Dynamics and Motion control

Lecture 7
Robustness to sensor noise and modelling errors

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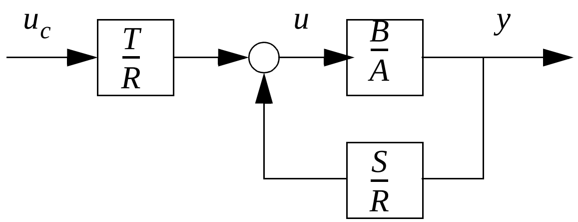
7.1. Lecture outline

- **1. Introduction**
- 2. The sensitivity and complementary sensitivity functions
- 3. Example

7.1.1. From Last lectures

- **Polynomial approach to pole placement design.**



- **Choose the control structure,** $\frac{S}{R}$**.**
- **Select the desired c.l. performance polynomial** $A_m A_o$ **from specifications.**
- **Solve the Diophantine equation** $A_m A_o = AR + BS$
- **Calculate the feed forward polynomial** $T = A_o t_0$
- **Approximate the discrete time controller with e.g. Tustin, or perform the design in discrete time from the beginning.**

7.1.2. Sensor noise and model errors

- **The pole placement and model following design techniques do not take sensor noise and model parameter errors into account.**
- **Sensor noise is always a critical factor when analog sensors are used.**
- **The process model which is the base for the pole placement is always an approximation and has errors.**
    - Nonlinear effects
    - Varying parameters due to operation, e.g., temperature varying friction.
    - Varying operational conditions, e.g., a machine runs sometimes with and sometimes without payload.

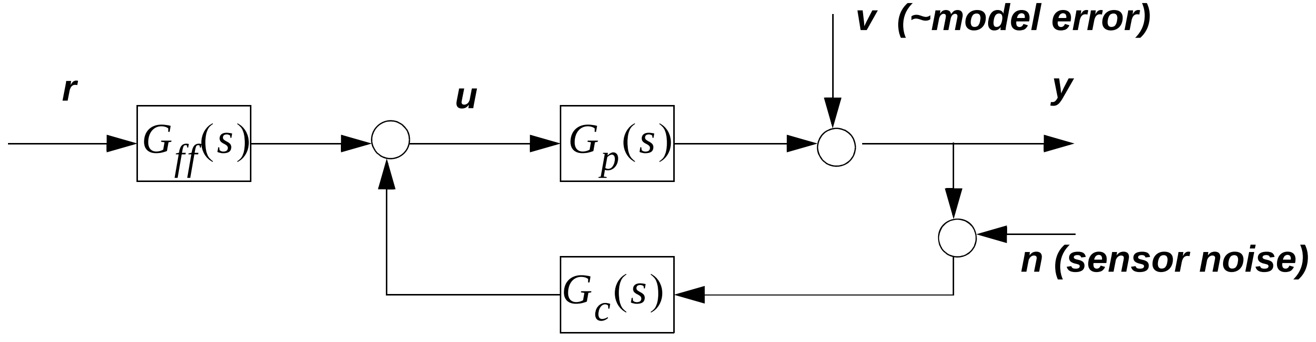
7.1.3. This lecture

- **Select $A_m(s)$ such that the c.l. response** $y = \frac{Bt_0}{A_m} u_c$ **has the desired properties of the c.l. response.**
- **Selecting the $A_o(s)$ part of the closed loop poles based on robustness and sensor noise.**
  - Introduction of the Sensitivity and Complementary sensitivity functions.
  - Comparison with state feedback.
  - Example

7.2. Lecture outline

- 1. Introduction
- **2. The sensitivity and complementary sensitivity functions**
- 3. Example

7.2.1. Parameter uncertainty and sensor noise



- **Uncertainty modelling of the real process,** $G_p(s) = G_p^n(s)[1 + \Delta_p(s)]$

  $G_p^n(s)$ is the nominal model used in control design. $\Delta_p(s)$ the relative error.

