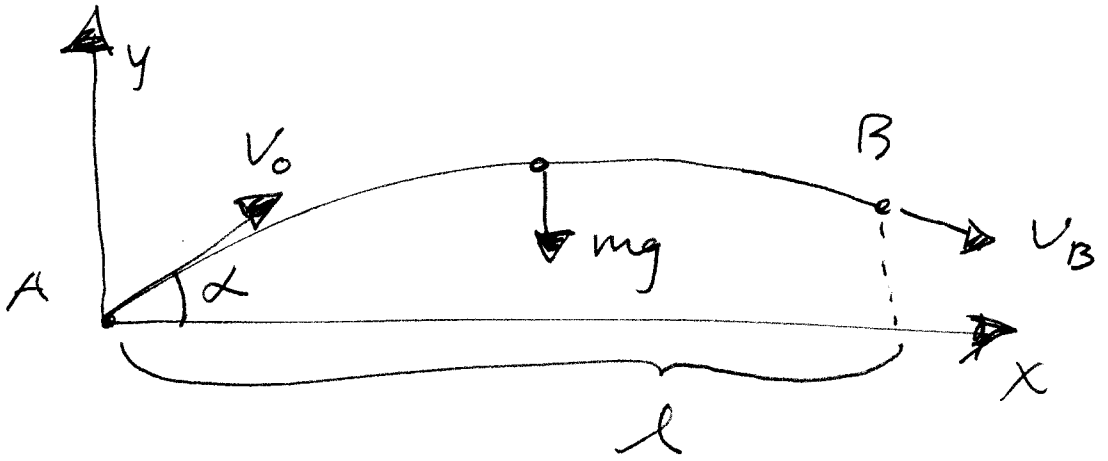
Lösningar till tentan 17/8 2009

1



$m\ddot{y} = -mg$

$\ddot{y} = -g$

$\dot{y} = -gt + C_1$ B.V.: $\begin{cases} t = 0 \\ \dot{y} = V_0 \sin\alpha \end{cases} \Rightarrow C_1 = V_0 \sin\alpha$

$\dot{y} = -gt + V_0 \sin\alpha$

$m\ddot{x} = 0$

$\ddot{x} = 0$

$\dot{x} = C_2$ B.V.: $\begin{cases} t = 0 \\ \dot{x} = V_0 \cos\alpha \end{cases} \Rightarrow C_2 = V_0 \cos\alpha$

$\dot{x} = V_0 \cos\alpha$

$x = V_0 \cos\alpha\, t + C_3$ B.V.: $\begin{cases} t = 0 \\ x = 0 \end{cases} \Rightarrow C_3 = 0$

$x = V_0 \cos\alpha\, t$

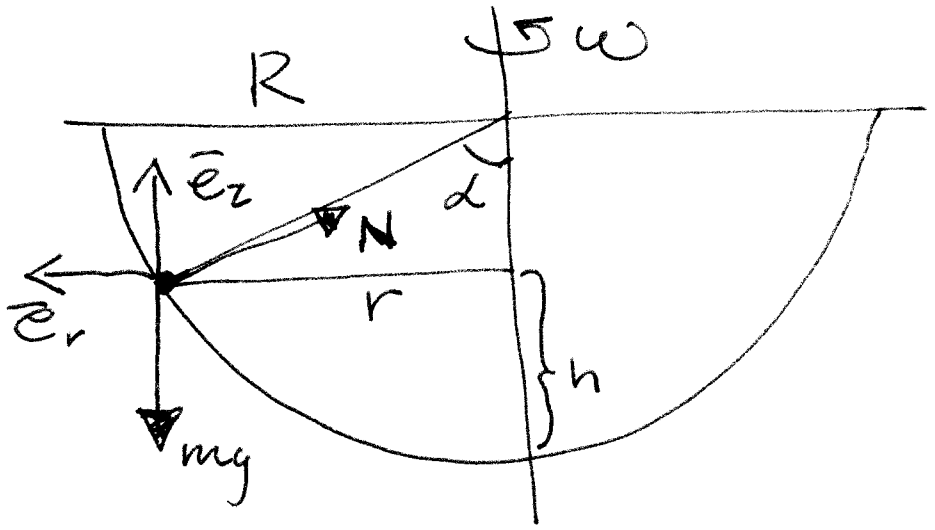
$\ell = V_0 \cos\alpha\, t_B \rightarrow t_B = \dfrac{\ell}{V_0 \cos\alpha}$ i B:

$\dot{x} = V_0 \cos\alpha$

$\dot{y} = -\dfrac{g\ell}{V_0 \cos\alpha} + V_0 \sin\alpha$

$$V_B = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{V_0^2 - 2g\ell \tan\alpha + \frac{g^2\ell^2}{V_0^2 \cos^2\alpha}}$$

