Model based control design

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Supplied as supplement to course book in
Automatic Control Basic course (Reglerteknik AK)

Objective: *To introduce some general approaches to model based tuning, and how in special cases they lead to PI or PID controllers*

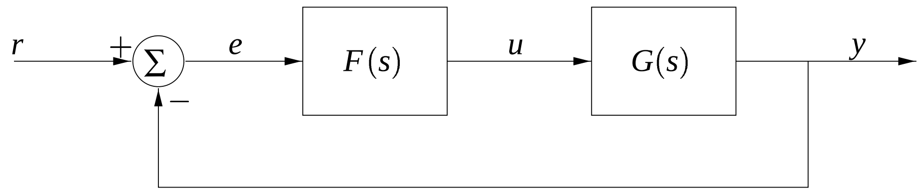

Figure 1: Block diagram of linear feedback control system

1 Introduction

Previously during the basic course in automatic control it is easy to get the impression that PID tuning typically should be performed by trial and error. One systematic method, based on only limited knowledge about the process, is introduced in Section 3.3 of Glad and Ljung.

In these notes we will discuss general tuning methods that take as an input a transfer function model of the process. Also notice that lead-lag design as described in Chapter 5 of Glad-Ljung could be regarded as another systematic method to design PID controllers (albeit leading to a series form – cf. Section 5.6 in Glad-Ljung).

2 Direct Synthesis

A typical block diagram for a feedback control system is depicted in Figure 1. The most straightforward design approach of all is to directly solve for the controller given a desired closed-loop transfer function.

Based on the block diagram we get

$$G_c(s) = \frac{Y(s)}{R(s)} = \frac{G(s)F(s)}{1+G(s)F(s)}$$

Then solving for $F(s)$ yields

$$F(s) = \frac{1}{G(s)} \frac{Y(s)/R(s)}{1-Y(s)/R(s)} \tag{1}$$

From this it can be observed that if perfect control is desired, i.e. $Y(s)/R(s) = 1$ it would require $F(s) = \infty$. A common choice is

$$\frac{Y(s)}{R(s)} = \frac{1}{\lambda s+1} \tag{2}$$

which gives a step response exponentially approaching the setpoint. The tuning then consists of selecting an appropriate value of λ. This can be done via trial and error (some rules of thumb can usually be given) or by computer simulation tests on the model before implementing on the real process.

Example 1 (First order system): Assume that the model is given by

$$G(s) = \frac{k_p}{\tau s + 1}$$

Then with the choice (2) the controller becomes

$$F(s) = \frac{\tau s + 1}{k_p} \frac{\frac{1}{\lambda s+1}}{1 - \frac{1}{\lambda s+1}} = \frac{\tau s + 1}{k_p \lambda s}$$

This is in fact a PI controller. If we compare to the standard form

$$F(s) = K\left(1 + \frac{1}{T_I s}\right) = \frac{K}{T_I}\frac{(T_I s + 1)}{s}$$

the controller obtained by the direct synthesis obviously corresponds to

$$T_I = \tau; \qquad K = \frac{T_I}{k_p \lambda} = \frac{\tau}{k_p \lambda}$$

Example 2 (Second order system): Assume that the process is given by

$$G(s) = \frac{k_p}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

Then direct application of (1) with the closed-loop system (2) gives

$$F(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k_p \lambda s} = \frac{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + 1}{k_p \lambda s}$$

This is an ideal PID controller

$$F(s) = K\left(1 + \frac{1}{T_I s} + T_D s\right) = K\left(\frac{T_D s + s + 1/T_I}{s}\right)$$

with

$$K = \frac{\tau_1 + \tau_2}{k_p \lambda}; \qquad T_I = \tau_1 + \tau_2; \qquad T_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$$

As pointed out already previously in the course it is, however, impossible to realize this controller in practice, having a pure derivative action. This problem can be re-solved by using a second order desired closed-loop system. For example

$$\frac{Y(s)}{R(s)} = \frac{1}{(\lambda s + 1)^2}$$

yields

$$\begin{aligned} F(s) &= \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k_p} \frac{\frac{1}{(\lambda s+1)^2}}{1 - \frac{1}{(\lambda s+1)^2}} = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k_p \lambda s(\lambda s + 2)} \\ &= \frac{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + 1}{2 k_p \lambda (\lambda/2 s + 1)} \end{aligned}$$

which is a PID controller with filtered derivative action. Comparing with the standard form

$$F(s) = K\left(1 + \frac{1}{T_I s} + \frac{T_D s}{\gamma s + 1}\right) = K\left(\frac{(\gamma + T_D)s^2 + (1 + \gamma/T_I)s + 1/T_I}{s(\gamma s + 1)}\right) \tag{3}$$

gives, after some straightforward calculations,

$$K = \frac{\tau_1 + \tau_2 - \gamma}{2\lambda k_p}; \quad T_I = \tau_1 + \tau_2 - \gamma; \quad T_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2 - \gamma}; \quad \gamma = \frac{\lambda}{2}$$

Notice that when choosing the model order of the desired closed-loop so that the controller is realizable, then direct synthesis actually designs the derivative filter parameter as well (contrary to most other approaches who use a γ linked to T_D, for example, as $\gamma = 0.1T_D$).

