2G1318 Queuing theory and teletraffic systems

Final exam, Thursday December 16, 2004, 14.00-19.00, in rooms E31-35

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Allowed help: Calculator, Beta mathematical handbook, and provided sheets of queuing theory formulas and Erlang tables.

1) The telecom market has crashed again and you decide to take a job at a civil engineering firm that designs buildings. You are assigned to the dimensioning of a commercial parking garage since you have taken queuing theory at KTH. Daytime parking is known to have an average duration of 40 minutes with a distribution that has a coefficient of variation of one; night time parking has an average duration of 6 hours and the distribution has a coefficient of variation that is ½.
   a) You posit that the arrival process could be modelled by a Poisson distribution. Give your rationale. What distributions should you select for the durations of parking in the day and the night times? Draw state diagrams for 0 up to 2 cars in the garage for the two cases (define the state variables, and label all transitions). (2p)
   b) Your first estimate is for a very large garage where every car could fit. What would the average occupancy would be if you estimate that there is one car arriving every 40 seconds for daytime parking and one car every 5 minutes at the night time. There is a single entrance to the garage. Define the queuing models that you use. (2p)
   c) Then you take as an actual dimensioning case to have the garage full 2 percent of the time during the day and 10 percent of the time during nights. Cars arriving to a full garage go elsewhere. Is it night or day time parking that you need to consider in order determining the needed number of parking spaces? (2p)
   d) You now introduce space outside the garage for cars waiting in the driveway for a vacated parking space when the garage is full. You assume that all arriving cars join the waiting line. The probability that an arriving car has to wait should not exceed ten percent for daytime parking. How many spaces are needed in the garage during daytime and what is the average number of waiting cars? What is the probability of waiting more than five minutes to enter? (3p)
   e) What are the departure processes of cars leaving the garage during the day and the night, respectively, given in Kendall's notation? Justify. (1p)

2) The relative frequencies of arithmetic operations needed to perform computing tasks sent to a processor fit the probability-density function:

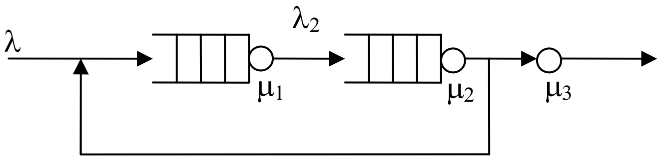
$$b(x) = 0.4\mu_1 e^{-\mu_1 x} + 0.6\mu_2 e^{-\mu_2 x}$$

   where x is the size of a task measured in number of instructions. Tasks arrive according to a Poisson process with 1000 per second; they are queued when the processor is occupied. The processor has a capacity of 1 MIPS.
   a) Calculate the average waiting time given that on average 1250 tasks can be processed and the coefficient of variation of the task size distribution is 1.1875. (3p)
   b) Tasks of type 1 (mean service rate μ_1) belong to the kernel and should be given priority over tasks of type 2 (mean service rate μ_2), so the system is modified to include non-preemptive priority scheduling. Calculate the average waiting times of tasks of type 1 and 2, and the average waiting time for an arbitrary task. (4p)
   c) Assume now that the processor performs garbage collection whenever there are no tasks to be performed. Tasks that arrive during the garbage collection have to wait until it is finished. A garbage collection cycle consists of two steps. First, the memory is scanned, this step takes 0.1 ms, and then the memory has to be defragmented. The second step takes an exponentially distributed amount of time with mean 0.9 ms. Derive a formula for the average waiting time of the two types of tasks, and calculate their corresponding expected waiting times. (Hint: Consider how the residual service/vacation time changes due to the introduction of the service vacations.) (3p)

3) Jobs arrive to a queuing system from three customers, each one of the customers generates jobs on average every 3 minutes according to a Poisson process when it does not have a job in the system already. The system has two servers and jobs that find both servers busy are dropped. The service of the jobs involves two stages, the time needed for each of them is on average 1 minute and is exponentially distributed.

   a) Give the Kendall notation of the system and draw the state diagram. (2p)
   b) Calculate the steady state probabilities. (3p)
   c) Calculate the probability that the system is in a blocking state and the probability that a job finds the system in a blocking state. (3p)
   d) Calculate the probability that the time spent in the system for a job that sees one job in the system upon arrival is more than 2 minutes. (2p)

4) Consider the following open queuing network.



Systems 1 and 2 are M/M/1 systems, while system 3 is an M/M/m/m system. Jobs served in system 2 are sent back to system 1 if they find system 3 busy. To analyse the system let us assume the usual independence assumptions, and $\lambda=1$, $\mu_1=2$, $\mu_2=4$, $\mu_3=3$ and m=1.

   a) Find the offered traffic to each of the three nodes in equilibrium. (2p)
   b) Calculate the probability that there are in total two customers in the system. (2p)
   c) Calculate the average time spent in the system for an arbitrary job. (3p)
   d) Find the highest λ for which the system is stable. (2p)
   e) You would like to decrease μ_1 to 1.25. What is the minimum number of servers, m, that you have to install in system 3 to keep the queuing systems in the network stable? (1p)

5) Identify the queuing models in the cases below and write the corresponding Kendall notations for the models. Also, answer the questions for each model. Motive your answers and be as specific as possible on the selection of the models.

   a) Files in a server have a length distribution of the form

   $$F(x) = 1 - e^{-(x^{\gamma})},\ x \geq 0,\ \gamma > 1$$

   (x is the file length and γ is a shape parameter of the distribution). Files are retrieved randomly by a very large population of users at a rate of 230 requests per minute; requests that cannot be served immediately are queued in a very large buffer. There are two processors that can retrieve and dispatch files in parallel to the users. The average time in the system from when a request arrives until the last bit of the file is sent is 13 seconds. What is the average number of requests in the system (including those being served)? (2p)
   b) A network consists of two nodes and has three customers circulating perpetually. The customers receive service with exponentially distributed service times with a mean of 1 second in node 1 and a mean of 2 seconds in node 2. Customers departing from node 1 return to it with probability 0.1 and enter node 2 otherwise; customers go from node 2 to node 1 when service is completed. What is the probability that all three customers are in node 1? (3p)
   c) A coffee shop offers wireless LAN access to the Internet. Customers surf on average twenty minutes (exponentially distributed). A new customer arrive every five minutes on average, with arrivals being a Poission process: There is always room for all and they get network access. What is the probability that a customer may download an MP3 file of 10.2 MB in less than 15 seconds, if the network has 11 Mb/s in total capacity and there are no new arrivals during that time? (3p)
   d) A conveyor brings one car chassis to a robot for painting every seven seconds. The robot needs exactly six second for painting one chassis; it remains idle when there are no chassis waiting. What is the state that a random observer finds the system in, and what is the state that an arriving chassis find the system in? (2p)