2



$\bar{e}_r$: $m(\ddot{r} - r\dot{\theta}^2) = -N\sin\alpha$ ①

$\bar{e}_z$: $0 = N\cos\alpha - mg$ ②

$r = R\sin\alpha$ $\dot{\theta} = \omega$ insatt i ① ger

$-mR\omega^2\sin\alpha = -N\sin\alpha \Rightarrow$

$N = mR\omega^2$ insatt i ②

$mR\omega^2\cos\alpha = mg$

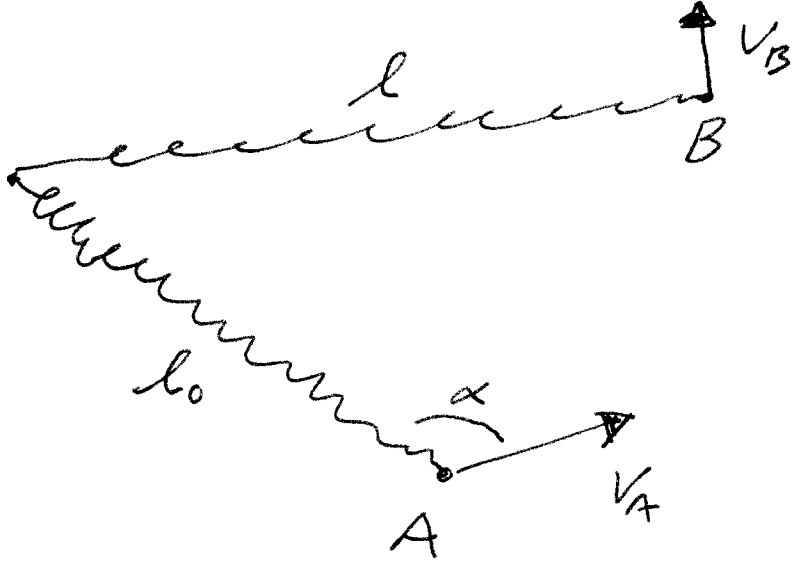
$R\cos\alpha = \frac{g}{\omega^2}$

$h = R - R\cos\alpha = R - \frac{g}{\omega^2}$

Om $\frac{g}{\omega^2 R} > 1$ (liten rotation) kan vi

att $h = 0$

3



Energikonservering ger

$$\frac{1}{2}mV_A^2 = \frac{1}{2}mV_B^2 + \frac{1}{2}k(l-l_0)^2 \quad \Longrightarrow$$

$$V_B = \sqrt{V_A^2 - \frac{k}{m}(l-l_0)^2}$$

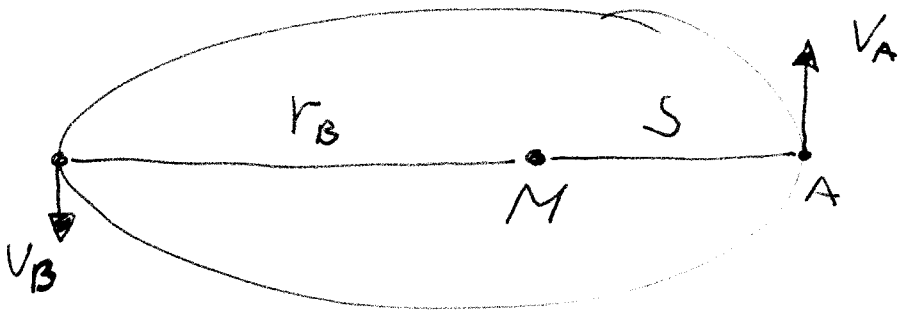
Rörelsemängdsmomentets bevarande ger

$$l_0 V_A \cos\beta = l V_B \qquad \text{där} \qquad \beta = \alpha - \frac{\pi}{2}$$

$$\cos\beta = \frac{l}{l_0}\frac{V_B}{V_A} = \frac{l}{l_0}\,\frac{\sqrt{V_A^2 - \frac{k}{m}(l-l_0)^2}}{V_A}$$

$$\alpha = \arccos\left(\frac{l}{l_0}\,\frac{\sqrt{V_A^2 - \frac{k}{m}(l-l_0)^2}}{V_A}\right) + \frac{\pi}{2}$$

4



$$V_B = \frac{V_A}{30}$$

Rörelsemängdsmomentats bevarande ger:

$$S V_A = r_B V_B \Rightarrow S V_A = r_B \frac{V_A}{30} \Rightarrow r_B = 30S$$

Energins bevarande ger:

$$\frac{1}{2} m V_A^2 - G\frac{mM}{S} = \frac{1}{2} m V_B^2 - G\frac{mM}{r_B} \Rightarrow$$

$$\frac{1}{2} V_A^2 - \frac{GM}{S} = \frac{1}{2}\left(\frac{V_A}{30}\right)^2 - \frac{GM}{30S} \Rightarrow$$

$$\frac{1}{2} V_A^2 \left(1 - \frac{1}{900}\right) = \frac{GM}{S}\left(1 - \frac{1}{30}\right)$$

$$V_A^2 \frac{899}{900} = \frac{2GM}{S}\frac{29}{30}$$

$$V_A^2 = \frac{58 \cdot 30}{899}\frac{GM}{S}$$

$$V_A = \sqrt{\frac{1740\ GM}{899\ S}}$$