Also observe that this design of γ often leads to larger values than the above default choice. For example $\tau_1 = \tau_2 = \tau$ with the choice $\lambda = \tau$ would lead to $T_D = \tau/6$ and $\gamma = \tau/2$, i.e. $\gamma = 3T_D$!

Non-invertible plants

We are confident most readers, after some practice, will agree that the direct synthesis method is easy to apply. Once a model for the process has been found, all that needs to be done is to decide on a desired closed-loop system and insert into (1). If the desired closed-loop is parametrized as above, then regardless of the process model order, there is only a single parameter λ left to fine tune the speed of the system.

You perhaps then immediately ask: What is the catch? Are there no drawbacks with this very computationally simple approach? One, which we will return to later in this chapter is that the response may be unnecessarily slow for input disturbances, due to that the controller cancels the plant.

The cancellation causes another, more important problem, though. Some processes cannot or should not be inverted. An example of the first category is processes with time delay, where inversion corresponds to predicting the input ahead of time. Example of processes that should not be inverted are ones with a zero in the right-half plane, since this would correspond to introducing an unstable pole in the controller. Even though it would be cancelled in the closed-loop transfer function it leads to an internally unstable process, i.e. infinite control signals would be required.

There is, however, a solution to this problem, and it is to not invert that part of the dynamics. One way to do that is to factorize the plant as

$$G(s) = G_+(s)G_-(s)$$

where $G_+(s)$ contains all the non-invertible dynamics (and has $G_+(0) = 1$). Then $G_+(s)$ is retained in the desired closed-loop system, i.e.

$$\frac{Y(s)}{R(s)} = G_+(s)H(s)$$

which after insertion into (1) results in the controller

$$F(s) = \frac{1}{G(s)}\frac{G_+(s)H(s)}{1-G_+(s)H(s)} = \frac{1}{G_-(s)}\frac{H(s)}{1-G_+(s)H(s)} \tag{4}$$

We will illustrate this by a couple of examples.

Example 3 (Right-half plane zero): Assume that we want to control the process

$$G(s) = \frac{k_p(-\beta s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$$

which has a zero in $s = -1/\beta$. As pointed out earlier, processes with RHP zeros are fundamentally hard to control since the initial response to a control action goes in the wrong direction(cf. Section 5.8 in Glad-Ljung). Think, for example, of a furnace where increasing the fuel flow may initially have a cooling effect, but of course eventually leads to an increase in temperature.

Factorizing $G(s)$ into

$$G_-(s) = \frac{k_p}{(\tau_1 s+1)(\tau_2 s+1)}, \quad G_+(s) = -\beta s+1$$

and using

$$H(s) = \frac{1}{\lambda s+1}$$

then by (4) gives

$$F(s) = \frac{(\tau_1 s+1)(\tau_2 s+1)}{k_p}\frac{\frac{-\beta s+1}{\lambda s+1}}{1-\frac{-\beta s+1}{\lambda s+1}} = \frac{(\tau_1 s+1)(\tau_2 s+1)}{k_p(\lambda+\beta)s}$$

Rewriting $F(s)$ as

$$\begin{aligned} F(s) &= \frac{1}{k_p(\lambda+\beta)}\left(\frac{\tau_1\tau_2 s^2+(\tau_1+\tau_2)s+1}{s}\right) \\ &= \frac{\tau_1+\tau_2}{k_p(\lambda+\beta)}\left(1+\frac{1}{(\tau_1+\tau_2)s}+\frac{\tau_1\tau_2}{\tau_1+\tau_2}s\right) \end{aligned}$$

This corresponds to an ideal PID controller with

$$K = \frac{\tau_1+\tau_2}{k_p(\lambda+\beta)}; \quad T_I = \tau_1+\tau_2; \quad T_D = \frac{\tau_1\tau_2}{\tau_1+\tau_2}$$

Notice that this design puts a limit on how much gain can be applied since

$$K \to \frac{\tau_1+\tau_2}{k_p\beta}$$

when $\lambda \to 0$. This is because the closed-loop system will go unstable if the gain is increased indefinitely. Also observe that the limiting K becomes smaller for larger β. The slower the

RHP zero is (i.e. the closer it is to the origin) the longer the response spends going the wrong way, and therefore the more severe the limitation on achievable control performance is!