- **Design goals**

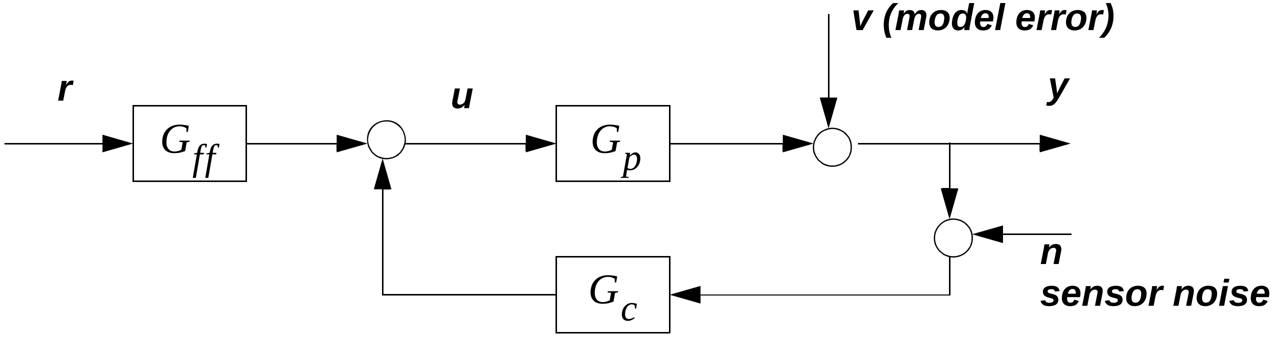
  - low influence of $\Delta_p(s)$, (low gain from *v -> y),* $\Delta_p(s)$ is unknown

  - low high frequency gain from *n -> y*, sensor noise normally high frequency

  - high gain (bandwidth) from *r -> y,* servo performance

- What are the requirements on $G_c(s)$ to achieve the goals ?

7.2.2. Sensitivity function

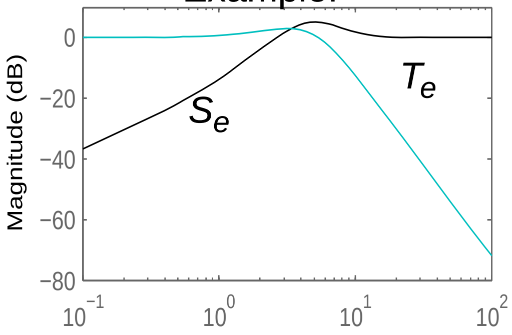


$$y = \frac{G_p G_{ff}}{1 + G_p G_c} r + S_e v + (1 - S_e) n \qquad \text{where:} \qquad S_e(s) = \frac{1}{1 + G_p G_c}$$

S_e is called the Sensitivity function and $T_e = 1 - S_e$ the complementary sensitivity function.

Regulator design is a compromise between S_e, model errors and T_e, sensor noise.

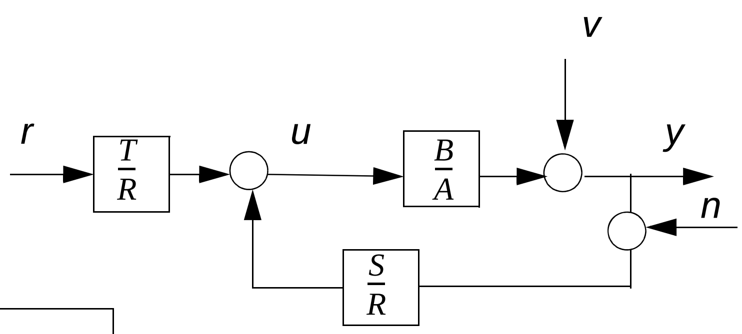
Example:



