LABORATORY 2: MODEL BASED CONTROL

1 Introduction

Laboratory 1 gave an introduction to the principles of feedback control and, in particular, PID control. The laboratory also gave some experience in the difficulties involved in tuning controllers based on trial and error. Laboratory 2 will show how feedback controllers that provide specific system properties can be derived in a systematic fashion. The derivation is based on a two-step procedure. First a model of the process is constructed based on a combination of physical insight and experimental data. The model is then used to construct a feedback control law which yields certain desired properties in the closed-loop system. Pole placement design as well as frequency domain loop shaping, i.e., lead-lag compensation, will be considered. In both cases will we show that the resulting control law can be implemented in the form of a PID controller.

The process

The laboratory investigation will be performed on the same tank system that was used in Laboratory 1. See also Figure 1.
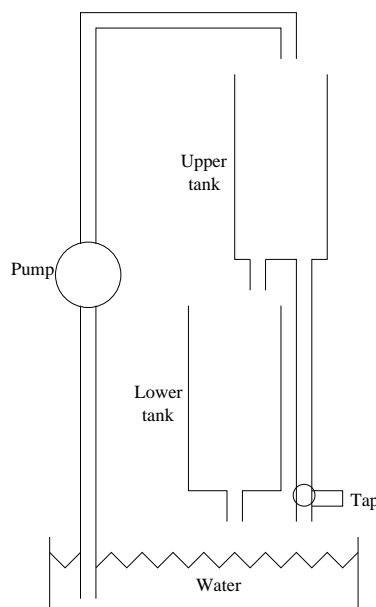

Figure 1: Coupled tanks with pump.

Preparations

You are supposed to have read this manual and also repeated Laboratory 1. Preparation tasks 2.1-2.7, 4.1, 4.2 and 5.1 in this manual should be solved before coming to the laboratory.

The following concepts should be well known and briefly explained:

Linearization:

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Poles and zeros:

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Frequency response:

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Crossover frequency:

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Phase margin:

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Bandwidth:

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Loopshaping, or lead-lag compensation:

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The laboratory will start with a small test based on the concepts above and the mandatory preparation tasks. To prepare for the test you should login to *Bilda* at *www.bilda.kth.se*, and take some preparatory tests. Note that the test itself will be based on questions drawn from the preparatory tests.

2 Modelling

2.1 Physical model of the process

Task 2.1 (*Preparation task*)
The relationship between the outflow velocity $v(t)$ [m/s] and the water level $h(t)$ [m] in a tank is given by Bernoullis law
$$v(t) = \sqrt{2gh(t)}$$

Write down the differential equations for the tank system in Figure 1 when the dynamics in the motor and the hoses are neglected. Let A_1 and A_2 be the cross-sectional areas of the tanks and a_1 and a_2 the effective outlet areas. Assume that the flow q from the pump is directly proportional to the voltage u applied to the motor.

Task 2.2 (*Preparation task*)
Assume that both tanks have the same cross-sectional area, i.e., $A_1 = A_2 = A$, and show that the model then can be written

$$\frac{dh_1(t)}{dt} = -\alpha_1\sqrt{2gh_1(t)} + \beta u(t)$$
$$\frac{dh_2(t)}{dt} = \alpha_1\sqrt{2gh_1(t)} - \alpha_2\sqrt{2gh_2(t)} \tag{1}$$

where $\alpha_1 = a_1/A$ and $\alpha_2 = a_2/A$. Note that the model now only contains the three parameters α_1, α_2 and β.

Task 2.3 (*Preparation task*)
Determine the parameters α_1, α_2 and β from the following construction data:

Diameter for the outlet holes: d $= 4.763 \cdot 10^{-3} m$
Diameter for the tanks: D $= 44.45 \cdot 10^{-3} m$

A pump voltage of $11V$ gives a flow of $50 \cdot 10^{-6} m^3/s$

Task 2.4 (*Preparation task*)
Determine the relationship between the levels in the upper and lower tank at steady-state conditions, i.e., determine
$$\gamma = \frac{h_2^0}{h_1^0},$$

for an equilibrium point
$$h_1 = h_1^0, \; h_2 = h_2^0, \; u = u^0. \tag{2}$$

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Task 2.5 (*Preparation task*)
Introduce the scaled deviation variables

$$\Delta x_1(t) = 400[h_1(t) - h_1^0]$$
$$\Delta x_2(t) = 400[h_2(t) - h_2^0]$$
$$\Delta u(t) = \frac{100}{15}[u(t) - u^0]$$

and linearize the system (1) around the equilibrium point (2). Note that the levels and the input have been scaled so that they are between 0 and 100.
Hint: Start by linearizing the system and then introduce the new variables $\Delta x_1(t)$, $\Delta x_2(t)$, and $\Delta u(t)$.

