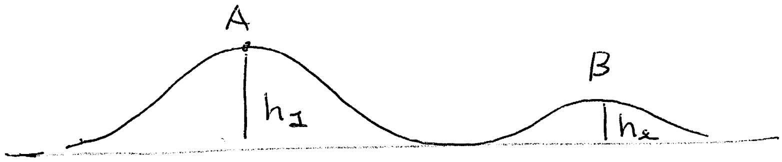
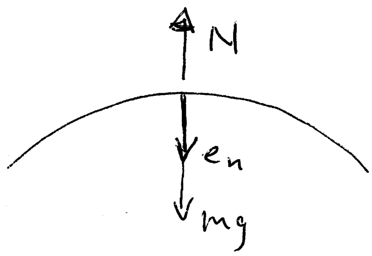
Lösningar 5C1102 27/8 05

1



Mekaniska energin konstant $\Rightarrow$ $mgh_1 = mgh_2 + \frac{1}{2}mv_B^2$

$$v_B^2 = 2g(h_1 - h_2)$$



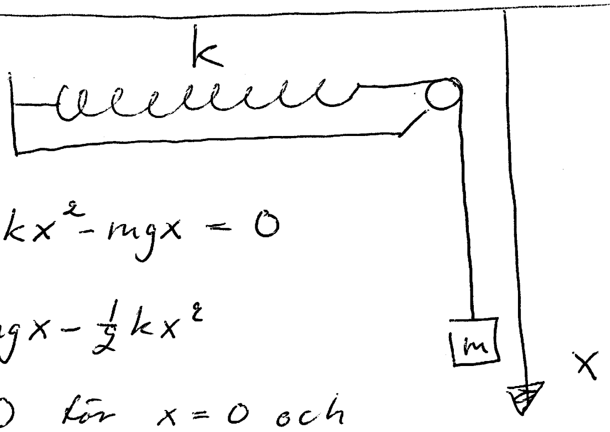
$$\bar{e}_n: \quad m\frac{v_B^2}{\rho} = mg - N$$

$$N = -m\frac{v_B^2}{\rho} + mg =$$

$$= mg\left(1 - \frac{2(h_1 - h_2)}{\rho}\right)$$

$$N \geq 0 \Rightarrow h_1 - h_2 \leq \frac{\rho}{2} \Rightarrow h_1 \leq h_2 + \frac{\rho}{2}$$

2



$$T + \frac{1}{2}kx^2 - mgx = 0$$

$$T = mgx - \frac{1}{2}kx^2$$

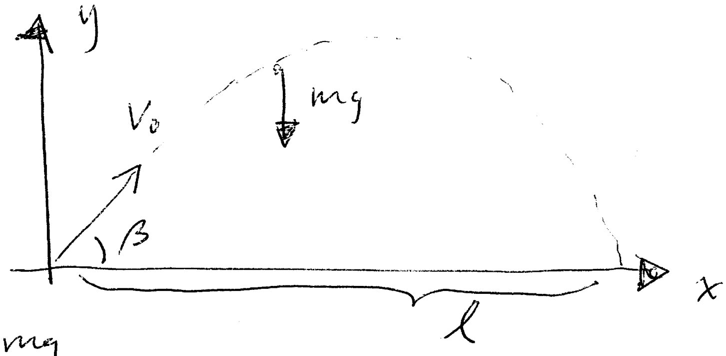
$T_{min} = 0$ för $x = 0$ och

$$x = \frac{2mg}{k}$$

$$\frac{dT}{dx} = mg - kx = 0 \Rightarrow x = \frac{mg}{k}$$

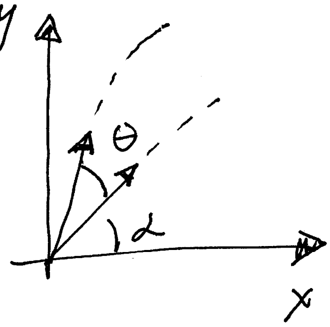
$$T_{max} = mg\frac{mg}{k} - \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{1}{2}\frac{(mg)^2}{k}$$

3



$m\ddot{y} = -mg$

$m\ddot{x} = 0$



B.V.: $x = y = 0$

$\dot{x} = v_0 \cos\beta$ då $t = 0$

$\dot{y} = v_0 \sin\beta$

Integrera två gånger! Detta ger:

