Basic Queuing Theory Formulas

Poisson distribution

$$P[X = k|T = t] = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, \qquad k = 0, 1, 2, \dots$$

Geometric distribution

$$P[X = k] = (1 - p)^{k-1} p, \qquad k = 1, 2, \dots$$

$$E[X] = \frac{1}{p}, \quad V[X] = \frac{1 - p}{p^2}$$

Exponential distribution

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Erlang distribution

$$f_X(x, r) = \begin{cases} \lambda^r \frac{x^{r-1}}{(r-1)!} e^{-\lambda x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$$E[X] = \frac{r}{\lambda}, \quad V[X] = \frac{r}{\lambda^2}$$

z-transform

$$F(z) = \sum_{n=0}^{\infty} f_n z^n$$

Laplace-transform

$$F(s) = \int_0^{\infty} f(t) e^{-st} \mathrm{d}t$$

Little's theorem

$$\overline{N} = \lambda_{eff} \overline{T}$$

M/M/1

$$\rho = \lambda / \mu$$

$$P_k = (1 - \rho) \rho^k$$

$$\overline{N} = \rho / (1 - \rho)$$

$$\overline{W} = \rho / (\mu - \lambda)$$

$$F_W(t) = 1 - \rho e^{-(\mu - \lambda) t}$$

$$F_T(t) = 1 - e^{-(\mu - \lambda) t}$$

M/M/m

$$\rho = \lambda / \mu$$

$$P_k = \begin{cases} P_0 \frac{\rho^k}{k!} & k \leq m \\ P_0 \frac{\rho^k}{m! m^{k-m}} & k \geq m \end{cases}$$

Erlang-C

$$D_m(\rho) = P(wait) = \frac{m E_m(\rho)}{m - \rho(1 - E_m(\rho))}$$

$$\overline{N} = \overline{N}_s + \overline{N}_q = \rho + \frac{\lambda}{m\mu - \lambda} P(wait)$$

$$\overline{W} = \frac{1}{m\mu - \lambda} P(wait)$$

$$F_W(t) = 1 - D_m(\rho)e^{-\mu(m-\rho)t}$$

M/M/1/S

$$\rho = \lambda/\mu$$

$$P_k = \frac{(1-\rho)\rho^k}{1-\rho^{S+1}}$$

$$P_S = P(blocking) = \frac{(1-\rho)\rho^S}{1-\rho^{S+1}}$$

$$\overline{N} = \frac{\rho}{1-\rho}(1-(S+1)P_S)$$

M/M/m/m

$$\rho = \lambda/\mu$$

$$P_k = \frac{\rho^k/k!}{\sum_{k=0}^{m}\frac{\rho^k}{k!}}$$

Erlang-B

$$E_m(\rho) = P(blocking) = \frac{\rho^m/m!}{\sum_{k=0}^{m}\frac{\rho^k}{k!}}$$

$$\overline{N} = \rho(1-P(blocking))$$

M/M/m/ m/ n

$$\rho = \lambda/\mu$$

$$P_k : Engset\ \ distribution$$

$$P_k(n) = \frac{\binom{n}{k}\rho^k}{\sum_{i=0}^{m}\binom{n}{i}\rho^i}$$

$$\text{Call blocking} : Engset\ \ formula$$

$$B_n(m,\rho) = P_m^*(n) = P_m(n-1) = \frac{\binom{n-1}{m}\rho^m}{\sum_{i=0}^{m}\binom{n-1}{i}\rho^i}$$

M/E_r/1

$$S^*(s) = \prod_{i=1}^{r}\frac{\mu_i}{s+\mu_i}$$

M/H_r/1

$$S^*(s) = \sum_{i=1}^{r}\alpha_i\frac{\mu_i}{s+\mu_i}$$

M/G/1

$$\overline{N} = \rho + \frac{1+C_s^2}{2}\frac{\rho^2}{1-\rho}, \qquad C_s^2 = \frac{\sigma_s^2}{\overline{S}^2}$$

$$\overline{W} = \frac{\lambda\overline{S^2}}{2(1-\rho)} = \frac{1+C_s^2}{2}\frac{\rho}{1-\rho}\overline{S}$$

$$G_N(z) = \frac{(1-\rho)(1-z)}{1-\frac{z}{S^*(\lambda(1-z))}}$$

$$W^*(s) = \frac{s(1-\rho)}{s-\lambda+\lambda S^*(s)}$$

M/G/1 with vacations

$$\overline{W} = \frac{\lambda\overline{S^2}}{2(1-\rho)} + \frac{\overline{V^2}}{2\overline{V}}$$

M/G/1 with non-preemptive priority

$$\overline{R} = \frac{1}{2}\sum_{k=1}^{n}\lambda_k\ \overline{S_k^2}$$

$$\overline{W}_i = \frac{\overline{R}}{(1-\rho_1-\ldots-\rho_{i-1})(1-\rho_1-\ldots-\rho_i)}$$

M/G/1 with preemptive priority

$$\overline{R}_i = \frac{1}{2}\sum_{k=1}^{i}\lambda_k\ \overline{S_k^2}$$

$$\overline{T}_i = \frac{(1-\rho_1-\ldots-\rho_i)\overline{S}_i + \overline{R}_i}{(1-\rho_1-\ldots-\rho_{i-1})(1-\rho_1-\ldots-\rho_i)}$$