The controller designed above is an ideal PID controller. As has been pointed out more than once by now, in practice we need to filter the derivative action. To design such a controller we should instead choose, for example,

$$H(s) = \frac{1}{(\lambda s + 1)^2}$$

which leads to the controller

$$\begin{aligned} F(s) &= \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k_p} \frac{\frac{-\beta s + 1}{(\lambda s + 1})^2}{1 - \frac{-\beta s + 1}{(\lambda s + 1})^2} = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k_p s(\lambda^2 s + 2\lambda + \beta)} \\ &= \frac{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + 1}{k_p(2\lambda + \beta)s(\lambda^2/(2\lambda + \beta)s + 1)} \end{aligned}$$

Then comparing with (3) yields, again after straightforward but somewhat tedious calculations,

$$K = \frac{\tau_1 + \tau_2 - \gamma}{k_p(2\lambda + \beta)}; \quad T_I = \tau_1 + \tau_2 - \gamma; \quad T_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2 - \gamma} - \gamma; \quad \gamma = \frac{\lambda^2}{2\lambda + \beta}$$

Example 4 (process with deadtime): A number of plants in process industry can be described by a first order plus deadtime model, i.e.

$$G(s) = \frac{k_p e^{-\theta s}}{\tau s + 1}$$

Hence it is important to be able to quickly design a good controller for such a system.

In order to use direct synthesis for the design we have two different options.

Rational approximation

To be able to use standard controllers we need to approximate $e^{-s\theta}$. One common approximation is the so-called first order Padé approximation

$$e^{-\theta s} \approx \frac{-\frac{\theta}{2}s + 1}{\frac{\theta}{2}s + 1}$$

which gives

$$G(s) \approx \frac{k_p(-\frac{\theta}{2}s + 1)}{(\tau s + 1)(\frac{\theta}{2}s + 1)}$$

We have now obtained a process model with a RHP zero and the design can be carried out as described in Example 3 above, leading to a PID controller.

Include deadtime in G_+

The other main alternative is to factorize the process into

$$G_+(s) = e^{-\theta s}; \qquad G_-(s) = \frac{k_p}{\tau s + 1}$$

Then application of direct synthesis with a

$$H(s) = \frac{1}{\lambda s + 1}$$

results in the controller

$$F(s) = \frac{\tau s + 1}{k_p} \frac{1}{\lambda s + 1 - e^{-\theta s}}$$

This controller will need to have a time delay built into the controller. This can, however, easily be achieved when implementing the controller in a computer (see Chapter 13 for more details). The closed-loop system becomes

$$Y(s) = G_+(s)H(s)R(s) = \frac{e^{-\theta s}}{\lambda s + 1} R(s)$$

Hence we get the same shape response as without deadtime only delayed θ time units. This is usually called deadtime compensation (or Otto Smith controller – cf. Section 7.4 in Glad-Ljung).

Still, the deadtime compensating controller requires a special implementation. Therefore people sometimes select to approximate $e^{-\theta s}$ at this stage by a first order Taylor series expansion, i.e.

$$e^{-\theta s} \approx 1 - \theta s$$

which leads to

$$F(s) = \frac{\tau s + 1}{k_p(\lambda + \theta)s}$$

This is obviously a PI controller with the parameters

$$K = \frac{\tau}{k_p(\lambda + \theta)}; \qquad T_I = \tau$$

Recently this has been suggested as a standard for tuning basic controllers in Swedish pulp and paper industry (together with a recommendation on possible choices of λ).

3 Internal Model Control

We will now describe a more general model based framework for controller design that involves the direct synthesis as a special case. It is based on the block diagram shown in Figure 2. To emphasize the difference between the model and the true process, we have here cements

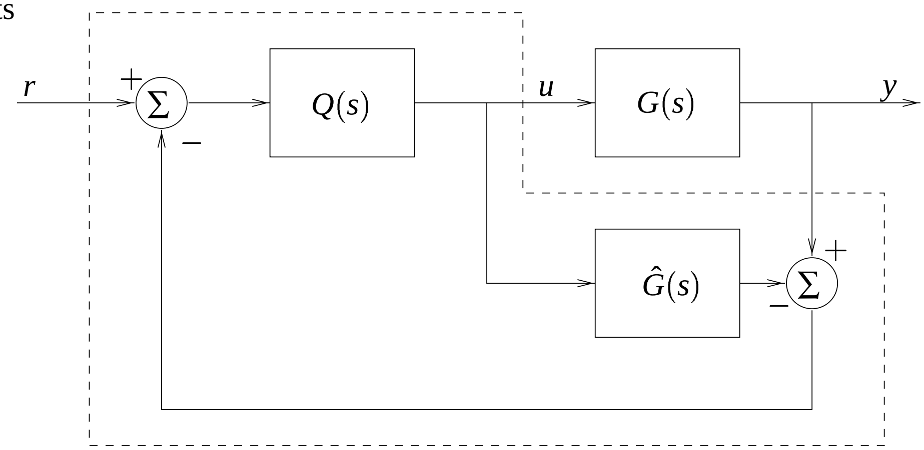

Figure 2: Block diagram internal model control (everything within dashed lines belongs to the controller)

introduced the notation $\hat{G}(s)$ for the plant model. Since the controller explicitly contains a model of the plant this is usually referred to as internal model control or IMC. The design is then to choose the transfer function $Q(s)$.