7.2.3. S_e and T_e in polynomial form

Control law

$$u(s) = \frac{T}{R}r - \frac{S}{R}y$$



Closed loop response

$$y = \frac{BT}{AR+BS}r + \frac{AR}{AR+BS}v - \frac{BS}{AR+BS}n$$

Pole placement gives

$$AR + BS = A_m A_o$$

Select $T = A_o t_0$

Then

$$y = \frac{Bt_0}{A_m}r + \frac{AR}{A_m A_o}v - \frac{BS}{A_m A_o}n$$

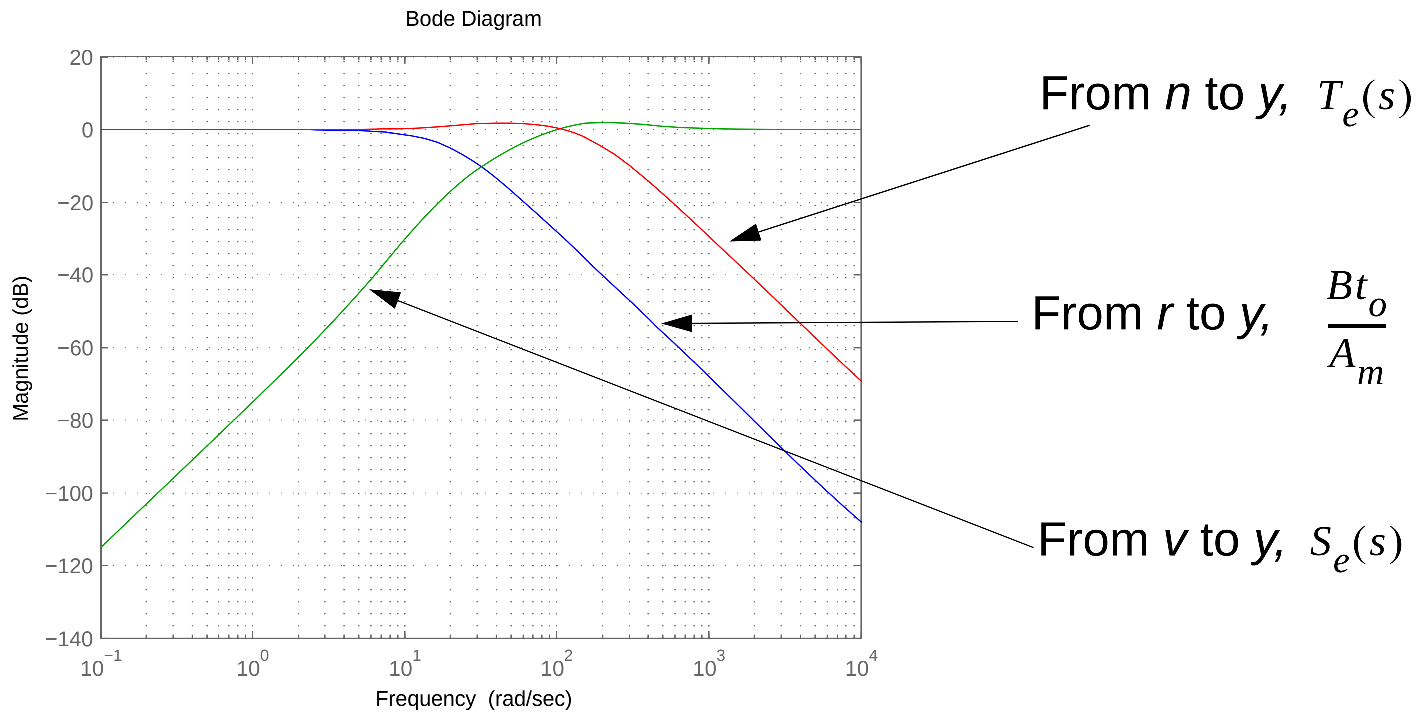
$$y = \frac{Bt_0}{A_m}r + S_e v - T_e n$$

Select A_m *and* t_0 *to get specified response from* r .

Select A_o *to get specified response from* v *and* n.

7.2.4. Frequency plane design of A_o

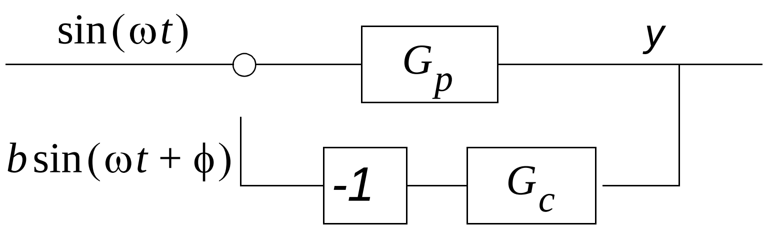
The response from command signal u_c, and the response from noise n, and disturbance v, can be independently designed.

