EP2200 Queuing Theory and Teletraffic Systems

Final exam, Saturday, December 18, 2010, 9.00-14.00

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Allowed help: Calculator, Beta mathematical handbook or similar, provided sheets of queuing theory formulas and Laplace transforms. Erlang tables are not attached since they are not needed to solve the exam problems.

1. Consider a loss system with 5 serving machines. Jobs arrive at the system in a Poisson fashion with an average rate, λ = 100 jobs/hour. The distribution of a job service duration is exponential with an average rate that depends on the total number of machines that are occupied, according to the following formula: $\mu_i = i^{1/2}\mu$, with $1/\mu$ = 3minutes. (That is, servers become faster when the number of jobs increases.)

a) Draw the state diagram and determine the probability that all serving machines are busy. What is the probability that an arriving job cannot be served by the system? (3p)

b) Give the distribution and the mean value of the blocking period. (1p)

c) Calculate the expected value of the length of the non-blocking period. (2p)

d) Determine the (average) utilization of each serving machine. (2p)

e) Compare the blocking probability with that of a pure M/M/5/5 system with job service rate equal to the mean job service rate of the considered system. (2p)

2. Consider a single server system with infinite buffer. Jobs arrive to the server according to a Poisson process with intensity λ per time unit. Arriving jobs belong to class 1 with probability *p*, and to class 2 with probability *1-p*. Assume *p*=0.8. The Laplace transform of the service time distribution for class 1 jobs is $S^*(s)=4/(s+2)^2$, and for class 2 jobs is $S^*(s)= 2/(s+2)$. The jobs need to wait in a FCFS queue if the server is busy.

a) Calculate the mean and the variance of the service time of an arbitrary job. (3p)

b) Determine the maximum allowable arrival intensity of jobs if the system has to remain stable. (2p)

Assume now the utilization of the server is 90%.

c) Calculate the average waiting time of an arbitrary job. (2p)

d) Implement a non-preemptive priority scheme between the two classes to reduce the average waiting time. What is the average waiting time of class 1 and class 2 jobs in this case, and what is the improved average waiting time of an arbitrary job? (3p)

3. Consider a post office with one customer arriving in every 5 minutes in average. One third of the customers post a letter or a parcel, which takes 2 minutes in average with exponential distribution, the others pick up a parcel, which takes 4 minutes in average, with an exponential distribution. There is one assistant serving the customers. Customers arriving when the assistant is busy, wait for their turn.

a) Give the Kendall notation of the system, the probability that the post office is empty, and the state transition diagram. (2p)

b) Calculate the average waiting time of the customers. (2p)

c) Calculate the average waiting time of customers arriving when the assistant is busy, but noone is waiting; and the probability that such a customer has to wait for more than 4 minutes. (3p)

d) Assume now, that the post office has two assistants serving the customers. Model the system with a Markov chain: define the states of the system and draw the state transition diagram. (3p)

4. Consider a dormitory kitchen where the kitchen is shared by 4 students in total but has only one hob for cooking and 1 place on the sofa (waiting position). Students that arrive at the kitchen

when the hob is occupied need to wait by sitting at the sofa, and when the sofa is occupied, the students return to their rooms (blocked). After finishing cooking, or when being blocked, the students study in their rooms for 4 hours on average, with exponentially distributed time. In addition, the students spend 40 minutes on average for cooking, also modeled as exponentially distributed.

a) Give the Kendall notation of the described queuing system and draw the state diagram. (1p)

b) Determine the probability that an arbitrary student arrives and finds the kitchen completely full (hob and sofa positions are occupied). (2p)

c) Calculate how many times, on average, a student cooks per day (24 hours). (2p)

Assume now that the students can remotely monitor the availability status of the cooking hob while sitting in their rooms. When the students decide to cook they first consult the monitoring system; if the hob is available they always go to the kitchen. If the hob is occupied, students are afraid that the sofa is also occupied, so they go to the kitchen with probability p=1/3, and start to study again otherwise. In addition, the students reduce their average cooking time to 20 minutes when they notice that there is a student waiting on the sofa.

c) Calculate the new probability that a student who goes to the kitchen gets blocked. (3p)

d) Calculate the probability that a student erroneously decides not to go to the kitchen after consulting the monitoring system. (2p)

5. Answer the following questions!

a) Consider k independent random variables with distribution Exp(λ). Prove that the minimum of these random variables is also exponentially distributed and give the parameter of the distribution. Give an example when this result is used in the analysis of Markovian queuing systems. (2p)

b) Consider an M/M/1 system with vacations. The vacation periods are exactly two time units, the arrival intensity is one arrival per time unit and the system utilization is 0.75. Calculate the average system time. (2p)

c) Consider an M/G/1 queuing system with infinite buffer. Service time is 1 minute with probability p = 0.5, otherwise it is 10 minutes. Jobs arrive at a Poisson fashion with 1 job every 10 minutes on average. Calculate the mean value of the busy period. (i.e. the server is busy) (3p)

d) Consider the queuing network below. Both queuing systems are M/M/1 ones, $\mu 1 = \mu 2$ and $\gamma 1 = \gamma 2$. The average number of customers in the queuing network is 2.5. What is the utilization of the queuing systems and what is the probability that a customer entering the network at queue 1 is served by both of the systems without waiting? (3p)