Task 2.6a (*Preparation task*)
Show that the linearized system can be described with the following transfer functions

$$\Delta X_1(s) = \frac{k}{1 + \tau s}\Delta U(s)$$
$$\Delta X_2(s) = \frac{\gamma}{1 + \gamma \tau s}\Delta X_1(s)$$

where $\tau = \frac{1}{\alpha_1}\sqrt{\frac{2h_1^0}{g}}$ and $k = 60\beta\tau$.

Task 2.6b (*Preparation task*)
Calculate the values of k, τ and γ when h_2^0=0.1 m.

Task 2.7 (*Preparation task*)

Assume that the process is steady state at $t = 0$, at which a step is made in the input signal with $\Delta u = 4$ scale units. The linearized model then gives

$$\Delta X_1(s) = \frac{k}{1 + \tau s} \frac{4}{s} \quad \Rightarrow$$
$$\Delta x_1(t) = 4k(1 - e^{-t/\tau})$$

When the system has settled in a new steady state, the level in the upper tank has changed with $4k$ units. At $t = \tau$ the change is

$$\Delta x_1(\tau) = 4k(1 - e^{-1}) = 0.632 \cdot 4k$$

Determine k and τ from the step response in the figure below.
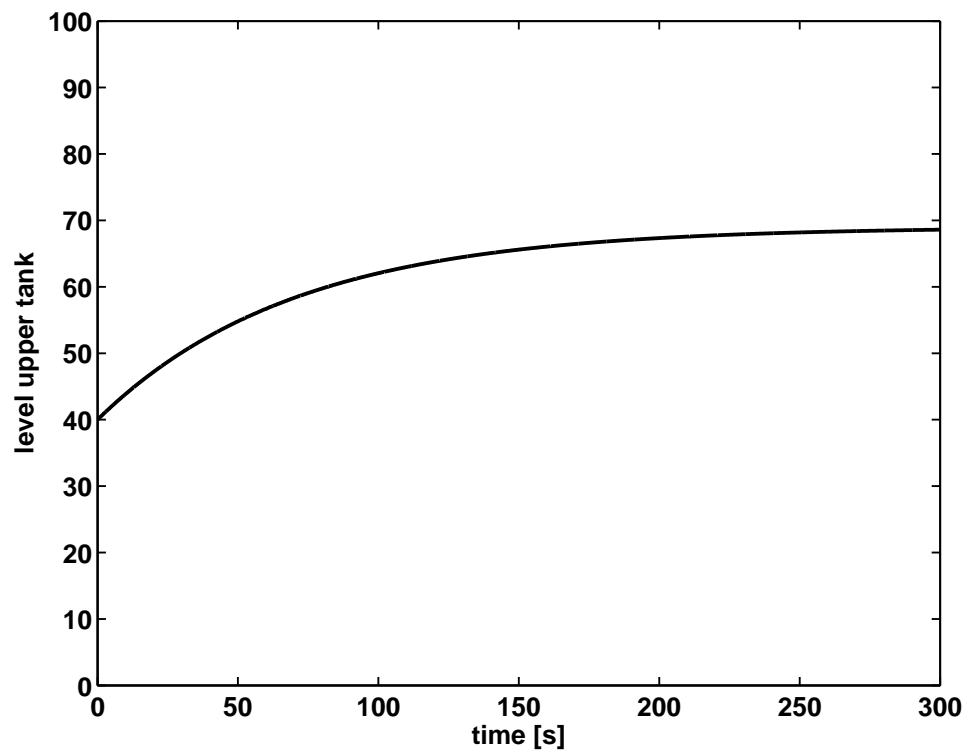
3 Experimental modelling

In the previous section we derived a mathematical model of the tank system based on physical insight. In this section we will determine the parameters in that model based upon experiments.

Task 3.1
Start the program by double clicking the Matlab icon on the desktop. The GUI is the same as used in Laboratory 1.