Let us start by comparing this block diagram to the standard one we have been using up until now (see e.g. Figure reffig5.2). The block diagram gives

$$U(s) = Q(s)(R(s) - (Y(s) - \hat{G}(s)U(s))) = Q(s)(R(s) - Y(s)) + Q(s)\hat{G}(s)U(s)$$

i.e.

$$U(s) = \frac{Q(s)}{1 - Q(s)\hat{G}(s)}(R(s) - Y(s))$$

Hence, since in the standard notation $U(s) = F(s)(R(s) - Y(s))$, IMC obviously corresponds to

$$F(s) = \frac{Q(s)}{1 - Q(s)\hat{G}(s)} \tag{5}$$

This means that essentially any conventional controller can be reformulated to IMC and vice versa. What is then the point in introducing the IMC concept? Firstly, by searching over all stable $Q(s)$ we cover all controllers that can stabilize $G(s)$. Secondly, it turns out that using the IMC block diagram it is easier to find the closed-loop system, which simplifies control design. Ideally, for a perfect model (i.e. $\hat{G}(s) = G(s)$) and no disturbances, the signal in the feedback path is zero. Hence the closed-loop system is simply found as

$$Y(s) = G(s)Q(s)R(s)$$

Notice that now the closed-loop system is linearly dependent on $Q(s)$, as opposed to $F(s)$ which can be found in both numerator and denominator of the closed-loop system.

Consequently, if we want the closed-loop system to be

$$Y(s) = H(s)R(s)$$

then we should use

$$Q(s) = \frac{1}{\hat{G}(s)} H(s)$$

Inserting this into (5) yields

$$F(s) = \frac{1}{\hat{G}(s)} \frac{H(s)}{1 - H(s)}$$

which is the direct synthesis controller.

Thus direct synthesis corresponds to the special choice of $Q(s)$ obtained when inverting $\hat{G}(s)$. The problem of non-invertible plants is then within IMC dealt with by using

$$Q(s) = \frac{1}{\hat{G}_-(s)} H(s)$$

leading to the same result as in direct synthesis above.

The advantage with IMC, compared to direct synthesis, is that it is more general, but also offers more insight into the analysis of the system.

As an example of the latter, recall the claim above that direct synthesis may give too slow control for input disturbances. A block diagram with input disturbance is shown in Figure 3. Ideally we have

$$Y(s) = H(s)R(s) + (1 - H(s))G(s)D(s)$$

Then, unless (1-H(s)) cancels part of $G(s)$, the original dynamics will still be present in the relationship between D and Y. This may be a major drawback if the aim is to speed up the response compared to the original one.

cements

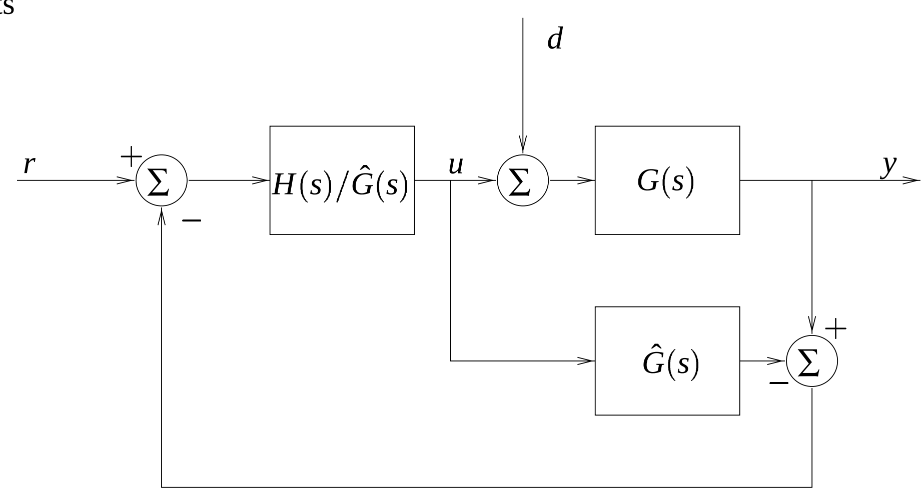

Figure 3: Block diagram of internal model control with input disturbance

Another drawback with IMC is that since a model of the process is run internally to the controller it can, without modification, only handle open-loop stable processes. These two drawbacks mean that for certain cases IMC cannot be (easily) applied, and we therefore need other model based approaches.

