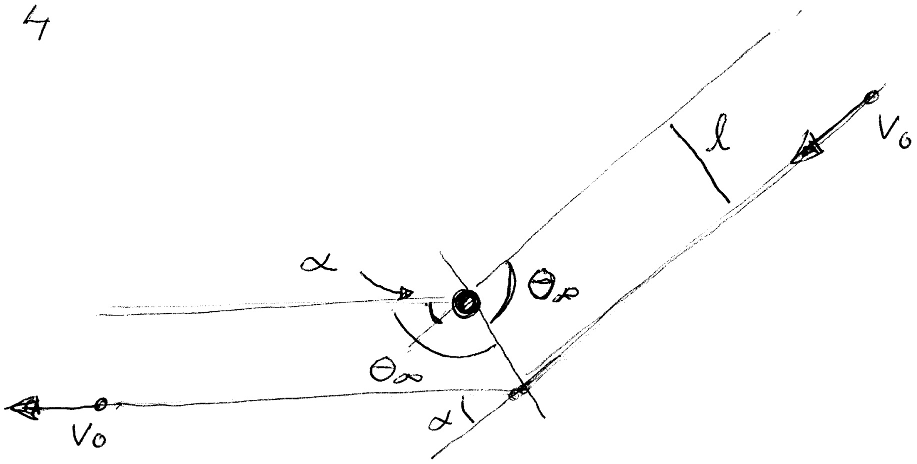
$$x(0) = 0 \Rightarrow A + \frac{mg}{2k} = 0 \Rightarrow A = -\frac{mg}{2k}$$

$$\dot{x}(0) = 0 \Rightarrow B = 0$$

Alltså fås

$$x(t) = \frac{mg}{2k}(1 - \cos\omega_n t)$$

4



Ur figuren fås $2\theta_\infty - \alpha = \pi \Rightarrow \alpha = 2\theta_\infty - \pi$

θ_∞ fås då $r \to \infty$: $r = \dfrac{ed}{1 + e\cos\theta}$

$r \to \infty \Rightarrow 1 + e\cos\theta \to 0 \to \theta_\infty = \arccos(-\frac{1}{e})$

$$E = \frac{(GM)^2 m}{2h^2}(e^2 - 1)$$

$E = \frac{1}{2} m v_0^2$ (Potentiella energin $\to 0$ då $r \to \infty$)

$h^2 = (l v_0)^2$ Detta ger:

$$\frac{1}{2}\cancel{m} v_0^2 (l v_0)^2 = \frac{\cancel{m}}{2}(e^2 - 1)(GM)^2 \Rightarrow$$

$$e^2 - 1 = \frac{v_0^4 l^2}{(GM)^2} \Rightarrow e = \frac{\sqrt{v_0^4 l^2 + (GM)^2}}{GM}$$

$$\theta_\infty = \arccos\left(-\frac{GM}{\sqrt{(GM)^2 + v_0^4 l^2}}\right)$$

$$\alpha = 2\arccos\left(-\frac{GM}{\sqrt{(GM)^2 + v_0^4 l^2}}\right) - \pi$$