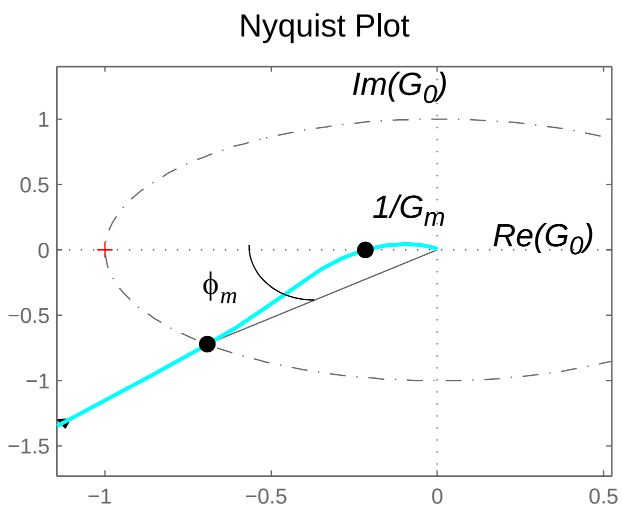


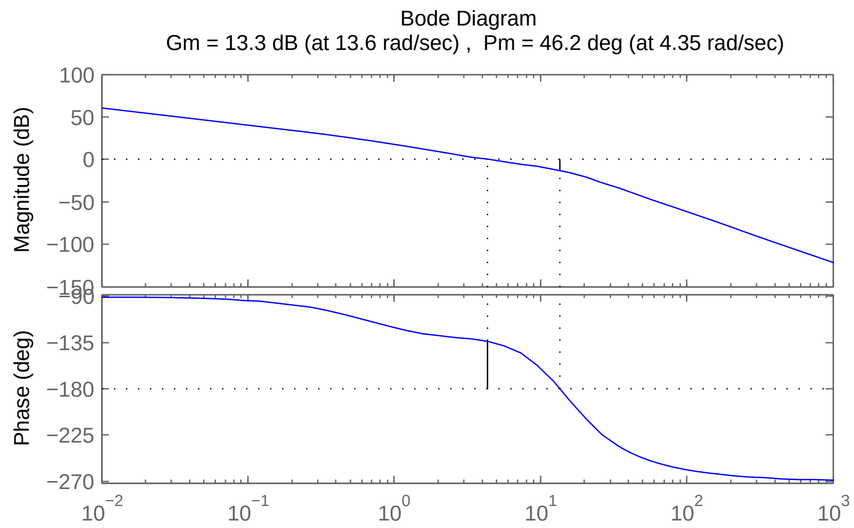
7.2.5. Stability margins against model unc.

-The phase margin, ϕ_m and gain margin, G_m can be shown in the Bode and Nyquist plots.

$-G_0 = -G_p G_c$ can be ϕ_m and G_m wrong without instability.





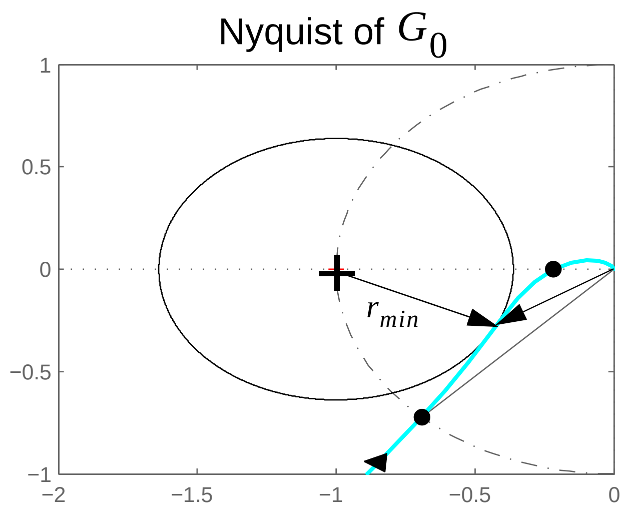


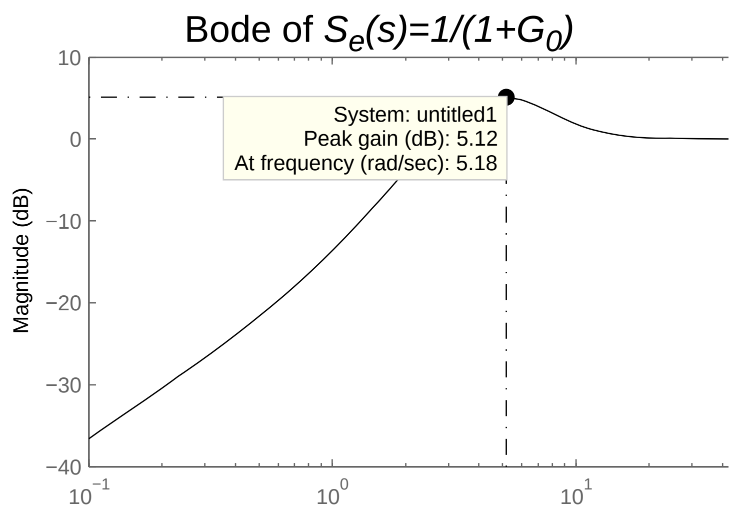
7.2.6. Sensitivity function as stability margin

Radius of the circle with centre in (-1,0) that is tangent to the loop gain $|G_0|$ is $r = |1 + G_0|$, the minimum radius, $r_{min} = |1 + G_0|_{min}$ can be used as a measure on the stability margin against parameter errors in the design model.