4 Pole placement

One rather straightforward model based approach, which is mentioned several times in Glad-Ljung, is to solve for the controller parameters to get a desired set of closed-loop poles.

This method overcomes the problem of slow input disturbance rejection and can easily handle unstable plants. The drawback, however, is that the design calculations can be rather complicated, in particular for higher order processes.

We will examplify pole placement by carrying out calculations for PI and PID design of a tank level example.

4.1 Example: tank level control

Consider a control problem that is very common in the process industry, namely level control. The problem is depicted in Figure 4.

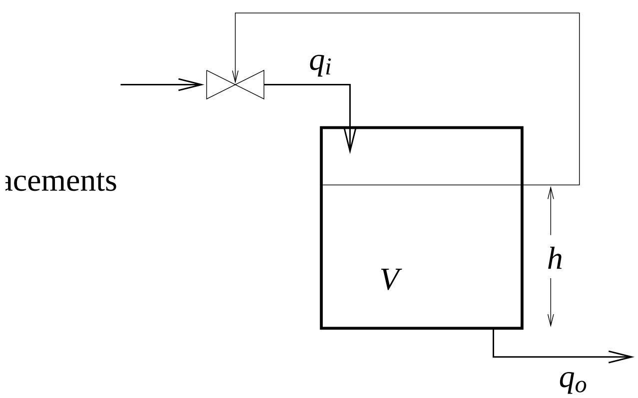

Figure 4: Level tank.

We have the following differential aquation for the volume V in the tank

$$\frac{dV}{dt} = q_i(t) - q_o(t)$$

where q_i and q_o are the inlet and outlet flows, respectively. Alternatively, if the level h is the desired output, assuming constant cross-sectional area of the tank A,

$$A\frac{dh}{dt} = q_i(t) - q_o(t) \tag{6}$$

The control is performed by manipulating the inlet flow rate via the valve. Assuming that the valve immediately delivers the demanded flow, we have in the general notation $u(t) = q_i(t)$ (and of course $y(t) = h(t)$). The main disturbance will be the outlet flow, which is typically determined by other control loops downstream. Hence we have $d(t) = q_o(t)$.

Laplace transformation now gives

$$Y(s) = \frac{1/A}{s}(U(s) - D(s)) = \frac{k_p}{s}(U(s) - D(s))$$

where $k_p = 1/A$. A blockdiagram of the system (including the controller yet to be determined) is shown in Figure 5. Notice that from a control theoretic point of view the outlet flow is really an input disturbance! Furthermore, observe that the process dynamics are given by a pure integrator, contrary to the tanks used in the lab with free flowing outlet (which are very rare in industry!) leading to a self-stabilizing level dynamics.

In an industrial control system the control signal and level output may be scaled to be expressed in 0 to 100 per cent (corresponding to valve fully closed or fully open). Then knowing A is not enough to determine the numerator of the process. The easiest way to deal with this problem is by assuming

$$G(s) = \frac{k_p}{s}$$

and determine k_p from a bump test like the one shown in Figure 6. Then

$$k_p = \frac{\Delta y}{\Delta u \Delta t}$$

PI control

First study tuning of a PI controller on the form

$$F(s) = K\left(1 + \frac{1}{T_I s}\right)$$

Straightforward calculations, based on the block diagram in Figure 5, yield

PSfrag replacements

$$Y(s) = \frac{Kk_p(s + 1/T_I)}{s^2 + Kk_p(s + 1/T_I)}R(s) - \frac{k_p s}{s^2 + Kk_p(s + 1/T_I)}D(s) \tag{7}$$

Suppose we want double poles in $s = -1/\lambda$, i.e. the desired characteristic function is

