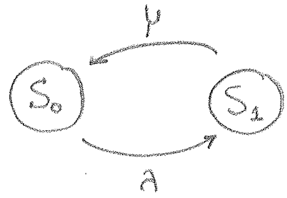
(1)

S_0 : Processor working properly
S_1 : Processor out of function

a)

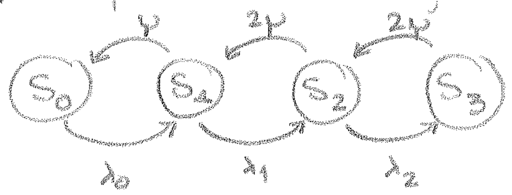


$\lambda = 1/10\text{hour}$
$\mu = 1/\text{hour}$ $\rightarrow$ $\rho = \frac{\lambda}{\mu} = 0{,}1$

B. Eq. : $\lambda P_0 = \mu P_1 \Rightarrow \lambda P_0 = \mu(1-P_0) \Rightarrow \dots$ $\begin{cases} P_0 = 10/11 \\ P_1 = 1/11 \end{cases}$

$Pr\{\text{out of function}\} = P_1$

b) S_i : i processors out of function



$\lambda_0 = 3 \cdot 1/30h = 1/10h$

$\lambda_1 = 2 \cdot 1/20h = 1/10h$

$\lambda_2 = 1/10h$

$\Downarrow$

$\lambda_0 = \lambda_1 = \lambda_2 = \lambda = 1/10h$
(homogeneous system)

B. Eq. :

$$\begin{cases} \lambda P_0 = \mu P_1 \\ \lambda P_1 = 2\mu P_2 \\ \lambda P_2 = 2\mu P_3 \\ P_1 + P_2 + P_3 = 1 \end{cases} \Rightarrow \begin{cases} P_0 = 4000/4421 \\ P_1 = 400/4421 \\ P_2 = 20/4421 \\ P_3 = \frac{1}{4421} \end{cases}$$

$Pr\{\text{unavailable}\} = P_3$

c) Processor waits, if it fails while system is on P_2 :

$$Pr(\text{wait}) = \frac{P_2}{P_0+P_1+P_2} = \frac{P_2}{1-P_3}$$

$\overline{T}_{WAIT} = 1/2\ h$ (2 servers)

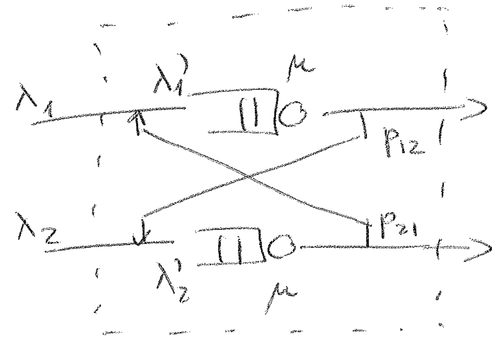
d)

$$\frac{\overline{T}_{UNAVAILABLE}}{\overline{T}_{AVAILABLE} + \overline{T}_{UNAVAILABLE}} = P_3 \Rightarrow \frac{0{,}5}{0{,}5 + \overline{T}_{AVAILABLE}} = P_3 \Rightarrow$$

$\Rightarrow \overline{T}_{AVAILABLE} = 2210\ h$

(2) $\lambda_1 = \lambda_2 = 1$, $\mu = 3$, $p_{12} = p_{21} = 0.5$

a) M/M/1 servers



$$\lambda_1' = \lambda_1 + p_{21}\lambda_2'$$
$$\lambda_2' = \lambda_2 + p_{12}\lambda_1'$$

$$\Rightarrow \lambda_1' = \lambda_2' = 2$$

b) Based on the independence of the queues:

$$P(\text{system empty}) = P_0^1 \cdot P_0^2 = (1-\rho_1)(1-\rho_2) = \left(1 - \frac{\lambda_1'}{\mu}\right)\left(1 - \frac{\lambda_2'}{\mu}\right) = \frac{1}{9}$$

c) Little theorem for the entire queuing nw:

$$\bar{T}_{sys} = \frac{\bar{N}}{\lambda_1 + \lambda_2} = \frac{\bar{N}_1 + \bar{N}_2}{\lambda_1 + \lambda_2} = \frac{4}{2} = 2$$

$$\bar{N}_i = \frac{\rho_i}{1-\rho_i} = 2$$

d) i. $$I = \frac{\lambda_1' + \lambda_2'}{\lambda_1 + \lambda_2} = 2$$

ii. Alt. solution:

Number of visits for one job: Geometric$\left(\frac{1}{2}\right)$

$$\Rightarrow I = \frac{1}{1/2} = 2$$

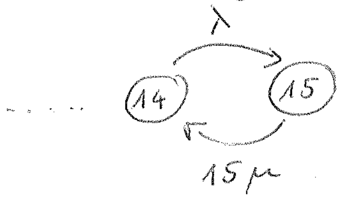
(3)

- M/M/m/m system

- $1/\lambda = 0.5$ min, $1/\mu = 5$ min $\Big\}$ $\rho = \frac{\lambda}{\mu} = 10$

a) $E_m(10) < 0.05 \overset{\text{Erlang tables}}{\Rightarrow} m \geq 15$

b) Mean blocking interval:



$\bar{T}_b = \frac{1}{k\mu} = \frac{1}{3}$ min

$\sim Exp(k\mu) = Exp(3)$

c) $N = \lambda_{eff} \cdot \bar{T} = \lambda_{eff} \cdot \frac{1}{\mu} = \frac{\lambda}{\mu}(1 - P_{block}) = \rho(1 - E_{15}(10)) = 9.63$

d) P(system empty) = P_0

$$P_k = \frac{\rho^k / k!}{\sum_{i=0}^{m} \rho^m / m!} \Rightarrow P_0 = \frac{\rho^0 / 0!}{\rho^m / m!} P_m \approx 4.7 \cdot 10^{-5}$$

e) Various answers accepted:

No: in an inf. pop. system the P(block) is always greater than $\emptyset$, even if it is small.

Yes: in the real telephone system the population is not infinite, so there could be a dedicated line for each pair of users.

④ $C = 10^7$ bits/sec

a) $\lambda_i = 100$ pps , $\lambda_{tot} = 300$ pps

Mean service time : $\bar{S} = \frac{1}{3} \cdot \frac{L_A + L_B + L_C}{C} = \frac{(8+16+24)10^3}{3 \cdot 10^7} = 1{,}6$ ms

Mean square service time :

$$\overline{S^2} = \frac{1}{3} \cdot \frac{2L_A^2 + 2L_B^2 + 2L_C^2}{C^2} \cong \ldots \cong 6 \cdot 10^{-6}$$

$$\rho = \lambda_{tot} \cdot \bar{S} = 300 \cdot 1{,}6 \cdot 10^{-3} = 0{,}48$$

$$M/G/1 \longrightarrow \bar{W} = \frac{\lambda_{tot} \cdot \overline{S^2}}{2(1-\rho)} \cong 1{,}723 \text{ ms}$$

$$\bar{T}_{SYSTEM} = \bar{S} + \bar{W} \cong 3{,}323 \text{ msec}.$$

b) LT : $W^*(s) = \frac{1}{3}\left(\frac{\mu_A}{s+\mu_A} + \frac{\mu_B}{s+\mu_B} + \frac{\mu_C}{s+\mu_C} \right)$; $\begin{cases} \mu_A = C/L_A \\ \mu_B = C/L_B \\ \mu_C = C/L_C \end{cases}$

c) $A > B > C$ (preemptive resume)

$$\bar{R}_A = \frac{1}{2} \lambda_A \overline{S_A^2} = \frac{1}{2} \cdot 100 \cdot 1{,}28 \cdot 10^{-6} = 0{,}064 \cdot 10^{-3}$$

$$\bar{R}_B = \bar{R}_A + \frac{1}{2} \lambda_B \overline{S_B^2} = \ldots = 0{,}32 \cdot 10^{-3}$$

$$\bar{R}_C = \bar{R}_A + \bar{R}_B + \frac{1}{2} \lambda_C \cdot \overline{S_C^2} = \ldots = 0{,}896 \cdot 10^{-3}$$

$$\rho_A = \lambda_A \bar{S}_A = 0{,}08$$

$$\rho_B = \lambda_B \bar{S}_B = 0{,}16$$

$$\rho_C = \lambda_C \cdot \bar{S}_C = 0{,}24$$

$$\bar{T}_A = \frac{(1-\rho_A)\bar{S}_A + \bar{R}_A}{1-\rho_A} = 0{,}869 \text{ msec}$$

