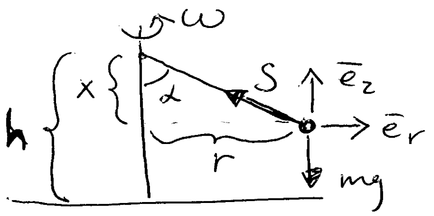
Lösningar tentan 24/5 08, SG1102

1



$$\left.\begin{array}{ll} \bar{e}_r: & -mr\omega^2 = -S\sin\alpha \\ \bar{e}_z: & 0 = S\cos\alpha - mg \end{array}\right\} \Rightarrow$$

$$S = \frac{mg}{\cos\alpha} \quad ; \quad \cancel{m}r\omega^2 = \frac{\cancel{m}g}{\cos\alpha}\sin\alpha \Rightarrow$$

$$\omega^2 = \frac{g}{r}\tan\alpha$$

$$\tan\alpha = \frac{r}{x}$$

Alltså fås

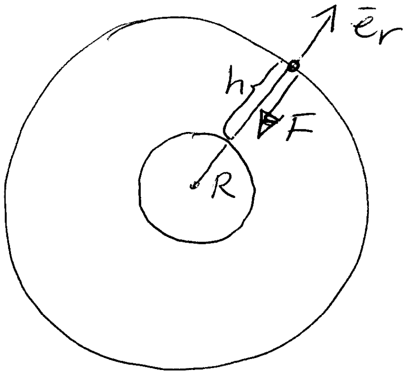
$$\omega^2 = \frac{g}{x}$$

För att partikeln inte ska nudda marken krävs att $x < h$

Alltså fås att

$$\omega^2 > \frac{g}{h} \Rightarrow \omega > \sqrt{\frac{g}{h}}$$

2



$$F = G\frac{Mm}{(R+h)^2}$$

$$N\mathrm{II} \quad \bar{e}_r: \quad -m(R+h)\omega^2 = -G\frac{Mm}{(R+h)^2}$$

$$\Rightarrow \quad (R+h)^3 = \frac{GM}{\omega^2}$$

Geostationär bana innebär att satellitens vinkelhastighet är densamma som jordens, ω_j.

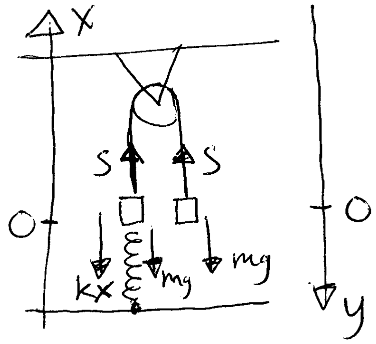
Alltså fås

$$(R+h)^3 = \frac{GM}{\omega_j^2} \quad ; \quad GM = gR^2 \Rightarrow$$

$$R+h = \left(\frac{gR^2}{\omega_j^2}\right)^{1/3} \Rightarrow$$

$$h = \left(\frac{gR^2}{\omega_j^2}\right)^{1/3} - R$$

3



$$\left.\begin{array}{ll} (1) & m\ddot{x} = S - kx - mg \\ (2) & m\ddot{y} = -S + mg \\ (3) & \ddot{x} = \ddot{y} \end{array}\right\} \Rightarrow$$

$$m(\ddot{x}+\ddot{y}) = -kx$$

$$2m\ddot{x} = -kx$$

$$(4) \quad \ddot{x} + \frac{k}{2m}x = 0$$

Lösning $\quad x = A\sin(\omega_n t + \alpha)$

där $\quad \omega_n = \sqrt{\frac{k}{2m}}$

För att svängningarna ska kunna fortgå måste linan vara spänd, d.v.s. $S \geqslant 0$.

(2), (3) och (4) ger

$$S = mg - m\ddot{x} = mg - m\left(-\frac{k}{2m}x\right) =$$

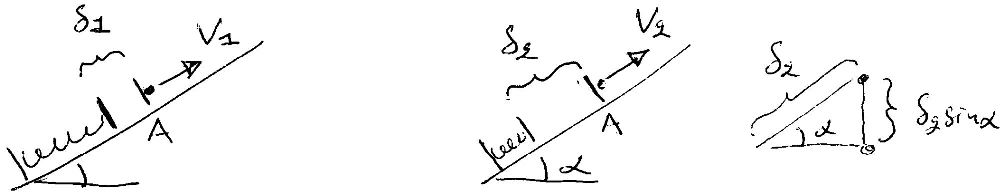
$$mg + \frac{k}{2}x = mg + \frac{k}{2}A\sin(\omega_n t + \alpha)$$

$$mg + \frac{k}{2}A\sin(\omega_n t + \alpha) \geqslant 0 \Rightarrow$$

$$mg - \frac{k}{2}A \geqslant 0 \Rightarrow mg \geqslant \frac{k}{2}A$$

$$A \leqslant \frac{2mg}{k}$$

4



$V_2 = V_1$

Med nollnivån för potentiella energin i A fås

$$\frac{1}{2}k\delta_1^2 - mg\,\delta_1\sin\alpha = \frac{1}{2}mV_1^2 \qquad (1)$$

$$\frac{1}{2}k\delta_2^2 - mg\,\delta_2\sin\alpha = \frac{1}{2}mV_2^2 = \frac{1}{2}m(2V_1)^2 \qquad (2)$$

(1) *ger att*

$$mV_1^2 = k\delta_1^2 - 2mg\,\delta_1\sin\alpha\,, \quad \text{insatt i (2) ger:}$$

$$\frac{1}{2}k\delta_2^2 - mg\,\delta_2\sin\alpha = 2\left(k\delta_1^2 - 2mg\,\delta_1\sin\alpha\right)$$

$$\delta_2^2 - \frac{2mg\sin\alpha}{k}\delta_2 - 4\delta_1^2 + \frac{8mg\,\delta_1\sin\alpha}{k} = 0$$

$$\delta_2 = \frac{mg\sin\alpha}{k} \pm \sqrt{\left(\frac{mg\sin\alpha}{k}\right)^2 + 4\delta_1^2 - \frac{8mg\delta_1\sin\alpha}{k}}$$

Vilken rot ska man välja? Prova med $\alpha = 0$!

Då fås $\delta_2(\alpha=0) = (\pm)\sqrt{4\delta_1^2} = 2\delta_1$

Man måste alltså välja den positiva roten.

$$\delta_2 = \frac{mg\sin\alpha}{k} + \sqrt{\left(\frac{mg\sin\alpha}{k}\right)^2 + 4\delta_1^2 - \frac{8mg\,\delta_1\sin\alpha}{k}}$$