$$(s + 1/\lambda)^2 = s^2 + 2/\lambda s + 1/\lambda^2 = 0$$

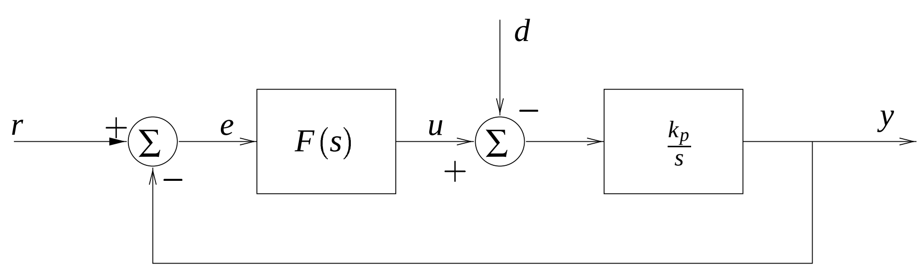

Figure 5: Block diagram of tank level control system

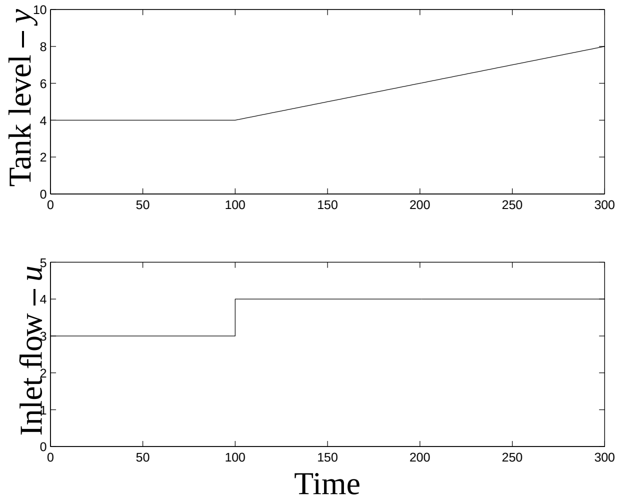

Figure 6: Bump test to determine slope k_p.

Comparison with (7) then leads to the following equations

$$\begin{aligned} Kk_p &= 2/\lambda \\ Kk_p/T_I &= 1/\lambda^2 \end{aligned}$$

The solution to the first equation above is obviously

$$K = \frac{2}{k_p\lambda}$$

and inserting this K into the second equation yields

$$T_I = 2\lambda$$

This is in fact quite a good way to tune a level controller. The number of tuning parameters has now been reduced to one, λ, with the unit time. This so-called "λ-tuning" results in the closed loop system

$$Y(s) = \frac{2\lambda s+1}{\lambda^2 s^2 + 2\lambda s + 1} R(s) - \frac{k_p \lambda^2 s}{\lambda^2 s^2 + 2\lambda s + 1} D(s)$$

Figure 7 shows the responses to first a unit setpoint step and then a unit step in the outlet flow for $\lambda = 50$. It can actually be calculated analytically that λ corresponds to the time when the level crosses the new setpoint (after a step in r) as well as the time between the disturbance step and the following peak error. This provides a good basis for understanding suitable values of λ in practice.

PID control

Finally we will also briefly discuss the potential benefits of PID control and again illustrate on the tank level control problem.

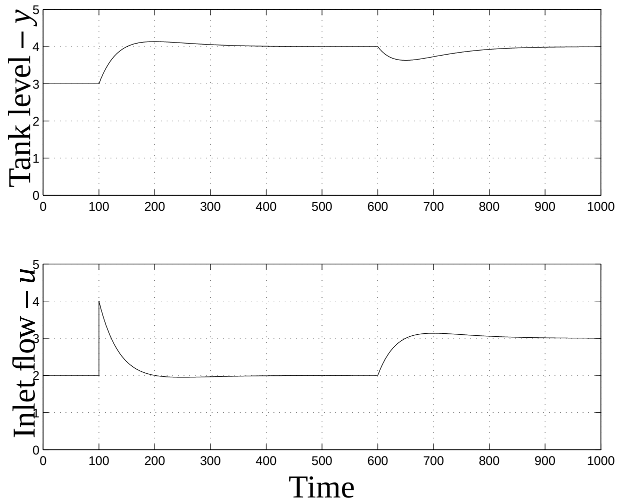

Figure 7: Step responses PI control tank level ($\lambda = 50$, $k_p = 0.02$). Unit steps in setpoint at $t = 100$ and in outlet flow at $t = 600$.

As suggested earlier in this chapter, an industrial PID controller typically has the form

$$F(s) = K\left(1 + \frac{1}{T_I s} + \frac{T_D s}{\gamma s + 1}\right) = \frac{K((\gamma + T_D)s^2 + (1 + \gamma/T_I)s + 1/T_I)}{s(\gamma s + 1)}$$

Again straightforward calculations give

$$\begin{aligned} Y(s) &= \frac{Kk_p((\gamma + T_D)s^2 + (1 + \gamma/T_I)s + 1/T_I)}{s^2(\gamma s + 1) + Kk_p((\gamma + T_D)s^2 + (1 + \gamma/T_I)s + 1/T_I)} R(s) - \\ & \quad \frac{k_p s(\gamma s + 1)}{s^2(\gamma s + 1) + Kk_p((\gamma + T_D)s^2 + (1 + \gamma/T_I)s + 1/T_I)} D(s) \end{aligned}$$

Hence, the characteristic equation is found to be

$$\gamma s^3 + (1 + Kk_p(\gamma + T_D))s^2 + Kk_p(1 + \gamma/T_I)s + Kk_p/T_I = 0$$