- Turn on the anti-windup protection.

- Display both tank levels by selecting them under $\boxed{\textbf{Displayed signals}}$.

- Manually change the level in the $\boxed{\textbf{lower}\ \text{tank}}$ to 40 ± 1. **This means that you have to find the control signal that gives this level in the lower tank**.

- Wait until the process is steady state and write down the u_0-value, $u_0 =$....... % (of the maximum input signal).

- Determine the γ-value for the process be calculating the ratio between the levels in the lower and upper tank. $\gamma =$.......

Task 3.2
$\boxed{\text{Note: In this task you should look at the } \textbf{upper} \text{ tank!}}$

- Change the input signal to $u_0 + 4$.

- Wait 100 seconds, and then use the $\boxed{\textbf{Capture}}$-button function to capture the step response.

- Determine k and τ in the same way as you did in task 2.7. Use the zoom-function in the capture-figure to improve the accuracy of your readings. $k =$....... $\tau =$.......

- Check with one of the assistants that the obtained values are reasonable.

Task 3.3
Are k, τ and γ equal to the ones that you derived in task 2.6b? Discuss the reasons for possible differences!

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4 Pole placement design

We have now derived a model of the system. This model shall next be used to construct a controller. The model-based controller design methods that we will study in this laboratory are pole placement design and loop shaping, or lead-lag design.

We will first study pole placement design for control of the level in the upper tank. Note from the model that the transfer function from ΔU to ΔX_1 is a first order system. Feedback control with a PI-controller, which also has one pole, then results in a closed-loop system with two poles. Since we have two free controller parameters, K and T_I, we can place the two poles anywhere in the complex plane, with the only restriction that they must be conjugate when containing an imaginary part.

Task 4.1 (*Preparation task*)
Show that, for a first order system $G(s) = k/(\tau s + 1)$, a pole placement at $s = -\beta \pm i\beta$ corresponds to a PI-controller with parameters

$$K = (2\beta\tau - 1)/k \qquad\qquad T_I = Kk/(2\beta^2\tau)$$

Note that we with this specific pole placement have restricted the closed loop poles to lie on the bisectors of the 3rd and 4th quadrant.

Task 4.2 (*Preparation task*)

We shall now study pole placement design for control of the level in the lower tank.

Note from the model that the transfer function from ΔU to ΔX_2 is a second order system. A PI-controller would thus yield three poles in the closed loop system, but with only two free controller parameters. To be able to place all poles we therefore need to employ a PID-controller.

The ideal PID-controller, in parallel form, is given by

$$F(s) = K[1 + \frac{1}{T_I s} + T_D s]$$

However, this controller is not suitable for implementation because it is not proper, i.e., it contains more zeros than poles, and furthermore, potentially highly sensitive to measurement noise due to the differentiation of the control error. To overcome these problems, commercial PID-controllers usually contain a filter for the D-part. In this laboratory, the PID-controller with filter is implemented as

$$F(s) = K[1 + \frac{1}{T_I s} + \frac{T_D N s}{s + N}]$$

This controller has two poles, and the closed-loop system for the level in the lower tank will hence contain four poles. Note that we now also have four parameters (K, T_I, T_D, N) so that all four poles can be placed arbitrarily in the complex plane.

We will here consider a design in which two poles are forced to be real and equal, corresponding to the closed-loop characteristic equation

$$(s + \chi)^2 (s^2 + 2\zeta\omega_0 s + \omega_0^2) = 0$$

Show in the figure below what the parameters χ, ζ and ω_o correspond to.
Hint: Introduce the angle α and define it so that $\zeta = \cos(\alpha)$.

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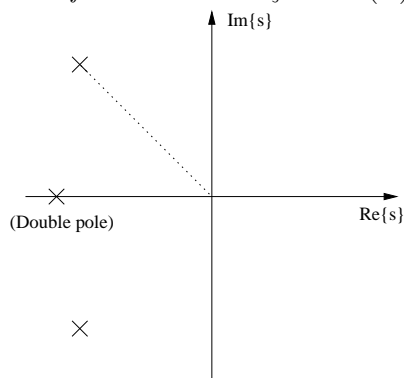
Task 4.3

In order to study how the choice of the poles affect the behavior of the closed-loop system, we will place the double real pole at $-\chi$ relatively far away from the origin compared to the two complex conjugate poles. Since poles far away from the origin are faster than poles closer to the origin, this means that the two complex poles will be the dominating ones.