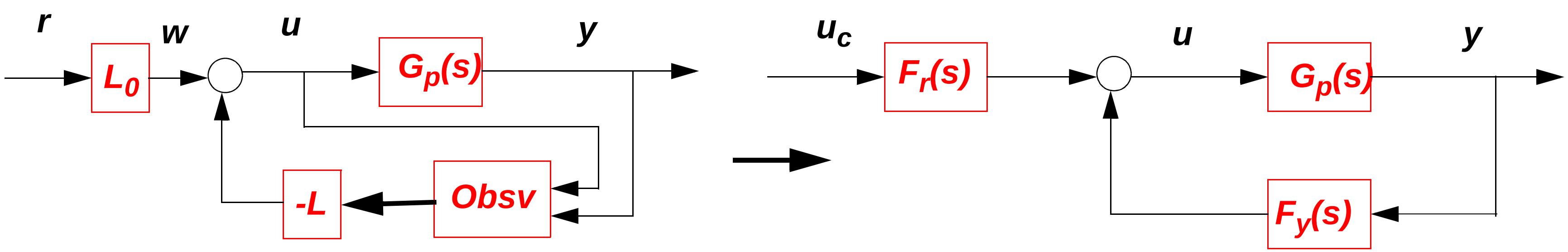
$|S_e|_{max} = |1/(1 + G_0)|_{max} = |1/r_{min}|$, $|S_e|_{max}$ is the same as stability margin.

Typical specification $|S|_{max} < (2-5)\mathrm{dB}$.





7.2.7. Connection between state feedback and S/R



The state observer $\dot{\hat{x}} = A\hat{x} + Bu + K(y - C\hat{x})$ with control law $u = -L\hat{x} + w$, gives

$$u = -\{L(sI - A + BL + KC)^{-1}K\}y + \{1 - L(sI - A + BL + KC)^{-1}B\}L_0 r = -F_y y + F_r r$$

see page 186 in Glad Ljung for a derivation.

AND: $S_e(s) = \dfrac{1}{1 + G_p F_y}$ and $T_e(s) = \dfrac{G_p F_y}{1 + G_p F_y}$

If the order of the polynomials $S(s)$, $T(s)$ and $R(s)$ are selected as, $\deg R = \deg G_p$ $\deg S = \deg T = \deg G_p - 1$.

Then:

1.) $F_y(s) = \frac{S(s)}{R(s)}$ and $F_r(s) = \frac{T(s)}{R(s)}$.

2.) The poles of the c.l. with state feedback $(A - BL)$ are the same as the c.l. polynomial $A_m(s)$, and the poles of the observer $(A - KC)$ are the same as for the polynomial $A_o(s)$.

Example: Selecting $\frac{S}{R} = \frac{s_1 s + s_0}{s^2 + r_1 s + r_0}$ and $T = A_o t_0$ for $G_p(s) = \frac{b}{s(s+a)}$, with

$A_m(s) = \det(sI - A + BL)^{-1}$ and $A_o(s) = \det(sI - A + KC)^{-1}$. Gives identical controllers.

However: selecting $R(s)$ one order higher than $S(s)$ i.e. designing a full state observer may give a discrete time controller with one sample delay.

Instead if, $\deg S = \deg R = \deg T = \deg G_p - 1$ then there is no delay in the controller

and if $A_m(s) = \det(sI - A + BL)^{-1}$ and $A_o(s) = \det(sI - A + KC)^{-1}$ where the observer is designed as a reduced order observer (Lueneburger observer) then again the two designs are identical. (see page 182 in Glad Ljung)

Advantage:

- no delay in discrete time

- one order lower controller gives less computations on the processor.

- simpler and more intuitive to include integral action in the controller.

- straight forward solution to the reduced observer comp. to s.s. design.

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7.3.1. Example how $A_o(s)$ influences the design

- Position controller using PID feedback $G_c(s) = \left(P + Ds + I\frac{1}{s}\right)$

- Process model $\frac{B}{A} = \frac{b}{s(s+a)}$

- Write it on polynomial form with a filter in denominator to get a proper feedback where $\deg S = \deg R$, $\frac{S(s)}{R(s)} = \frac{s_2 s^2 + s_1 s + s_0}{s(s+r_0)}$.

- The Integral term increases the order of the polynomials $S(s)$, $T(s)$ and $R(s)$ with one compared to the order without integral action.

- C.I. $AR + BS = s^4 + (r_0 + a)s^3 + ar_0 s^2$, 4:th order polynomial

- The c.l. transfer function is $y = \frac{BT}{AR + BS} r + \frac{AR}{AR + BS} v + \frac{BS}{AR + BS} n$.

