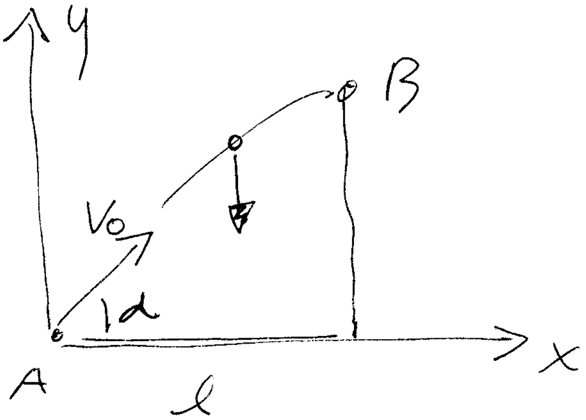
Lösningar, tentan 25/5 2010, SG1102

1)



NII : $m\ddot{y} = -mg$

$m\ddot{x} = 0$

Integrering två gånger ger med begynnelsevillkor:

$y = -\frac{1}{2}gt^2 + v_0 \sin\alpha \, t$

$x = v_0 \cos\alpha \, t$

I B fås

$l = -\frac{1}{2}g t_B^2 + v_0 \sin\alpha \, t_B$ (1)

$l = v_0 \cos\alpha \, t_B$ (2)

$0 = -g t_B + v_0 \sin\alpha$ (3)

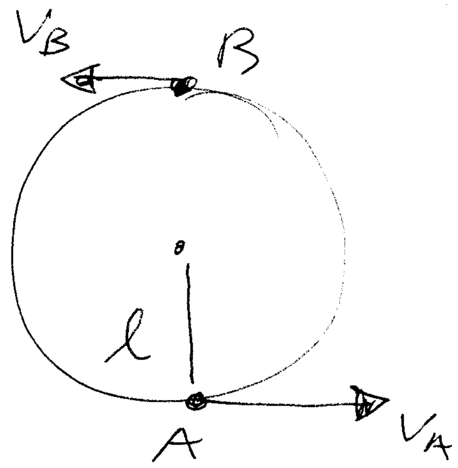
(3) ger $t_B = \frac{v_0 \sin\alpha}{g}$, insatt i (3) och (4) ger:

$$\left.\begin{array}{l} l = \frac{1}{2} \frac{v_0^2 \sin^2\alpha}{g} \\ l = \frac{v_0^2 \sin\alpha \cos\alpha}{g} \end{array}\right\} \quad \tan\alpha = 2 \quad \alpha = \arctan 2$$

$\tan\alpha = 2 \Rightarrow \frac{\sin^2\alpha}{1-\sin^2\alpha} = 4 \Rightarrow \sin^2\alpha = \frac{4}{5}$

$l = \frac{1}{2} \cdot \frac{4}{5} \frac{v_0^2}{g} \Rightarrow v_0 = \sqrt{\frac{5gl}{2}}$

2)



$$V_B = \frac{1}{2} V_A$$

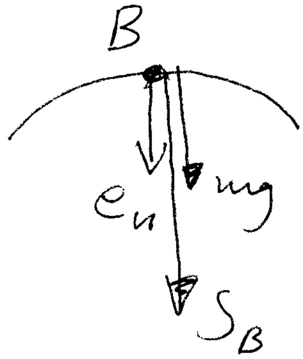
Energiprincipen ger:

$$\frac{1}{2} m V_A^2 = \frac{1}{2} m \left(\frac{1}{2} V_A\right)^2 + 2mgl \Rightarrow$$