$$\bar{T}_B = \frac{(1-\rho_A-\rho_B)\bar{S}_B + \bar{R}_B}{(1-\rho_A)(1-\rho_A-\rho_B)} = 2{,}197 \text{ msec}$$

$$\overline{T_C} = \frac{(1-\rho_A-\rho_B-\rho_C)\,\overline{S_C} + \overline{R_e}}{(1-\rho_A-\rho_B)(1-\rho_A-\rho_B-\rho_C)} = 5{,}425 \text{ msec}$$

d) Little:

$$\overline{N_A} = \lambda_A \overline{T_A} = 0{,}0869$$

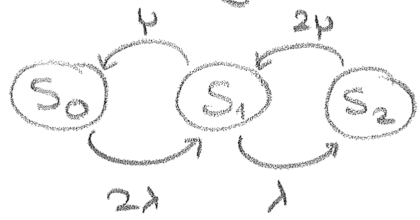
$$\overline{N_B} = \lambda_B \overline{T_B} = 0{,}2197$$

$$\overline{N_C} = \lambda_C \overline{T_C} = 0{,}5425$$

e) Same as in c) : $\overline{W_A} = \overline{T_A} - \overline{S_A} = 0{,}869 - 0{,}8 = 0{,}069$ msec

(5)

a) State diagram: S_i : i customers in the system



$\lambda = 10$
$\mu = 20$

B. Eq.:
$$\begin{cases} 2\lambda P_0 = \mu P_1 \\ \lambda P_1 = 2\mu P_2 \\ P_0 + P_1 + P_2 = 1 \end{cases} \longrightarrow \begin{cases} 20 P_0 = 20 P_1 \\ 10 P_1 = 40 P_2 \\ P_0 + P_1 + P_2 = 1 \end{cases} \longrightarrow \begin{cases} P_0 = 4/9 \\ P_1 = 4/9 \\ P_2 = 1/9 \end{cases}$$

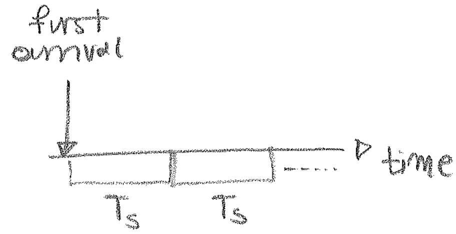
$Pr\{busy\} = P_2$

$Pr\{call\ blocking\} = 0$ (2 servers, 2 customers)

b) M/D/2/2

$\lambda = 10\ sec^{-1}$

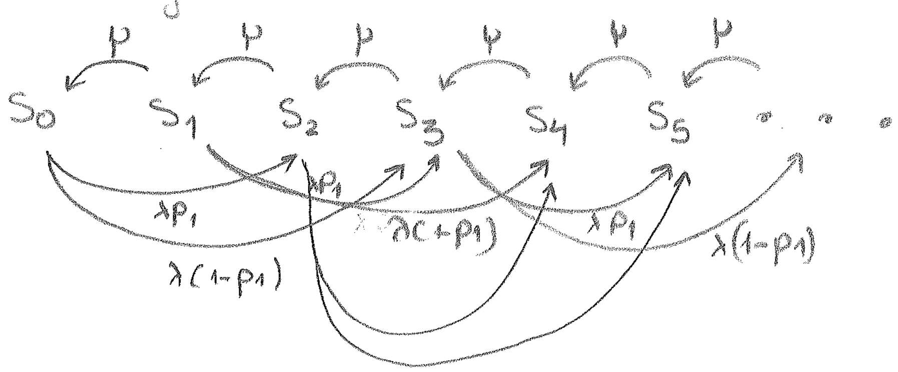
$T_s = 10^{-2}\ sec$



$Pr(\text{3rd packet blocked}) = Pr\{\text{more than 1 arrivals in } T\}$

$= 1 - Pr\{0 \text{ or } 1 \text{ arrivals}\} = 1 - e^{-\lambda T_s} - \lambda T_s \cdot e^{-\lambda T_s} = 4{,}678 \cdot 10^{-3}$

c) S_i : i stages left in total



d) $\overline{T_s} = 10 sec$, $\mu = 1/10$, $\rho < 1 \Longleftrightarrow \lambda < \mu = 1/10\ sec^{-1}$

System with vacations: Vacation intervals do not affect the stability $\Rightarrow \lambda < 1/10$ as well.