EP2200 Queuing theory and teletraffic systems

Final exam, Thursday December 17, 2009, 14:00-19:00

Available teachers: Ioannis Glaropoulos, 08 790 4251

Allowed help: Calculator, Beta mathematical handbook or similar, provided sheets of queuing theory formulas and Erlang tables

1. Consider a processor that takes care of a service. Assume however that the processor suffers from random failures that can put it out of function. The reparation time of the processor is assumed to be exponentially distributed with a mean of 1 hour. The time period between the point that the processor is repaired until its next failure (*time to failure*) is also exponentially distributed with mean of 10 hours.

a) Draw the state diagram and calculate the percentage of time in steady state that the processor is out of function. (2p)

To increase the reliability of the system we install 3 processors that work in parallel. When only one processor is operating, then its time to failure is exponentially distributed with a mean of 10 hours, as above. When 2 processors are operating, then they share the workload and for both of them the mean time to failure becomes 20 hours. Similarly, when all the 3 processors are operating, their mean time to failure becomes 30 hours. Assume finally that there exist 2 service units, so two processors can simultaneously be under reparation. Consider the system in steady state.

b) Draw the state diagram and calculate the percentage of time the system is unavailable (all the processors are out of order). (3p)

c) Calculate the probability that an arbitrary processor that fails has to wait for its reparation to begin and the mean value of the waiting time. (3p)

d) Compute the mean value of the period between two system failures. (2p)

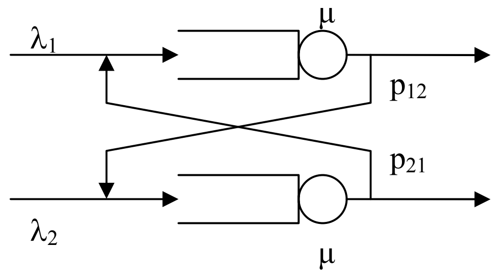
2. Consider the open queuing network on the figure below. The queuing systems in the queuing network have single server, exponential service time with parameter $\mu=3$, and infinite buffer capacity. There are two streams of customers arriving to the queuing network, according to Poisson process. The arrival intensities are $\lambda_1=\lambda_2=1$ customer per second. The probability that after service a customer moves to the other queue is $p_{12}=p_{21}=0.5$. Consider the queuing network in steady state.

a) Calculate the arrival intensities to the two queues of the queuing network. (2p)

b) Calculate the probability that the queuing network is empty at an arbitrary point of time. (3p)

c) Calculate the average time a customer spends in the queuing network. (3p)

d) Calculate the average number of services a customer receives before leaving the network. (2p)



3. You have to dimension a telephone link between Stockholm and Göteborg. Calls arrive according to a Poisson process, the mean call interarrival time is 0.5 min. The length of the calls is exponentially distributed, with a mean of 5 minutes.

a) Calculate the offered load and the required number of lines to keep the call blocking below 5%. (3p)

b) Give the distribution and the mean value of the time the system remains in blocking state. (2p)

c) Calculate the average number of ongoing calls. (2p)

d) Calculate the probability that all the lines are idle at an arbitrary point of time. (2p)

e) Is it possible to ensure zero blocking probability by increasing the number of lines? (1p)

4. Consider a communication link with fixed capacity C=10Mbits/sec. Assume that the link has infinite buffer and serves 3 independent packet streams. In all streams, A, B and C, packet length follows an exponential distribution with a mean of L_A, L_B and L_C, respectively. Packets from every stream arrive at a Poisson fashion with rates $\lambda_A=\lambda_B=\lambda_C=\lambda$. Assume a typical FCFS service scheme. Assume L_A=1kByte, L_B=2kByte, L_C =3kByte and λ=100 packets per second. Consider the system in steady state.

a) Calculate the mean time an arbitrary packet spends in the system (waiting and service time). (2p)

b) Give the Laplace transform of the service time distribution. (2p)

For the rest of the problem assume that a preemptive resume priority scheme is implemented, with stream A being the highest priority and stream C being the lowest priority stream.

c) Calculate the mean system time for packets of streams A, B and C. (2p)

d) Calculate the mean number of packets of streams A, B and C in the system. (2p)

e) Assume that service scheme is changed to LCFS (Last Come First Served). Calculate the mean waiting time for packets of stream A. (2p)

5. Answer the following questions!

a) Assume an M/M/2/2/2 system with λ=10 per customer and μ=20. Calculate the probability that all servers are busy and the probability of call blocking. (3p)

b) Consider an M/D/2/2 system with λ= 10 arrivals per second and service time $T_s=10^{-2}$ seconds. The first packet arrives and finds the system empty. Calculate the probability that the third packet arrival is blocked. (2p)

c) Assume a queuing system with one server, unlimited buffer and Poisson arrivals with parameter λ. With probability p_1 the service time of a customer follows an Erlang-2 distribution with a mean of $2/\mu$, otherwise it follows an Erlang-3 distribution with a mean of $3/\mu$. Draw the state transition diagram! (3p)

d) Consider a job processing system with a single processor. The service time is exponentially distributed; the mean service time is 10s. What arrival rates keep the system stable? Now assume that the processor starts a maintenance process whenever it gets idle. The maintenance takes exactly 5s. What are the acceptable arrival rates in this case? Motivate your answer. (2p)