$$\frac{3}{8} V_A^2 = 2gl \Rightarrow \quad V_A^2 = \frac{16}{3} gl$$

$$V_B^2 = \frac{4}{3} gl$$

I högsta punkten fås

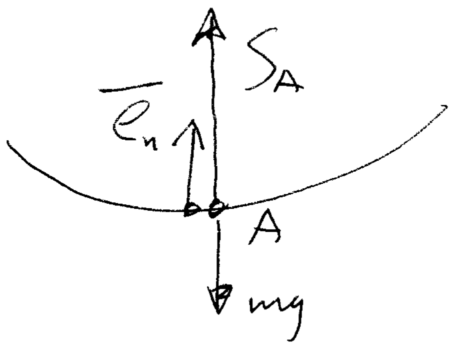


$$m \frac{V_B^2}{l} = S_B + mg$$

$$S_B = \frac{4}{3} mg - mg = \frac{mg}{3}$$

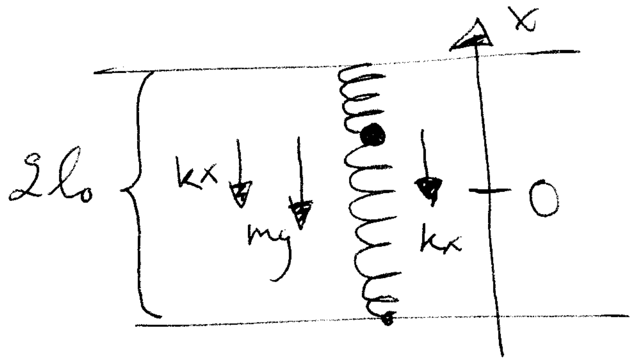
I lägsta punkten fås

$$m \frac{V_A^2}{l} = S_A - mg$$



$$S_A = \frac{16}{3} mg + mg = \frac{19}{3} mg$$

3)



a) $m\ddot{x} = -2kx - mg$

$\ddot{x} + \frac{2k}{m}x = -g \Rightarrow \ddot{x} + \omega_n^2 x = -g \; ; \; \omega_n^2 = \frac{2k}{m}$

$x = x_h + x_p$

$x_p = -\frac{g}{\omega_n^2} = -\frac{gm}{2k}$

$x_h = A\cos(\omega_n t) + B\sin(\omega_n t)$

$x = A\cos(\omega_n t) + B\sin(\omega_n t) - \frac{gm}{2k}$

$\dot{x} = \omega_n\left[-A\sin(\omega_n t) + B\cos(\omega_n t)\right]$

$\dot{x}(0) = 0 \Rightarrow B = 0$

$x = A\cos(\omega_n t) - \frac{gm}{2k}$

$x(0) = d \Rightarrow d = A - \frac{gm}{2k} \rightarrow A = d + \frac{gm}{2k}$

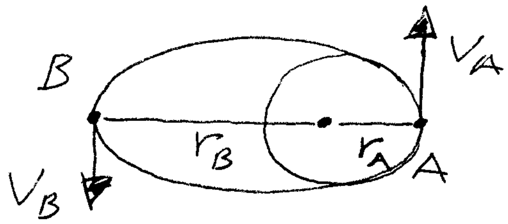
$\mathbf{x} = \left(d + \frac{gm}{2k}\right)\cos(\omega_n t) - \frac{gm}{2k}$

b) Vändläget fås då $\cos(\omega_n t) = -1$

$x_{min} = -d - \frac{gm}{k}$

4)

$$r_A = \frac{a(1-e^2)}{1+e} = a(1-e)$$



$$r_B = \frac{a(1-e^2)}{1-e} = a(1+e)$$

Rörelsemängdsmomentets bevarande:

$$V_B\, a(1+e) = V_A\, a(1-e) \Rightarrow V_B = \frac{1-e}{1+e} \qquad 1$$

Energins bevarande:

$$\frac{1}{2} m V_A^2 - \frac{mgR^2}{a(1-e)} = \frac{1}{2} m V_B^2 - \frac{mgR^2}{a(1+e)} \qquad 2$$

1 och 2 ger:

$$\frac{1}{2} V_A^2\left(1 - \left(\frac{1-e}{1+e}\right)^2\right) = \frac{gR^2}{a}\left(\frac{1}{1-e} - \frac{1}{1+e}\right) \Rightarrow$$

$$\frac{V_A^2\, 2e}{(1+e)^2} = \frac{gR^2\, 2e}{a(1-e)(1+e)} \Rightarrow V_A^2 = \frac{gR^2(1+e)}{a(1-e)}$$

Cirkulär bana ger

$$V_A'^2 = \frac{gR^2}{r_A} = \frac{gR^2}{a(1-e)}$$

Hastighetsändringen:

$$V_A - V_A' = \sqrt{\frac{gR^2(1+e)}{a(1-e)}} - \sqrt{\frac{gR^2}{a(1-e)}}$$

$$= R\sqrt{\frac{g}{a(1-e)}}\left(\sqrt{1+e} - 1\right)$$