- Select $AR + BS = A_m A_o$ and $T = A_o t_0$ where $A_m(s) = s^2 + 2\zeta_m \omega_m s + \omega_m^2$ and $A_o(s) = s^2 + 2\zeta_o \omega_o s + \omega_o^2$.

- Calculate t_0 by computing the dc-gain from r to y,

$$y = \frac{BT}{AR + BS} r = \frac{BA_o t_0}{A_m A_o} r = \frac{Bt_0}{A_m} r = \frac{Bt_0}{s^2 + 2\zeta_m \omega_m s + \omega_m^2} r,$$ selecting $t_0 = \frac{\omega_m^2}{B}$ gives

$$y = \frac{\omega_m^2}{s^2 + 2\zeta_m \omega_m s + \omega_m^2} r$$ which has unit gain.

- The complete response becomes $y = \frac{\omega_m^2}{A_m} r + \frac{AR}{A_m A_o} v + \frac{BS}{A_m A_o} n$.

- Specifications:

  - *Rise time = 0.2 s* (5% - 95% of final value) without saturation!

    -> $\omega_m = 30, \zeta_m = 0.9$

  - There is electric noise from the sensor at *50 Hz*.

    -> $T_e(2\pi 50j) < 20dB$

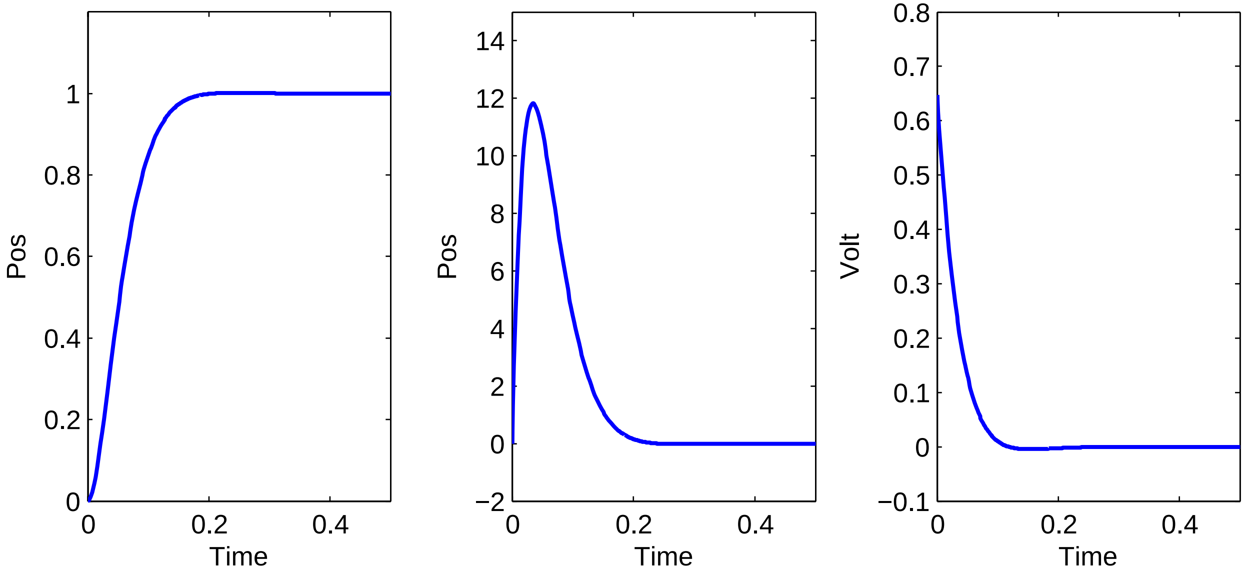
  - The inertia can change from a nominal value to twice this value due to changes in payload.

    -> Sufficiently phase and gain margins $|S_e|_{max} < Xdb$

- $A_m(s)$ is given from rise time.