1. Change $\boxed{\textbf{Tank}}$ to $\boxed{\textbf{lower}\text{ tank}}$.

2. Click on the popup-menu $\boxed{\textbf{Control}}$ and choose Pole placement. This will bring up a new graphical user interface for pole placement design.
   Note that the control is still implemented as a PID.

3. Enter the system parameters; k, τ and γ.

4. Choose χ=0.5, ζ=0.7 and ω_0=0.1.

5. Click on $\boxed{\textbf{Transform and transfer corresponding PID-parameters}}$ to calculate and implement the corresponding PID-parameters.

Set the reference value to 40 and wait until the system is steady state. Change the reference from 40 to 50. Write down the rise time T_r and the overshoot M in the table below.

Now open the tap and measure the response time T_{resp} until the effect of the disturbance has been accounted for, which here is considered to occur when the level remains between 49 and 51.

Close the tap again and change the reference to 40. Wait until the system is steady state and then repeat the experiment for the other pole placements given in the table below.

χ	ζ	ω_0	T_r	M	T_{resp}
0.5	0.7	0.1			
0.5	0.7	0.2			
0.5	0.3	0.2			
0.5	0.0	0.2			

How does M, T_r and T_{resp}, respectively, depend on ζ and ω_0? Compare with what you would expect from theory.

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If it is desirable to have no overshoot, how would you suggest to place the poles?

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Try the above suggestion and consider whether you get any overshoot. Can you explain the result that you get?
Hint: consider the closed-loop zeros.

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In pole placement design we limit ourselves to consider the location of the system poles only. In many cases this will yield acceptable results. However, as seen above, the zeros of the closed-loop can in some cases also have a significant impact on the closed-loop response. In these cases, a design method which takes a more general approach to the system properties is preferred. One such approach is shaping of the systems frequency response, through frequency domain loop shaping (lead-lag compensation), which is considered next.

If any of the assistants are available, then this is a good opportunity for partial reporting of your results. Otherwise, continue working and report later.

Partial report accepted by:

5   Frequency domain loop shaping

The frequency domain is well suited for systematic model based design of controllers in order to achieve desired system properties such as speed of response, damping and steady-state control error. In the frequency domain these properties are represented by the bandwidth ω_B, resonance peak M_p and steady-state gain of the closed-loop system $G_c(0)$, respectively. Since the relationship between the controller and the open-loop system properties is more transparent than for the corresponding closed-loop system, we usually prefer to shape the open-loop frequency response. The related properties for the frequency response of the open-loop system $G_o(s) = F(s)G(s)$ are the crossover frequency ω_c, the phase margin Φ_m and steady-state gain $G_o(0)$, respectively.

To shape the open-loop frequency response we will here employ a lead-lag controller with single lead- and lag-elements

$$F(s) = K \underbrace{\frac{\tau_D s + 1}{\beta \tau_D s + 1}}_{F_{lead}} \underbrace{\frac{\tau_I s + 1}{\tau_I s + \gamma}}_{F_{lag}}$$

The lead-element F_{lead} is used to "lift" the phase so as to obtain a desired phase margin Φ_m. The maximum phase lift as a function of β is shown in Figure 5.1. The lag-element F_{lag} is used to increase the loop-gain at low frequencies in order to reduce the steady-state control error. If a zero steady-state error is required, then $\gamma = 0$. The gain K is used to provide the desired crossover frequency ω_{cd}.
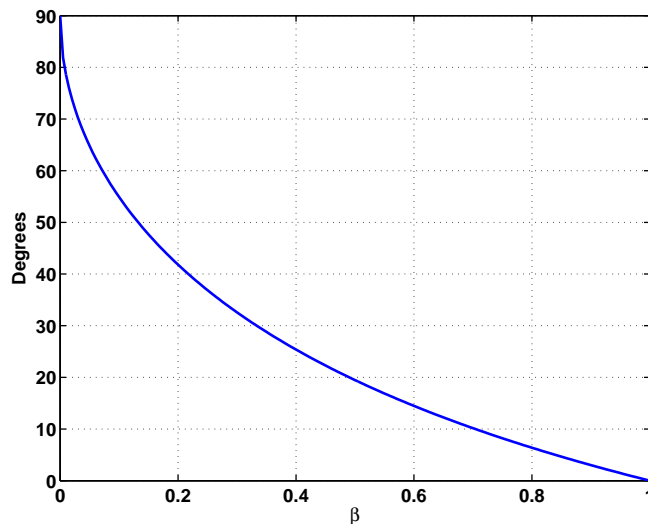
Figure 5.1: Maximum phase increase.