$y = -\frac{1}{2} g t^2 + v_0 \sin\beta \, t$

$x = v_0 \cos\beta \, t$

Vid nedslaget fås: $x = l$, $y = 0$

$0 = -\frac{1}{2} g t^2 + v_0 \sin\beta t$

$l = v_0 \cos\beta t$ $\Rightarrow$ $t = \frac{2v_0}{g} \sin\beta$

$l = \frac{2v_0^2}{g} \sin\beta \cos\beta =$

$= \frac{v_0^2}{g} \sin(2\beta)$

1. $\beta = \alpha = \frac{\pi}{4}$ $\Rightarrow$ $l_1 = \frac{v_0^2}{g}$

2. $\beta = \alpha + \theta$ $\quad l_2 = \frac{v_0^2}{g} \sin(2(\alpha + \theta))$

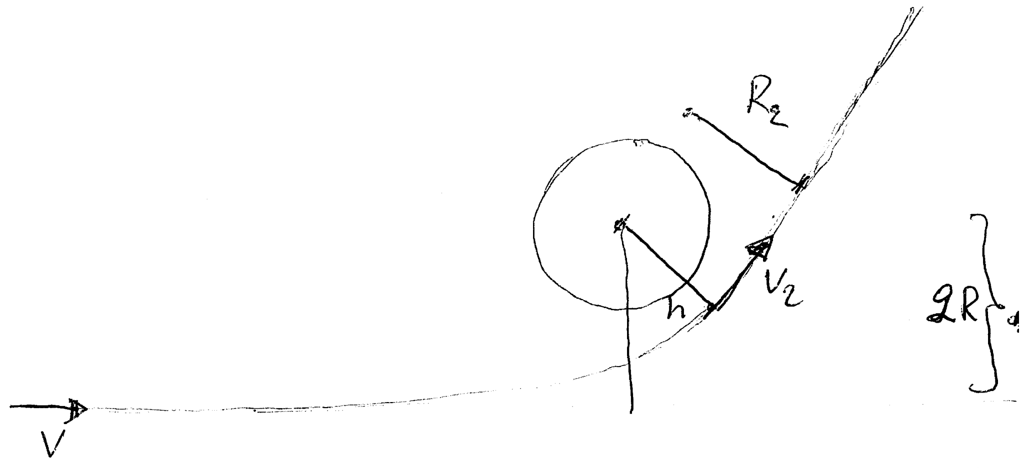
$l_2 = \frac{1}{2} l_1 \Rightarrow \sin(2(\alpha + \theta)) = \frac{1}{2} \Rightarrow$

$2(\alpha + \theta) = \frac{\pi}{6}$ eller $2(\alpha + \theta) = \frac{5\pi}{6}$

Första lösningen ger $\theta < 0$ ej relevant

Andra lösningen ger $\theta = \frac{5\pi}{12} - \alpha = \frac{\pi}{6} = 30^\circ$

4



Rörelsemängdsmomentet bevaras : $V m 2R = V_2 m R_2$ ①

Energin bevaras : $\frac{1}{2} m V^2 = \frac{1}{2} m V_2^2 - g m \frac{R^2}{R_2}$ ②

(Potentiella energin → 0 då avståndet är stort)

② $V_2 = \frac{2R}{R_2} V$ insatt i ① ger

$$\frac{1}{2} V^2 = \frac{1}{2} \frac{4R^2}{R_2^2} V^2 - g \frac{R^2}{R_2} \Rightarrow$$

$$R_2^2 + \frac{2gR^2}{V^2} - 4R^2 = 0$$

$$R_2 = -\frac{gR^2}{V^2} \overset{+}{(-)} \sqrt{\left(\frac{gR^2}{V^2}\right)^2 + 4R^2}$$

$$h = R_2 - R = R\left(-\frac{gR}{V^2} - 1 + \sqrt{\left(\frac{gR}{V^2}\right)^2 + 4}\right)$$

b) Kometen passerar om $h \geq 0$ →

$$\left(\frac{gR}{V^2}\right)^2 + 4 \geq \left(\frac{gR}{V^2} + 1\right)^2 \Rightarrow$$

$$\frac{2gR}{V^2} \leq 3 \Rightarrow V \geq \sqrt{\frac{2gR}{3}}$$