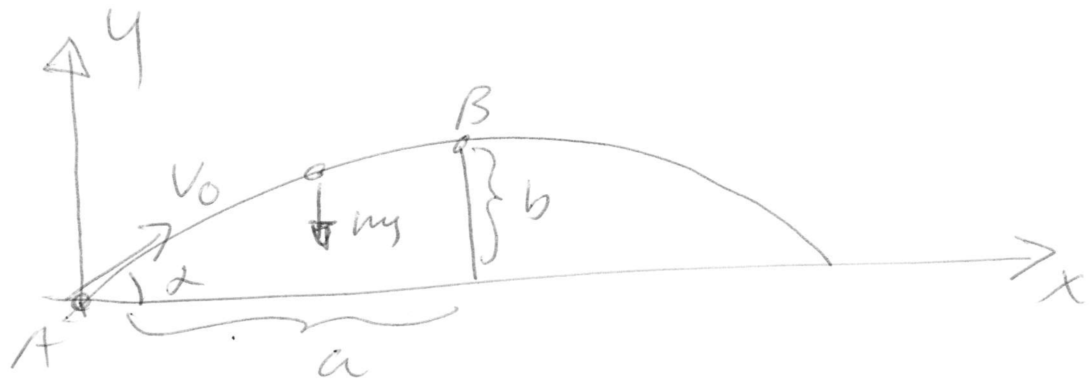
Lösningar till tentan 25/5 2009, SG1102

1.



$m\ddot{y} = -mg$ ⟹ $\dot{y} = -gt + C_1$ B.V ⟹ $C_1 = v_0 \sin\alpha$

$m\ddot{x} = 0$ ⟹ $\dot{x} = C_2$ $C_2 = v_0 \cos\alpha$

$\dot{y} = -gt + v_0 \sin\alpha$ ⟹ $y = -\frac{1}{2}gt^2 + v_0 \sin\alpha\, t + C_3$

$\dot{x} = v_0 \cos\alpha$ ⟹ $x = v_0 \cos\alpha\, t + C_4$

B.V. $\begin{cases} t = 0 \\ x = y = 0 \end{cases}$ ⟹ $C_3 = C_4 = 0$

$y = -\frac{1}{2}gt^2 + v_0 \sin\alpha\, t$

$x = v_0 \cos\alpha\, t$

I B: $x = a$, $y = b$, $\dot{y} = 0$

$0 = -gt + v_0 \sin\alpha$ (1)

$b = -\frac{1}{2}gt^2 + v_0 \sin\alpha\, t$ (2)

$a = v_0 \cos\alpha\, t$ (3)

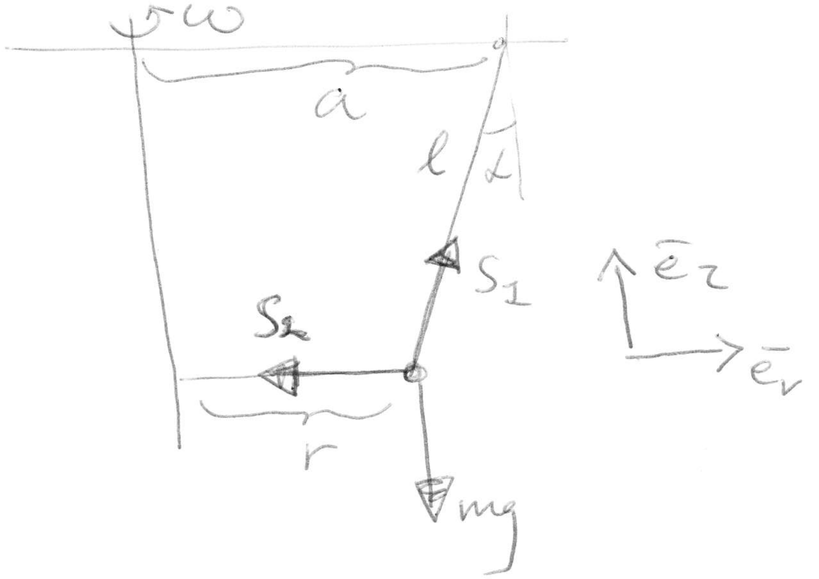
(1) ⟹ $t = \dfrac{v_0 \sin\alpha}{g}$

Insatt i (2) och (3) ger

$$\left.\begin{aligned} b &= \frac{1}{2}\frac{v_0^2 \sin^2\alpha}{g} \\ a &= \frac{v_0^2 \sin\alpha \cos\alpha}{g} \end{aligned}\right\} \Rightarrow \frac{b}{a} = \frac{1}{2}\tan\alpha \Rightarrow$$

$$\tan\alpha = 2\frac{b}{a} \qquad \alpha = \arctan\left(\frac{2b}{a}\right)$$

2.