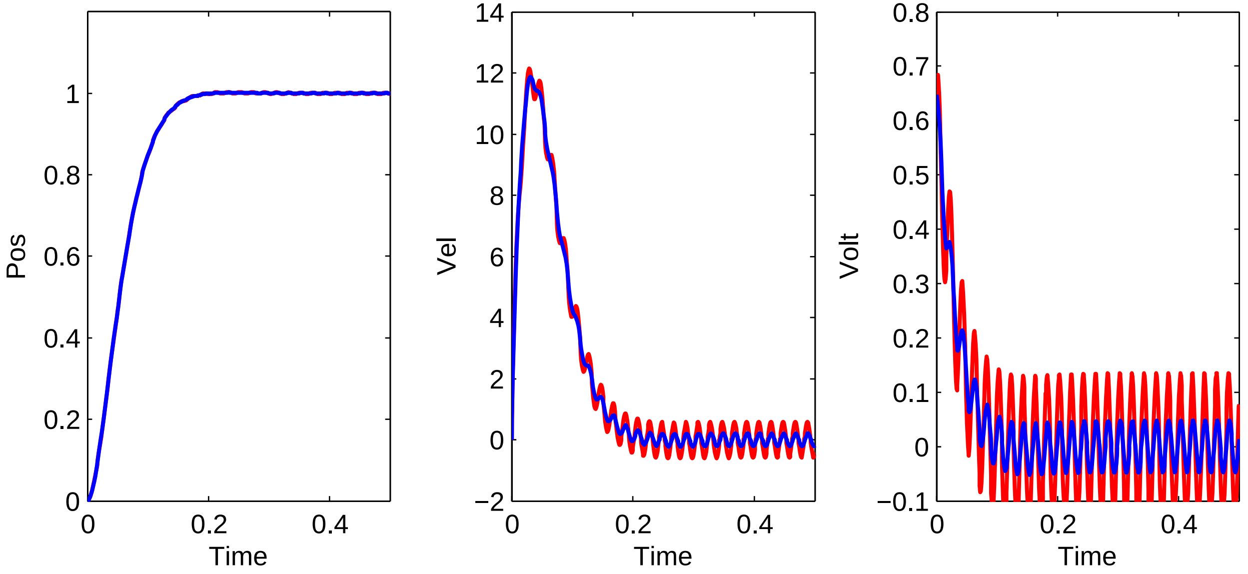
- Test different observer polynomials $A_o(s) = s^2 + 2\zeta_o\omega_o s + \omega_o^2$ if it is possible to satisfy the other two specifications.

- The nominal step response without sensor noise and increased inertia does **not** depend on $A_o(s)$! $y = \frac{BT}{AR + BS} = \frac{\omega_m^2}{s^2 + 2\zeta_m\omega_m s + \omega_m^2}$

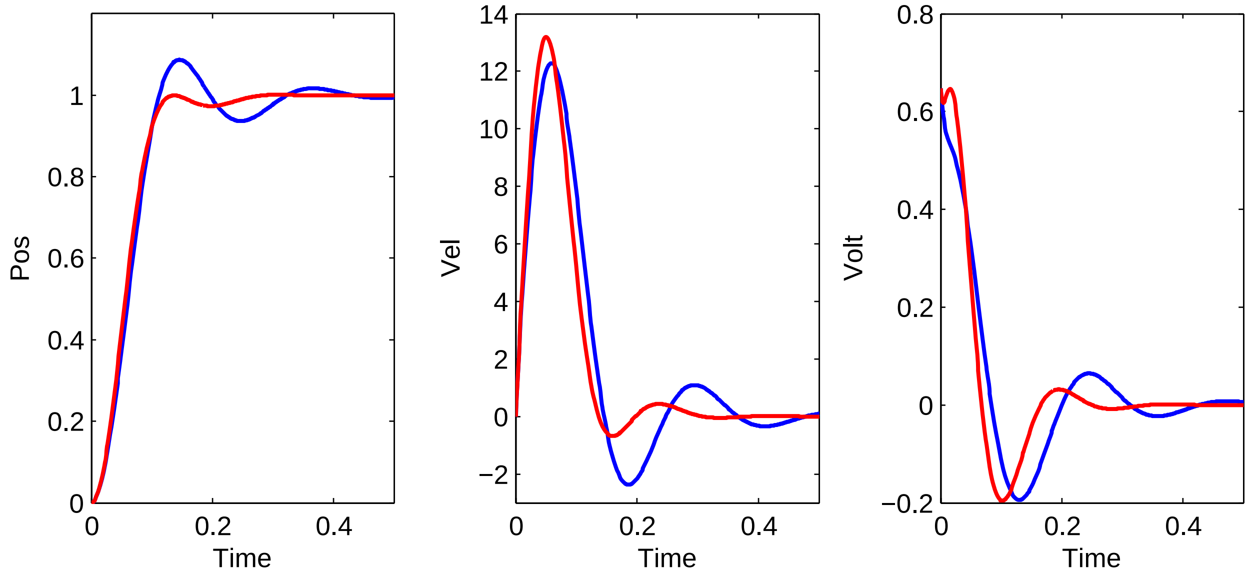


- Rise time OK!