Suppose we want to place all three poles in $s = -1/\lambda$, i.e. we would like to obtain the characteristic equation

$$(s + 1/\lambda)^3 = s^3 + 3/\lambda s^2 + 3/\lambda^2 s + 1/\lambda^3 = 0$$

Dividing the first equation by γ and comparing coefficients yields

$$\begin{aligned} (1 + Kk_p(\gamma + T_D))/\gamma &= 3/\lambda \\ Kk_p(1 + \gamma/T_I)/\gamma &= 3/\lambda^2 \\ Kk_p/(T_I\gamma) &= 1/\lambda^3 \end{aligned}$$

The first thing to notice is that we have four unknown parameters, but only three equations (since there are only three poles). One parameter has to be fixed. Using the standard choice

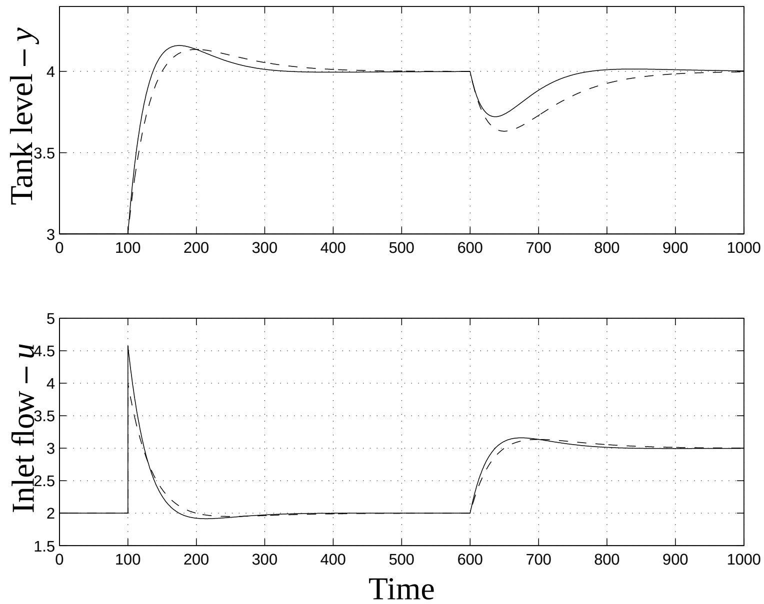

Figure 8: Step responses of tank level ($\lambda = 50$, $k_p = 0.02$). Solid – PID control, dashed – PI control.

$\gamma = \alpha T_D$ (where α is still to be determined) yields (after some calculations)

$$K = \frac{\alpha T_I T_D}{k_p \lambda^3}; \quad T_I = 3\lambda - \alpha T_D$$

where T_D is the real valued solution to the equation

$$-\alpha(1+\alpha)T_D^3 + 3\lambda(1+\alpha)T_D^2 - 3\lambda^2 T_D + \lambda^3/\alpha = 0$$

For $k_p = 0.02$, the choices $\lambda = 50$ and $\alpha = 1.5$ lead to the parameters

$$K = 1.54; \quad T_I = 33.0; \quad T_D = 78.0; \quad \gamma = 117$$

Figure 8 shows a simulation of the tank level under PID control, where also the previous PI control results above have been plotted. By adding derivative action the output is brought back to the setpoint after the disturbance almost twice as fast, with a negligible increase in control effort.

5 Example: distillation column

We will finish this chapter by comparing IMC and pole placement on a simplified model of a distillation column. Figure 9 depicts a typical distillation column with the variables

x_F	–	feed composition
F	–	feed flow rate
y_D	–	distillate composition
L	–	reflux
x_B	–	bottom composition
V	–	boil-up

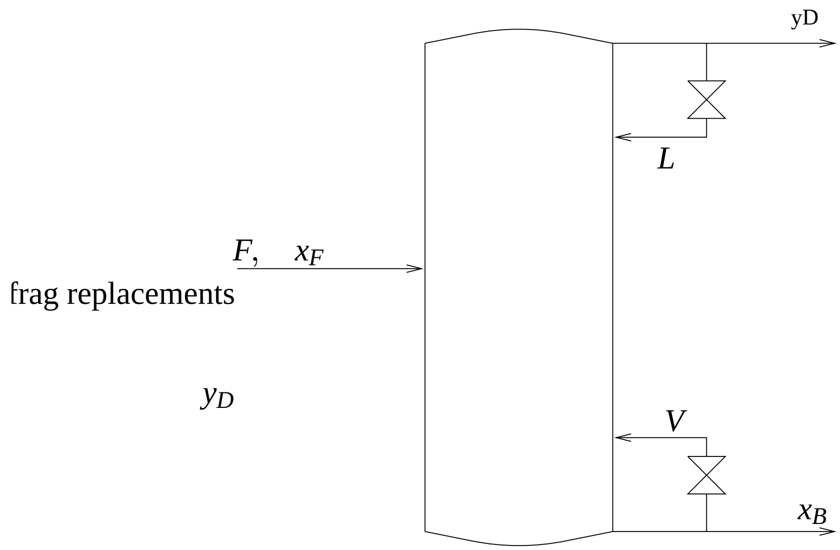

Figure 9: A distillation column

A linearized (and simplified) model of the column with two control signals L and V looks like