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Task 5.1 (*Preparation task*)

The lead-lag controller

$$F(s) = K \frac{\tau_D s + 1}{\beta \tau_D s + 1} \frac{\tau_I s + 1}{\tau_I s + \gamma}$$

with $\gamma = 0$, can be implemented with a PID-controller on parallel form

$$F_{pid}(s) = K_{pid}[1 + \frac{1}{T_I s} + \frac{T_D N_{pid} s}{s + N_{pid}}]$$

Write both controllers on the normal form

$$F(s) = \frac{as^2 + bs + c}{s^2 + ds}$$

and derive the equations that relate the parameters so as to make the two above controllers equal.

Note: You are not expected to solve the system of equations.

Task 5.2 We shall now employ loopshaping, or lead-lag, design to derive a controller for the level in the lower tank.

First study the Bode diagram for the uncompensated system G, which can be generated through the following steps

1. Click on the popup-menu $\boxed{\textbf{Control}}$ and choose Leadlag. This will bring up a new graphical user interface for lead-lag design. *Note that the control law is implemented as a PID.*

2. Enter the system parameters; k, τ and γ.

3. Plot the Bode diagram for G.

Determine the crossover frequency ω_c and the phase marginal Φ_m for the uncompensated system G. $\qquad \omega_c = ...$ $\qquad \Phi_m = ...$

Task 5.3 Now use the controller F to design an open-loop system FG with crossover frequency $\omega_{cd} = 0.2$ rad/sec and phase margin $\Phi_m = 70°$. This can be done as described below

1. Calculate the required phase margin increase. (Recall that the lag-part will decrease the phase with approx. 5.7°). Required phase margin increase = ...

2. Determine the β-value that gives this phase margin increase. (See figure 5.1). $\beta = ...$

3. Choose the parameter τ_D such that the maximum phase lift is obtained at the desired crossover frequency ω_{cd}, i.e., $\tau_D = \frac{1}{\omega_{cd}\sqrt{\beta}}$ ....

4. Choose the parameter $\tau_I = 10/\omega_{cd} = ...$

5. Plot the bode diagram for $L=FG$ with controller gain $K = 1$ and read the amplitude $|FG|_{dB}$ in decibel at ω_{cd}. Calculate the corresponding amplitude $|FG|=10^{|FG|_{dB}/20}$. $\boxed{\text{An example: if } |FG|_{dB} = -12 \text{ then } |FG| = 10^{-12/20}. \text{ Note the minus sign!}}$ Determine the controller gain K that gives an amplitude of 1 at the desired crossover frequency, ω_{cd}. $\qquad K = ...$

6. Plot the Bode diagram for $L = FG$ with the determined controller gain K and verify that you have obtained the desired frequency response.

7. Click on $\boxed{\textbf{Transform and transfer corresponding PID-parameters}}$ to calculate and implement the corresponding PID-parameters.

We will consider the performance of the obtained controller by performing experiments similar to those performed for the pole placement design above.

Set the reference value to 40 and wait until the system is steady state. Change the reference from 40 to 50. Write down the rise time T_r and the overshoot M in the table below.

Next open the tap and measure the response time T_{resp} until the effect of the disturbance has been accounted for.

Finally, close the tap again and change the reference to 40. Wait until the system is steady state and then repeat the experiment for the other combinations listed in the table below.

ω_c	Φ_m	T_r	M	T_{resp}
0.2	70			
0.2	60			
0.2	50			
0.4	60			
0.8	60			

How does T_r, M and T_{resp} depend on ω_c and Φ_m, respectively? Compare with what you would expect from theory.

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6 Summary

Task 6.1

- What are the advantages and disadvantages with model based control design compared to trial-and-error tuning of a PID-controller.

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- Summarize the results you found with pole placement design in this laboratory and compare with what you would expect from theory.

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- Summarize the results you found with lead-lag design and compare with what you would expect from theory.

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