- Design two different observer polynomial, $\omega_o = 2\omega_m$ and $\omega_o = 4\omega_m$.
- 1.) Inspect the step response with sensor noise, $n = 0.01\sin(2\pi 50t)$
- The red line is with the faster observer and the blue line with the slower observer.



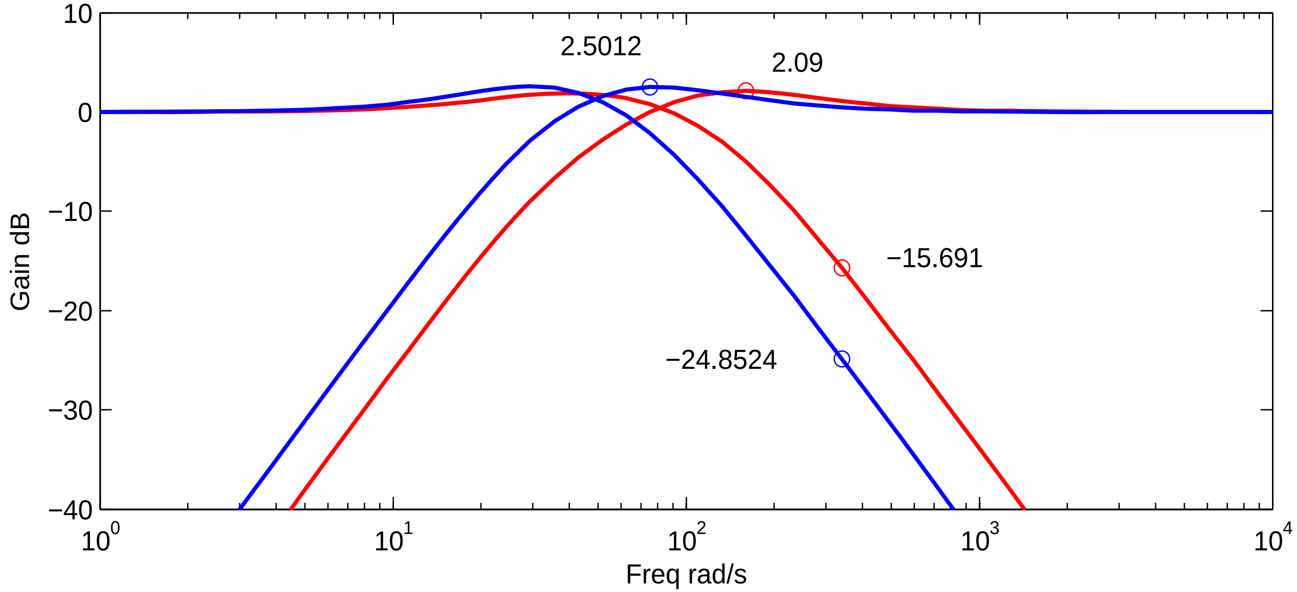
- 2.) Inspect the step response when the inertia is increased twice. This could e.g. happen when the machine picks up a payload.
- Red line is the fast observer polynomial. Blue line is the slow observer polynomial.



7.3.2. Frequency response for the example

- Calculate the Sensitivity function $S_e(s) = \frac{AR}{AR + BS}$ and the Complementary Sensitivity function $T_e(s) = 1 - S_e(s) = \frac{BS}{AR + BS}$.

- Blue line is the slower and red line the faster observer. *(50 Hz = 314 rad/s)*



7.3.3. Robust control design methods.

- **Find a fixed parameter controller that satisfies some c.l. specifications for a process with uncertain physical parameters.**
  - Example: $G_p(s, k, \tau) = \dfrac{k}{\tau s + 1}$ where $k \in [1, 5]$ and $\tau \in [0.01, 0.05]$.
- **Frequency plane, Quantified feedback theory, QFT.**

A good reference is:
Quantitative feedback design of linear and nonlinear control systems
Oded Yaniv
Kluwer Academic Publisher, ISBN 0-7923-8529-2

- **Complex plane, using pole region assignment**

A good reference is:
Robust Control -systems with uncertain physical parameters

Jurgen Ackerman Springer-Verlag, ISBN 0-387-19843-1

7.3.4. Summary

- Regulator design is about giving and taking
  -better robustness against parameter variations gives higher sensitivity to noise, and the other way around.

- Tune $A_o(s)$ to achive an appropriate compromise.

- Pole placement can be done with state space models or with transfer functions. The result can depending on the choice of *S(s)* and *R(s)* be the same.

- Selecting $\deg S = \deg R = \deg G_p - 1$ gives something similar to a reduced order observer and less delay in the implementation.