$$\begin{bmatrix} \Delta Y_D(s) \\ \Delta X_B(s) \end{bmatrix} = \frac{1}{\tau s+1} \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} \Delta L(s) \\ \Delta V(s) \end{bmatrix}$$

where the gains k_{ij} usually are such that the corresponding matrix is close to singular. This makes controlling both y_D and x_B to desired setpoints a very hard control problem. It is in fact possible, though. In this example, and often in practice, we will, however, concentrate on control of one output; the distillate.

Typically the top left corner of the system can be

$$\Delta Y_D(s) = \frac{0.9}{200s+1} \Delta L(s)$$

where the time unit (and hence also the time constant) is in minutes.

IMC

Direct application of direct synthesis/IMC yields

$$F(s) = \frac{200s+1}{0.9\lambda s}$$

which is a PI controller with

$$K = \frac{200}{0.9\lambda}; \qquad T_I = 200$$

For example the choice $\lambda = 10$ gives $K = 22.2$.

Pole placement

Given the PI controller

$$F(s) = K\left(\frac{s + 1/T_I}{s}\right)$$

the closed-loop system becomes

$$G_c(s) = \frac{0.9K(s + 1/T_I)}{s(200s+1) + 0.9K(s + 1/T_I)}$$

Hence, we have the characteristic equation

$$200s^2 + (1 + 0.9K)s + 0.9K/T_I = 0$$

or after dividing by 200

$$s^2 + (1 + 0.9K)/200s + 0.9K/(T_I 200) = 0$$

Now assuming that we want poles in, for example,

$$s = -0.05 \pm 0.05i$$

gives a desired characteristic equation

$$(s + 0.05 + 0.05i)(s + 0.05 - 0.05i) = s^2 + 0.1s + 0.005$$

Comparing coefficients then gives the following equations

$$\begin{aligned} \frac{1 + 0.9K}{200} &= 0.1 \\ \frac{0.9K}{T_I 200} &= 0.005 \end{aligned}$$

which is solved by

$$K = 21.1; \qquad T_I = 19$$

Simulation comparison

Let us now compare the two solutions by simulating step responses. An important disturbance to the system is varying feed composition, which goes through essentially the same dynamics as the reflux, i.e. we have

$$\Delta Y_D(s) = \frac{0.9}{200s+1}\Delta L(s) + \frac{k_F}{200s+1}\Delta X_F(s)$$

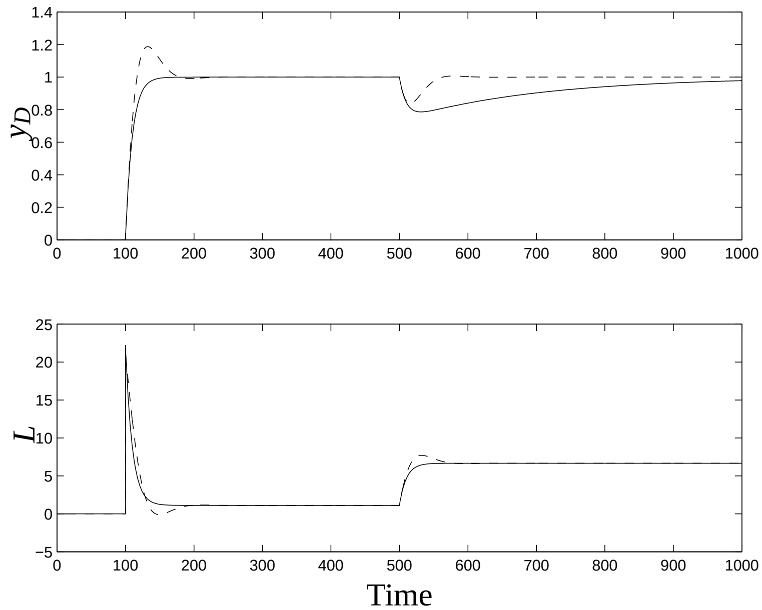

Figure 10: Step responses for distillation column. Solid – IMC, dashed – pole placement.

Figure 10 shows step responses for the two controllers for $k_F = 1$, first to a unit step in setpoint and then to a negative five unit step in Δx_F. IMC gives a very good response to the setpoint step However, since for a distillation column there is enough control power to speed up the system several times, it is a good example of a process where IMC gives too sluggish disturbance rejection.

Pole placement on the other hand handles the disturbance much better without having to apply larger peak control signal, but instead gives a significant overshoot to the setpoint step. If necessary, though, that is a problem which can be handled separately by filtering the setpoint.