$\bar{e}_r$: $m(\ddot{r} - r\dot{\theta}^2) = -S_2 + S_1 \sin\alpha$ (1)

$\bar{e}_z$: $0 = S_1 \cos\alpha - mg$ (2)

$\ddot{r} = 0$; $r = a - l\sin\alpha$; $\dot{\theta} = \omega$

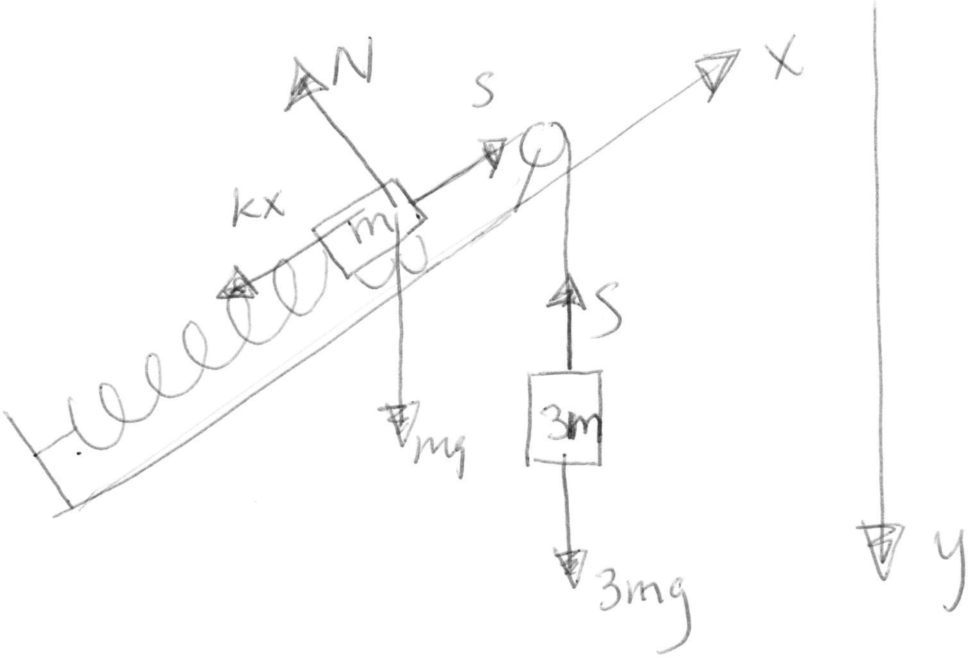
(2) $\Rightarrow$ $S_1 = \dfrac{mg}{\cos\alpha}$

Sätt in i (1) och utnyttja övriga samband:

$-m(a - l\sin\alpha)\omega^2 = -S_2 + mg\tan\alpha \Rightarrow$

$S_2 = mg\tan\alpha + m(a - l\sin\alpha)\omega^2$

3.



NII: $m\ddot{x} = -kx + S - mg\sin\beta$ (1)

$3m\ddot{y} = -S + 3mg$ (2)

$\ddot{x} = \ddot{y}$ (3)

(2) och (3) ger $S = 3mg - 3m\ddot{x}$

insatt i (1) ger

$$m\ddot{x} = -kx + (3mg - 3m\ddot{x}) - mg\sin\beta$$

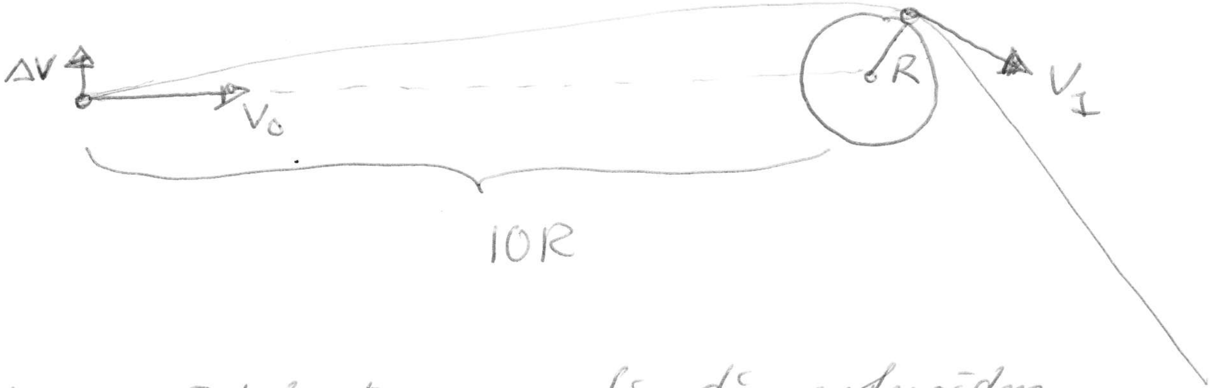
$$4m\ddot{x} + kx = mg(3 - \sin\beta)$$

$$\ddot{x} + \frac{k}{4m}x = \frac{g}{4}(3 - \sin\beta)$$

$$\omega_n = \sqrt{\frac{k}{4m}} = \frac{1}{2}\sqrt{\frac{k}{m}}$$

$$T_n = \frac{2\pi}{\omega_n} = 4\pi\sqrt{\frac{m}{k}}$$

4.



Minsta värdet för ΔV fås då asteroiden tangerar jorden.

Rörelsemängdsmomentet bevaras:

$$10Rm\Delta V = RmV_1 \quad (1)$$

Energin bevaras:

$$\frac{1}{2}m(V_0^2+\Delta V^2) - \frac{mgR^2}{10R} = \frac{1}{2}mV_1^2 - \frac{mgR^2}{R} \quad (2)$$

(1) ger: $V_1 = 10\Delta V$, insatt i (2) ger:

$$\frac{1}{2}(V_0^2+\Delta V^2) - \frac{gR}{10} = \frac{1}{2}(10\Delta V)^2 - gR \Rightarrow$$

$$100\Delta V^2 - \Delta V^2 = V_0^2 + 2gR - \frac{gR}{5}$$

$$99\Delta V^2 = V_0^2 + \frac{9}{5}gR$$

$$\Delta V^2 = \frac{V_0^2}{99} + \frac{gR}{55}$$

$$\Delta V = \sqrt{\frac{V_0^2}{99} + \frac{gR}{55}}$$

Detta är minsta hastighetsförändringen som krävs för att undvika kollision med jorden